A NON-LINEAR TRANSPORTATION PROBLEM WITH ADDITIONAL CONSTRAINTS IN FUZZY ENVIRONMENT

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Abstract : This paper deals a non-linear transportation problem with an additional impurity constraint in addition with standard availability and demand constraints. Here transportation cost categorized into two parts: one part is for the amount of transportation and another part is due to the distance of transportation. More-over the fixed unit costs are imprecise ones. The problem is optimized by max-min criteria suggested by Bellman and Zadeh [5] through generalized reduced gradient method. Finally, a numerical example is taken into consideration to verify the model.

Index Terms- **Non-linear transportation problem, impurity constraints, fuzzy programming technique**.

1. INTRODUCTION

The classical transportation problem (Hitchcock transportation problem) is one of the sub-classes of linear programming problem in which all the constraints are of equality type or of inequality type. In classical form, the problem minimizes the total cost of transporting the product which is available at some sources and is required to various destinations. The unit costs i.e. the cost of transporting one unit from a particular supply point to a particular demand point, the amounts available at the supply points and the amounts required at the demand points are the parameters of the transportation problem. Such transportation problem often referred to as transhipment problem [1] also.

In today's competitive market, the pressure on organisation to find the better ways of delivering to customers becomes stronger. In this consequence, it is effective" where the organisation is situated", i.e., location of the sources in respect to the location of destination. Such transportation model provides a distance frame work which is also cost effective in reality. For the first time, this conception helps us to modelled a non-linear transportation problem (NLTP).

In conventional transportation problem, it is assumed that decision maker is sure about the precise values transportation costs, availabilities, demands of the commodity. But in real world, all these parameters may not be known precisely due to several uncontrollable factors, so fuzzy decision-making method is needed here, which is first introduced by Bellman Zadeh [5], Zimmermann [20] showed the fuzzy programming technique with some suitable membership functions to solve multi-objective linear programming problems. The results obtained by fuzzy linear programming lead to efficient solutions, too. Bit et al. [3] by using linear membership function, applied the fuzzy programming technique to solve multi objective transportation problem. In 1999, Biswal and Verma [4] used fuzzy programming technique to find the optimal compromise solution of a nonlinear multi objective transportation problem. Jimenez and Verdegay [11] presented fuzzy programming techniques for solving different uncertain solid transportation problem. Later on, various researchers (cf. [2], [13], [15]) discussed additive fuzzy programming techniques for multi-objective uncertain STP.

A procedure for solving a fuzzy solid transportation problem was presented by Fuzzy programming and additive fuzzy programming techniques for multi-objective transportation problems were discussed in [2]. G. Maity and S.K. Roy [14] develops a mathematical model for a transportation problem consisting of a multi-objective environment with nonlinear cost and multi-choice demand. D. Dutta and A.S. Murthy [9] was introduced fuzzy transportation problem with additional restriction. In the recent years, the solid transportation problems in fuzzy environment widely published in various styles (cf. Jana et al [21], Khanra et al [22], Dalman [8]). The multi-objective time transportation problem with additional impurity restriction was studied by Singh and Saxena[16]. Charnes and Cooper [6] developed the models for industrial applications of linear programming problem and managed them with numerical illustrations. The goal programming approach was introduced by Ignizio [10] in the mathematical models.The goal programming approach was widely used by several authors in STP and multi-objective STP. Metev and Gueorguieva (cf. [13] [18]) used nonlinear programming for finding a weakly efficient set of solutions. C. Sudhagar and K. Ganesan [17] has been proposed a new method for dealing with Fuzzy Integer Linear Programming Problems. H. Dalman [8] presented an uncertain Multi-Objective Multi-Item Solid Transportation Problem based on uncertainty theory. Chang [7] provided a novel approach for mixed integer fractional polynomial programming problems. Ramik [15] solving fuzzy linear programming in duality theory.

In this paper, a transportation problem is considered under the joint decisions of the locations of origins and amount of transportation. In this way a non-linear transportation problem is formulated consisting of two terms: first part is due to the unit transportation cost occurred with respect to the amount of transportation and second part is varying with distance from origin to destination. Such a non-linear transportation problem (NLTP) is modelled with an impurity constraint, which is another new concept in the era of transportation with imprecise cost parameters. The imprecise model converted into a deterministic ones using Bellman-Zadeh's max-min composition. Finally, a numerical example has been taken to illustrate the model.

2. MATHEMATICAL MODEL FORMULATION

2.1 Notations: The following notations are used throughout the paper
Index sets: i index for source ($i = 1, 2, \ldots m$) and j index for destination ($j=1, 2,\ldots,n$)

Parameters:
c_{ij}^{o} transportation cost per unit amount transported from $i-th$ source to $j-th$ destination.
c_{ij}^{1} transportation cost per unit distance from $i-th$ source to $j-th$ destination.
a_i total available supply for each source (or origin) i.
b_j total demand required for the $j-th$ destination.
(p_j, q_j) position of the $j-th$ destination.
d_{ij} distance from i -th source to $j-th$ destination.
Decision variables:
w_{ij} units transported from $i-th$ origin to $j-th$ destination.
(x_i, y_i) position of the $i-th$ origin.

Objective functions:
Z total transportation cost from $i-th$ origin to $j-th$ destinations.

2.2 Problem Formulation:
Let us consider a transportation problem with m origins O_i ($i = 1,2,\ldots,m$) and n destinations D_j ($j = 1,2,\ldots,n$), in which the positions (x_i,y_i) of origins to be decided with respect to the positions of destinations (p_j, q_j). The amount w_{ij} transported from $i-th$ origin to $j-th$ destination need to decide by the decision maker.

Objective functions: The aim of this problem is to minimize the total transportation cost which is accompanied on the amount of transportation and distance of transportation. From the above discussions, we develop mathematical formulation of objectives as follows:

$$Min\ \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{o}\, w_{ij} + \sum_{i=}^{m}\sum_{j=}^{n} \tilde{c}_{ij}^{1}\, d_{ij} y_{ij} \tag{1}$$

The cost coefficient associated with distances are not deterministic number but imprecise in nature so the corresponding objective function $\tilde{Z}$ becomes imprecise. Generally, the cost related to distance will be paid if the transportation activity is assigned from *i – th* source to *j – th* destination. In view of this fact, we introduce the following variable:

$$y_{ij} = \begin{cases} 1\ if\ w_{ij} \neq 0 \\ 0\ if\ w_{ij} = 0 \end{cases}$$

And the distance function is defined as:

$$d_{ij} = \sqrt{(x_i - p_j)^2 + (y_i - q_j)^2}$$

Constraints: Traditionally there are two types of constraints in a TP, source constraint and destination constraint. As the quantities exit, from a source cannot exceed the supply capacity of products, we have

$$\sum_{j=1}^{n} w_{ij} \leq a_i\ \forall i \tag{2}$$

The quantity of product received in a destination should not be less than its demand, that is

$$\sum_{i=1}^{m} w_{ij} \geq b_j\ \forall j \tag{3}$$

Consider one unit of the commodity transported from the $i-th$ supply point contains f_i units of impurity. The total impurity at $i-th$ origin is $f_i w_{ij}$. Demand point j cannot receive more than g_j units of impurity. That is, we must require

$$\sum_{j=1}^{n} f_i w_{ij} \leq g_j\ \forall j \tag{4}$$

When total supply $\sum_{i=1}^{m} a_i$ is equals to total demand (total flow) $\sum_{j=1}^{n} b_j$, the resulting formulation is called a balanced transportation problem. It is natural to require the non-negativity of decision variable that is:

$$w_{ij} \geq 0\ \forall i, j \tag{5}$$

Imprecise cost coefficient: In this paper, the fuzzy costs $\tilde{c}_{ij}^{1} = (\alpha_{ij}, \beta_{ij})$ are subnormal of fuzzy numbers having strictly increasing linear membership functions (see Fig.1). Where α_{ij} as the least cost associated with the amount to be shipped from $i-th$ origin to $j-th$ destination and β_{ij} as the least cost associated with the amount to be shipped from $i-th$ origin to $j-th$ destination at the highest quality of product. Without loss of generality, it is assumed that $\beta_{ij} \geq \alpha_{ij} > 0$.

$$\mu_{ij}(c_{ij}) = \begin{cases} q_{ij} & c_{ij} \geq \beta_{ij} \\ q_{ij}\dfrac{(c_{ij}-\alpha_{ij})}{(\beta_{ij}-\alpha_{ij})} & \alpha_{ij} \leq c_{ij} \leq \beta_{ij} \\ 0 & otherwise \end{cases} \quad (6)$$

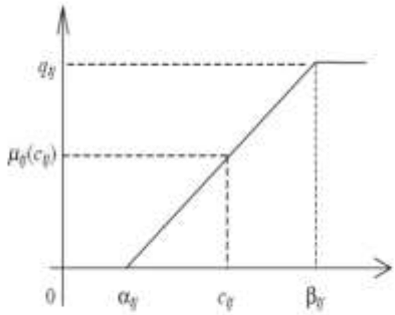

Figure 1: Membership function of $\tilde{c}_{ij}$

3. Solution Procedure:

The problem described in section-2.2 is solved using following fuzzy programming technique.

Step-1: The transportation costs of many real-world applications are not deterministic numbers. Consider a manufacturing company, which provides different product for the different warehouses and transported to different destinations. In that case, the company usually restricted the transported cost c_{ij}^{o} from $i{-}th$ origin to $j-th$ destinations and the transported cost c_{ij}^{1} vary from the distance where the product or goods can be shipped from $i - th$ warehouses to $j - th$ market. Here, we assume a minimum cost for the amount of product shipped from $i - th$ origin to $j - th$ destination. We use the notation $< \alpha_{ij}, \beta_{ij} >$ to denote $\tilde{c}_{ij}^{1}$. Matrix $\tilde{c}_{ij}^{1}$ shown as follows $[\tilde{c}_{ij}^{1}] = [< \alpha_{ij}, \beta_{ij} >]_{m\times n}$

The matrix $[q_{ij}]$ is defined by $[q_{ij}] = [q_{ij}]_{m\times n}$ where q_{ij} represents the highest quality of product associated with the amount transported from $i - th$ warehouses to $j - th$ market and $0 < q_{ij} \leq 1$.

Step-2: Let $\tilde{c}_T$ denote the total cost and the number a and b are defined as the lower and upper bounds of the total cost, respectively. We define the membership function of $\tilde{c}_T$ as the linear monotonically decreasing function in Eq. (7). Numbers 'a' and 'b' are constants and subjectively chosen by the manager. We may take 'a' as the minimum cost of the transportation problem with $\alpha_{ij}'s$ as costs and 'b' is the maximum costs of the transportation problem with $\beta_{ij}'s$ as costs, the demand and supply values in both being same as those of problem. The membership function of the total cost is

$$\mu_T(\tilde{c}_T) = \begin{cases} 1 & if\ c_T \leq a \\ \dfrac{(b-z_1)}{(b-a)} = \dfrac{(b-c_T)}{(b-a)} & b \leq c_T \leq b \\ 0\,, & c_T \geq b \end{cases} \quad (7)$$

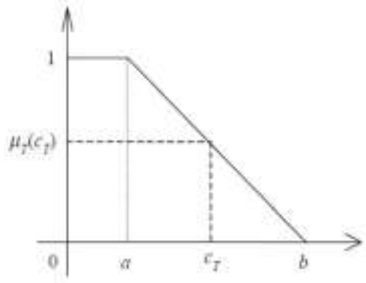

Figure 2: Membership function of $\tilde{c}_T$

Step-3: As per Bellman-Zadeh's criterion [5], which maximize the minimum of the membership functions corresponding to that solution i.e.

$$Max\left\{Min\left(\mu_{ij}, \mu_T(\tilde{c}_T)\right)\right\} \quad (8)$$

It is needed to determine w_{ij}, which is an element of a feasible solution W of the given objective function Eq. (1).

Then we can represent the problem as follows:

$$Max\ \left\{Min\left(\mu_{ij}, \mu_T(c_T)\right); w_{ij} > 0\right\}$$

Subject to,

$$\sum_{j=1}^{n} w_{ij} \leq a_i\,, \quad i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} w_{ij} \geq b_j\,, \quad j = 1,2,\dots,n$$
$$\sum_{j=1}^{n} f_i w_{ij} \leq g_j\,, \quad j = 1,2,\dots,n$$
$$w_{ij} \geq 0\,,\quad i = 1,2,\dots,m,\quad j = 1,2,\dots,n \tag{9}$$

Step-4: We further restrict the transportation cost to be less than or equal to β_{ij} since any expense exceeding β_{ij} is useless. By membership function of Eq. (6) and Eq. (7) we can further represent Eq. (9) as the following equivalent model.

$$\text{Max } \lambda$$

Subject to,

$$\lambda \leq q_{ij}\frac{(c_{ij}-\alpha_{ij})}{(\beta_{ij}-\alpha_{ij})} \quad i = 1,2,\dots,m\,,\quad j = 1,2,\dots,n$$
$$\lambda \leq \frac{b-z_1}{b-a}$$
$$\sum_{j=1}^{n} w_{ij} \leq a_i \quad i = 1,2,\dots,m$$
$$\sum_{i=1}^{m} w_{ij} \geq b_j \quad j = 1,2,\dots,n$$
$$\sum_{j=1}^{n} f_i w_{ij} \leq g_j \quad j = 1,2,\dots,n$$
$$w_{ij} \geq 0\,,\quad i = 1,2,\dots,m,\quad j = 1,2,\dots,n$$
$$c_{ij}^{\lambda} \leq \beta_{ij},\ \ \forall i = 1,2,\dots,m,\ j = 1,2,\dots,n \qquad 0 \leq \lambda \leq 1 \tag{10}$$

Where c_{ij}^{λ} denote the λ-cut of $\tilde{c}_{ij}^{1}$. In Eq. (10), since w_{ij}, c_{ij}^{λ} and λ are all decision variables, it can be treated as a mixed integer nonlinear programming model.

We first define the set $E = (i, j)$ as the set of all pairs (i, j) where w_{ij} is an element of the feasible solution W of Eq. (1) and confine our discussion based on E. Then, we can simplify Eq. (10) as follows

$$Max\ \lambda$$

Subject to

$$\lambda \leq q_{ij}\frac{(c_{ij}^{\lambda}-\alpha_{ij})}{(\beta_{ij}-\alpha_{ij})} \quad for\ (i,j) \in E \tag{11}$$
$$\lambda \leq \left\{\frac{b-\sum_{(i,j)} c_{ij}^{o}\, w_{ij} - \sum_{(i,j)} c_{ij}^{\lambda} d_{ij} y_{ij}}{b-a}\right\} \tag{12}$$
$$c_{ij}^{\lambda} \leq \beta_{ij},\quad for\ (i,j) \in E$$

We let $h_{ij} = \beta_{ij} - c_{ij}^{\lambda} \geq 0$. Then Eq. (11) and Eq. (12) can be expressed as follows

$$Max\ \lambda \tag{13}$$

Subject to

$$\lambda \leq q_{ij}\frac{(\beta_{\text{ij}}-\alpha_{\text{ij}}-h_{\text{ij}})}{(\beta_{ij}-\alpha_{ij})}\ for\ (i,j) \in E \tag{14}$$
$$\lambda \leq \left\{\frac{b-\sum_{(i,j)} c_{ij}^{o}\, w_{ij} - \sum_{(i,j)}(\beta_{\text{ij}}-h_{\text{ij}}) d_{ij} y_{ij}}{b-a}\right\} \tag{15}$$
$$h_{\text{ij}}, \lambda, y_{ij} \geq 0\quad for\ (i,j) \in E \tag{16}$$

Theorem 1. Let λ_w be the optimal value of Eq.(13) to Eq. (16) suppose $b < \frac{\sum_{(i,j)} c_{ij}^{o} w_{ij} + \sum_{(i,j)}(\beta_{\text{ij}}-h_{\text{ij}}) d_{ij} y_{ij}}{1-min_{(i,j)} q_{ij}}$.

Then $\lambda_w = q_{ij}\frac{(\beta_{\text{ij}}-\alpha_{\text{ij}}-h_{\text{ij}})}{(\beta_{ij}-\alpha_{ij})}\ for\ (i,j) \in E = \frac{b-\sum_{(i,j)} c_{ij}^{o} w_{ij} - \sum_{(i,j)}(\beta_{\text{ij}}-h_{\text{ij}}) d_{ij} y_{ij}}{b-a}$

Proof: The problem Eq. (13) to Eq. (16) can be written into a linear programming model as

$$Max\ \lambda \tag{17}$$

Subject to

$$h_{\text{ij}} + \lambda\frac{\beta_{\text{ij}}-\alpha_{\text{ij}}}{q_{ij}} \leq (\beta_{\text{ij}}-\alpha_{\text{ij}})\ \ for\ (i,j) \in E \tag{18}$$
$$-\sum_{(i,j)} h_{\text{ij}} d_{ij} y_{ij} + (b-a)\lambda \leq b - \sum_{(i,j)} c_{ij}^{o}\, w_{ij} - \sum_{(i,j)} \beta_{\text{ij}} d_{ij} y_{ij} \tag{19}$$

$$\lambda, h_{\text{ij}} \geq 0, \quad for\ (i,j) \in E$$

We obtain the dual problem of the above problem as

$$Min \sum\nolimits_{(i,j)} (\beta_{\text{ij}} - \alpha_{\text{ij}}) v_i + \left\{ b - \sum\nolimits_{(i,j)} c_{ij}^o\, w_{ij} - \sum\nolimits_{(i,j)} \beta_{ij} h_{ij} y_{ij} \right\} v_{n+1} \tag{20}$$

subject to

$$v_i - v_{n+1} \geq 0\,, \quad for\ (i,j) \in E \tag{21}$$

$$\sum\nolimits_{(i,j)} \frac{\beta_{\text{ij}} - \alpha_{\text{ij}}}{q_{ij}} v_i + (b-a) v_{n+1} \geq 1 \tag{22}$$

$$v_i, v_{n+1} \geq 0, \quad for\ i = 1,2,\dots,n$$

Let $s_1, s_2, \dots, s_{n+1}$ be the slack variables of Eq. (18) and Eq. (19) respectively. Similarly, let $u_1, u_2, \dots, u_{n+1}$be the surplus variable of Eq. (21) and Eq. (22) respectively.

Since

$$b < \frac{\sum_{(i,j)} c_{ij}^o\, w_{ij} + \sum_{(i,j)} (\beta_{\text{ij}} - h_{\text{ij}}) d_{ij} y_{ij}}{1 - min_{(i,j)} q_{ij}}$$

we have

$$min_{(i,j)} q_{ij} > \frac{b - \sum_{(i,j)} c_{ij}^o\, w_{ij} + \sum_{(i,j)} (\beta_{\text{ij}} - h_{\text{ij}}) d_{ij} y_{ij}}{b - a}$$

By Eq. (15) we have $\lambda < min_{(i,j)} q_{ij}$ and $\forall h_{\text{ij}} > 0$. Based on the complementary slackness theorem, we obtain the surplus variables $u_1 = u_2 = \cdots = u_n = 0$.

Hence $v_i - v_{n+1} = 0$ for $i = 1,2,\dots,n$. and $v_1 = v_2 = \cdots = v_n = v_{n+1}$. If $v_1 = v_2 = \cdots = v_n = v_{n+1} = 0$, there is a contradiction to Eq. (22). Therefore, we have $v_1 = v_2 = \cdots = v_n = v_{n+1} > 0$,and again by the complementary slackness theorem, we find the slack variable $s_1 = s_2 = \cdots = s_{n+1} = 0$. Thus, the theorem is proved.

In most of the real-world problems, the upper bound condition of the total cost $\tilde{c}_T$

$$b < \frac{\sum_{(i,j)\epsilon E} c_{ij}^o\, w_{ij} + \sum_{(i,j)} (\beta_{\text{ij}} - h_{\text{ij}}) d_{ij} y_{ij}}{1 - min_{\forall (i,j)} q_{ij}}$$

can be just satisfied. Therefore, we concentrate our discussion in this situation.

Theorem 2. Let λ_{w} be the optimal value of Eq. (11) to Eq. (15) and b $< \frac{\sum_{(i,j)} c_{ij}^o w_{ij} + \sum_{(i,j)} (\beta_{\text{ij}} - h_{\text{ij}}) d_{ij} y_{ij}}{1 - min_{\forall (i,j)} q_{ij}}$. Also let $\gamma_{ij} = \frac{\beta_{\text{ij}} - \alpha_{\text{ij}}}{q_{ij}}$ for i=1,2,…,m, j=1,2,…,n. Then $\lambda_{\text{w}} = \frac{b - \sum_{(i,j)} c_{ij}^o w_{ij} + \sum_{(i,j)} \alpha_{\text{ij}} d_{ij} y_{ij}}{b - a + \sum_{(i,j)} \gamma_{ij} d_{ij} y_{ij}}$

Proof: By theorem 1, assuming the solution to be non-degenerate, we have

$$\lambda_w = \frac{(\beta_{\text{ij}} - \alpha_{\text{ij}} - h_{\text{ij}}) d_{ij} y_{ij}}{\gamma_{ij} d_{ij} y_{ij}}$$

$$= \frac{b - \sum_{(i,j)} c_{ij}^o\, w_{ij} - \sum_{(i,j)} (\beta_{\text{ij}} - h_{\text{ij}}) d_{ij} y_{ij}}{b - a}$$

Hence, by componendo and dividendo, we get

$$\lambda_w = \frac{b - \sum_{(i,j)} c_{ij}^o\, w_{ij} - \sum_{(i,j)} (\beta_{\text{ij}} - h_{\text{ij}}) d_{ij} y_{ij} + \sum_{(i,j)} (\beta_{ij} - \alpha_{ij} - h_{ij}) d_{ij} y_{ij}}{b - a + \sum_{(i,j)} \gamma_{ij} d_{ij} y_{ij}}$$

$$= \frac{b - \sum_{(i,j)} c_{ij}^o\, w_{ij} - \sum_{(i,j)} \alpha_{ij} d_{ij} y_{ij}}{b - a + \sum_{(i,j)} \gamma_{ij} d_{ij} y_{ij}} \tag{23}$$

Step-5: Using max-min criteria, the considered problem can be restarted as,

$$Max \left\{ \frac{b - \sum_{(i,j)} c_{ij}^o\, w_{ij} - \sum_{(i,j)} \alpha_{ij} d_{ij} y_{ij}}{b - a + \sum_{(i,j)} \gamma_{ij} d_{ij} y_{ij}} \right\} \tag{24}$$

Subject to

$$\sum_{j=1}^{n} w_{ij} \leq a_i\,, i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} w_{ij} \geq b_j\,, j = 1,2,\dots,n$$

$$\sum_{j=1}^{n} f_i w_{ij} \leq g_j\,, j = 1,2,\dots,n$$

$$w_{ij} \geq 0\,, i = 1,2,\dots,m, j = 1,2,\dots,n \tag{25}$$

This is a linear fractional programming problem and its optimal solution is obtained by generalized reduced gradient technique. (Kanti, Swarup [18]).

Now $h_{ij}, for\ (i,j) \in E$ can be obtained from $\lambda_w = \frac{\beta_{ij}-\alpha_{ij}-h_{ij}}{\gamma_{ij}}\ for\ (i,j) \in E$. Then the fuzzy costs corresponding to the maximal value of λ are given by $c_{ij}^{\lambda} = \beta_{ij} - h_{ij}$

5. Numerical Example

Consider non-linear transportation problem with 2-origins, 2-destination with the following input data:

Table-1: Input data of unit transportation cost $[c_{ij}^{o}, < \alpha_{ij}, \beta_{ij} >]$

i/j	1	2	Demand
1	[13,< 5,13>]	[15,< 6,11 >]	4
2	[10,< 4,13 >]	[12,< 2,13 >]	4
Availability	6	2	

Table-2: Others Input data

Min impurity	Max impurity	location of destination	q_{ij}	γ_{ij}
$f_1 = 1$ $f_2 = 2$	$g_1 = 5$ $g_2 = 8$	(4,8) (7,9)	$q_{11} = 0.8, q_{12} = 0.5$ $q_{21} = 0.9, q_{22} = 0.9$	$\gamma_{11} = 10,\quad \gamma_{12} = 10$ $\gamma_{21} = 10,\quad \gamma_{22} = 10$

Table-3: Output associated with least and highest cost

Model	Optimal solution w_{ij}	Unknown location (x_i, y_i)	Distance	Total cost
Minimization of α_{ij}'s cost	$w_{11} = 3.23, w_{12} = 2.76$ $w_{21} = 0.76, w_{22} = 1.23$	$(x_1, y_1) = (5.38,8)$ $(x_2, y_2) = (4,9.1)$	$d_{11} = 1.33, d_{12} = 1.9$ $d_{21} = 0.0,, d_{22} = 3.0$	130
Maximization of β_{ij}'s cost	$w_{11} = 3.23, w_{12} = 2.76$ $w_{21} = 0.76, w_{22} = 1.23$	$(x_1, y_1) = (7,9)$ $(x_2, y_2) = (5.3,9)$	$d_{11} = 3.16, d_{12} = 0.0$ $d_{21} = 1.65,, d_{22} = 1.68$	190

Hence from Eq. (24) to Eq. (25) the reduced fractional programming problem is

$$Max \left\{ \frac{190 - 13w_{11} - 15w_{12} - 10w_{21} - 12w_{22} - 6.5y_{11} - 11.4y_{12} - 6y_{22}}{60 + 13y_{11} + 19y_{12} + 306y_{22}} \right\} \quad (26)$$

Subject to

$w_{11} + w_{12} \leq a_1,\ w_{21} + w_{22} \leq a_2, w_{12} + w_{22} \geq b_2,\ f_1 w_{11} + f_2 w_{12} \leq g_1, f_2 w_{12} + f_2 w_{22} \leq g_2$

For $(i,j) \in E$, we have,

$$\lambda_w = \frac{\beta_{ij} - \alpha_{ij} - h_{ij}}{\gamma_{ij}}$$

so that, $h_{ij} = \beta_{ij} - \alpha_{ij} - \lambda_w \gamma_{ij}$

The optimal solution of problem Eq. (26) which is a fractions programming, problem is solved and obtained results are shown below; Therefore, we have,

Table-4: Output or optimum results

Model	Optimal solution		Value of h_{ij}		max λ	fuzzy cost corresponding λ	Z
Maximize λ	$w_{11} = 3.23$ $w_{21} = 0.76$	$w_{12} = 2.76$ $w_{22} = 1.23$	$h_{11} = 3.1$ $h_{21} = 4.1$	$h_{12} = 0.1$ $h_{22} = 6.1$	0.49	$c_{11}^{0.49} = 9.9, c_{12}^{0.49} = 10.9$ $c_{21}^{0.49} = 8.9, c_{22}^{0.49} = 6.9$	160.33

6. Conclusion

In this paper, a non -linear transportation problem (NLTP) is formed in terms of the location of the origin (source). The model is constructed with one additional impurity constraints and imprecise cost parameters. Such a fuzzy non-linear transportation problem is converted to a fractional programming problem using Bellman-Zadeh's max-min criteria. Thus, the article has an emerging practical implication in reality. The model can be extended in different types of environment also can be solved following different soft computing method. In this content, the article can be extended in near future.

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