Electric field effects on dynamic behavior of atmospheric aerosols

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Abstract: An investigation has presented to analyze the effects of electric field on atmospheric aerosols with and without couple stress. The dynamic behavior of velocity profiles of aerosols in the atmosphere and the skin friction coefficients are studied. The impact of dimensionless couple stress parameter and electric number have been observed by graphical illustrations.

Index Terms: couple stress fluid, velocity profiles, atmospheric fluid, skin friction coefficient

1. Introduction

The behavior of air pollutants in the atmosphere is governed by the process of molecular diffusion and convection. It is mainly affected by various meteorological conditions such as wind, temperature inversion, foggy atmosphere, etc. This process is also affected by several removal mechanisms such as chemical transformation, dry deposition, greenbelt plantation, etc. [1] Recently, theoretical analysis have been initiated to investigate the dynamic behaviour of secondary aerosols by chemical reactions in atmosphere. Reactions of gases on aerosol surfaces play an important role in the chemistry of the troposphere.

Electric fields are used extensively in different industrial problems, particularly in those concerned with chemical, electrical, electronic and drug industries for various separation processes. Waterman [41] reviewed the process of using electric field to improve coalescence and found these techniques to be quite effective in the removal of water from oil. Williams and Bailey [42] examined coalescence of poorly conducting drops in the presence of an electrical field both theoretically and experimentally. Schmidt [30] performed experiments with different aerosols and observed that the application of an electric field had the effect of reducing sedimentation time. A detailed calculation of the aerosol particles and size distribution suggested that this phenomenon of reduction of sedimentation time is due to the electric field enhancing aerosol particle coalescence.

Studies of predicting the collision frequencies of settling mechanisms have been carried out in the past. Also, the deposition of nanoparticles under different conditions was investigated in the numerical studies by Sun et al., [35], Liu and Lin[20], Yin and Lin [43]. Wang et al., [40] used a trajectory analysis for estimating the aggregation rates and found that the electric fields can enhance the gravitational settling of charged particles.

Atmospheric electricity abounds in the environment; some traces of it are found less than four feet from the surface of the earth, but on attaining greater height it becomes more apparent. Benjamin Franklin [3] was the first to
design an experiment to prove the electrical nature of lightening. Lemonnier [19] discovered that even when there are no clouds, the so-called fair weather condition, a weak electrification exists in the atmosphere. He also found some evidence that the electrification varied from night to day. The conductivity of the atmosphere increases exponentially with altitude. The amplitudes of the electric and magnetic components depend on seasons, latitude and height above the sea level. The greater the altitude the more atmospheric electricity abounds. The presence of the earth's surface influences the concentration of ions, aerosols and radioactive particles, through its control over the wind, temperature and water vapour distributions.

The broad subjects of Oceanography, Meteorology, Atmospheric Science and Limnology all contain some important features of fluids. Several investigations are carried out on the problem of hydrodynamic flow of a viscous incompressible fluid considering various variations in the problem. Mention may be made of the studies of Greenspan and Howard [10,11], Holton [16], Walin [39], Hayat and Hutter [14], Singh et al., [31,32] and Guria et al., [12,13]. The problem of electrohydrodynamic flow of fluids is studied by many researchers viz. Ng et al.,[27,28,29], Vedin and Ronnmark [38], Haque and Araj [15] and Tiwari et al., [37].

In recent years, the study of a chemically reactive pollutant has generated considerable attention because of its harmful effects. Kazuhide Ito and Hiroshi Harashima [18] investigated coupled CFD analysis of size distributions on indoor secondary organic aerosol derived from ozone.

These studies have motivated to investigate the velocity profiles and skin friction of aerosols in a poorly conducting atmospheric fluid with and without couple stress through a horizontal channel under the effect of electric field.

Sometimes, the primary pollutants may not cause pollution of the air but secondary pollutants produce air borne pollutants. The study of freely suspended atmospheric pollutants which execute microrotation is called micropolar fluid. When microrotation balances natural vorticity of the micropolar fluid is called couple stress fluid.

The study of non-Newtonian fluid has attracted much attention, because of their practical applications in engineering, industry and various natural phenomena. In the category of non-Newtonian fluids, couple stress fluid has distinct features, such as polar effects in addition to possessing large viscosity. Malashetty et al., [21,22] have discussed the effect of rotation on the set of double diffusive convection in a Darcy porous medium saturated with a couple stress fluid. Das [5,6] considered the effect of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid. Bakr [2] studied the effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid. Cai-Wan Chang-Jian and Chao-Kuang Chen [4] investigated a dynamic analysis of the rub-impact rotor supported by two couple stress fluid film journal bearings. Malashetty et al., [22] have discussed the onset of convection in a couple stress fluid saturated porous layer using a thermal non-equilibrium model. Gaikwad et al., [8] investigated an analytical study of linear and non-linear double diffusive convection with Soret and Dofour effects in couple stress fluid. The effect of surface roughness on the hydrodynamic lubrication of porous step slider bearings with couple stress fluids, has been studied by Naduvanamani and Siddangouda.
The objective of this paper is to present the mathematical model on dynamic behavior of atmospheric aerosols with and without couple stress under the influence of electric field.

2. Mathematical Formulation

Consider a two dimensional geometry as shown in Figure 1. It consists of flow through a horizontal channel extended to infinity on both directions. The x-axis is taken along the walls and the y-axis perpendicular to it. The couple stress fluid is filled in the channel with embedded electrodes of different potentials at y=0 and y=h.

\[ \phi = \frac{V}{h} (x-x_0) \]

\[ \phi = \frac{V}{h} x \]

\[ \frac{\partial \bar{q}}{\partial t} + (\bar{q} \cdot \nabla) \bar{q} = -\nabla p + \mu \nabla^2 \bar{q} - \lambda \nabla^4 \bar{q} + \rho_e \vec{E} \]

where \( \lambda \) is a couple stress parameter.

The Conservation of charges

\[ \frac{\partial q_e}{\partial t} + (q_e \cdot \nabla) q_e + \nabla \cdot \vec{J} = 0 \]

Maxwell’s equation

\[ \nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} \] (Gausslaw)
\[ \nabla \times \vec{E} = 0 \text{ (Faraday’s law)} \]  
\[ \vec{J} = \sigma \vec{E} \text{ (Ohm’s law)} \]  

where \( \vec{q} \) the velocity, \( p \) the pressure, \( \rho \) the density, the electric \( \vec{E} \) field, \( \rho_e \) the electric charge density, the current density. \( \vec{J} \)

The above equations are solved using the following boundary conditions on velocity and potential are,

\[ u = 0 \text{ and } T = T_1 \text{ at } y = b \]
\[ u = 0 \text{ and } T = T_0 \text{ at } y = 0 \]  

\[ \text{The couple stress condition, } \frac{d^2u}{dy^2} = 0 \text{ at } y = 0 \text{ and } y = b \]  

\[ \phi = \frac{V}{h} x \text{ at } y = 0 \]  
\[ \phi = \frac{V}{h} (x - x_0) \text{ at } y = b \]  

Boundary conditions on velocity represents the no-slip conditions of the couple stresses at the solid boundaries. In cartesian form, using the above approximation equation (3) becomes

\[ 0 = -\frac{\partial p}{\partial x} + \mu \nabla^2 u - \lambda \nabla^4 u + \rho_e E_x \]  

In a poorly conducting fluid, the electrical conductivity is assumed to vary linearly with temperature and increases with temperature in the form

\[ \sigma = \sigma_0 \left[ 1 + \alpha_h (T_b - T_0) \right] \]  

where \( \alpha_h \) is the volumetric coefficient of expansion and \( \sigma \) is the electrical conductivity.

We assume the flow is fully developed and unidirectional in the x – direction. This means the velocity is independent of time and all physical quantities except pressure and concentration are independent of x, so that the velocity and temperature will be functions of y only. Using the following dimensionless quantities,

\[ y^* = \frac{y}{h}; \quad u^* = \frac{u}{V \frac{E_x}{h}}; \quad \rho_e^* = \frac{\rho_e}{\varepsilon_0 V \frac{h}{h^2}}; \quad x^* = \frac{x}{h}; \]  

where \( V \) is electric potential, we get electric potential through electrodes.

We assume that the fluid with pollutants is isotropic and homogenous so that molecular diffusivity D, viscosity \( \mu \) are all constants.
\[
\frac{d^4 u}{dy^4} - a^2 \frac{d^2 u}{dy^2} - a^2 W e P e \tilde{E}_x = a^2 P
\]  
(14)

where \( W_e = \frac{\varepsilon_0 V^2}{\mu}, \quad l = \frac{\lambda}{\sqrt{\mu a}} \) is the couple stress parameter.

Equation (7) becomes, \( \nabla \cdot \mathbf{j} = 0 \)  
(15)

Using equation (1) we get,
\[
\sigma (\nabla^2 \phi) + \nabla \phi \cdot \nabla \sigma = 0
\]  
(16)

The boundary conditions on velocity, couple stress, temperature and electric potential after dimensionless are
\[
u = 0 \quad \text{at} \quad y = 0, 1
\]  
(17)

\[
\frac{d^2 u}{dy^2} = 0 \quad \text{at} \quad y = 0, 1
\]  
(18)

\[
\begin{align*}
\theta &= 0 \quad \text{at} \quad y = 0, 1 \\
\end{align*}
\]  
(19)

\[
\begin{align*}
\phi &= x \quad \text{at} \quad y = 0 \\
\phi &= x - x_0 \quad \text{at} \quad y = 1 \\
\end{align*}
\]  
(20)

In a poorly conducting fluid, \( \sigma << 1 \) and hence any perturbation on it is negligible and hence any perturbation on it is negligible and hence it depends on the conduction temperature \( T_b \) namely,
\[
\frac{d^2 T_b}{dy^2} = 0
\]  
(21)

with the boundary conditions
\[
\begin{align*}
T_b &= T_0 \quad \text{at} \quad y = 0, 1 \\
\end{align*}
\]  
(22)

\[
T_b - T_0 = \Delta T
\]  
(23)

Therefore equation (16) becomes \( \sigma_0 [I + \alpha_0 \Delta T y] = \sigma_0 (I + \alpha y) = \sigma_0 e^{\alpha_0 y} \)
\[
\sigma \approx e^{\alpha y}
\]  
(24)

where \( \alpha = \alpha_0 \Delta T \).

Then (16) using (24) we get
\[
\frac{d^2 \phi}{dy^2} + \alpha \frac{d\phi}{dy} = 0
\]  
(25)

Its solution satisfying the boundary condition (20) is
\[
\phi = x - \frac{x_0}{1 - e^{-\alpha}} \left[ 1 - e^{-\alpha} \right]
\]  
(26)
Using the dimensionless quantities and equation (25), equations (5), (6) and (7) reduce to

\[ \rho_e = \nabla \cdot \bar{E} = -\nabla^2 \phi = - \frac{x_0 \alpha^2 e^{-\alpha y}}{1 - e^{-\alpha}} ; \quad \bar{E}_x = -1 \]

Therefore,

\[ \rho_e \bar{E}_x = \frac{x_0 \alpha^2 e^{-\alpha y}}{1 - e^{-\alpha}} \]

(27)

We consider two cases. First case in the presence of couple stress and the second without couple stress.

**Case 1**: \(a \neq 0\) (with couple stress)

The solution of equation (14) satisfying the condition (17) and (18) is

\[ u = k_1 k_2 y - k_3 y + k_4 (y - y^2) - k_5 e^{\alpha y} + k_6 e^{-\alpha y} + k_7 e^{\alpha y} - k_8 e^{-\alpha y} \]

(28)

where

\[ k_1 = \left( \frac{a_0}{\alpha^2 (\alpha^2 - a^2)} \right) ; \quad k_2 = \left( 1 - e^{-\alpha} \right) ; \quad k_3 = \left( \frac{a_0}{\alpha^2 (\alpha^2 - a^2)} \right) ; \quad k_4 = \frac{P}{2} ; \]

\[ k_5 = \frac{P}{2} ; \quad k_6 = \frac{1}{2 (\sin \alpha h \alpha)} \left( \frac{a_0 (e^\alpha - e^{-\alpha})}{\alpha^2 (\alpha^2 - a^2)} + \frac{a^2 P (e^\alpha - 1)}{a^4} \right) ; \]

**Case 2**: \(a = 0\) (without couple stress)

To find the velocity from equation (14) satisfying the condition (17) and (18) is

\[ u = \frac{a_0}{\alpha^2} e^{-\alpha y} + \frac{P}{2} y^2 + \frac{a_0}{\alpha^2} y - \frac{P}{2} y - \frac{a_0}{\alpha^2} e^{-\alpha y} - \frac{a_0}{\alpha^2} \]

(29)

where \( \alpha_0 = \frac{w_x x_0 \alpha^2}{e^{-\alpha} - 1} \)

The analytical results for both with and without couple stress are depicted graphically through figures.

To find skin friction,

In many practical applications it is advantageous to know the skin friction at the boundaries. These can be determined once we know the velocity. The skin friction \( \tau \) at the walls is defined as

\[ \tau = \mu \frac{d \bar{u}}{d y} \]

making this dimensionless using the scale for \( \bar{u} \) used earlier we get.
\[
\tau = \left( \frac{du}{dy} \right)_{y=0 \text{ and } 1}
\]

where \( \frac{du}{dy} \) can be obtained using (28) and (29).

The analytical results for both cases (with and without couple stress) are represented through figures.

3. Results and Discussions

A mathematical modelling of velocity profiles and the skin friction of atmospheric aerosols with and without couple stress under the influence of electric field is discussed. The graphical illustrations are as follows.

Figure 2. Velocity profiles \( u \) with \( y \) for different values of electric number \( \text{we} \) in the presence of couple stress.

Figure 3. Velocity profiles \( u \) with \( y \) for different values of electric number \( \text{we} \) in the presence of couple stress.

Figure 2 and 3 represents the effect of electric number on the velocity profiles of aerosols in the presence and absence of couple stress. It is observed that the velocity increases with the increase in electric number for both with and without couple stress.

Figure 4. Skin friction coefficient \( \tau \) with \( y \) for different values of \( \text{we} \).

Figure 5. Skin friction coefficient \( \tau \) with \( y \) for different values of \( \text{we} \).
for different values of electric number we 

in the presence of couple stress 

values of couple stress parameter in the presence of electric number

Figure 6. skin friction coefficient $\tau$ with $y$ for different values of electric number we in the absence of couple stress.

The coefficient of skin friction on the lower and upper surface for different values of electric number are illustrated through figure 4,5and 6. Figure 4 and 5 depicts the effects of couple stress on skin coefficient with $y$. It is seen that the skin friction coefficient increases with increase in couple stress parameter. From the figure 6, It is found that the skin friction coefficient decreases with increase in electric number in the absence of couple stress parameter.

5. Conclusion

The general introduction of couple stress fluid and atmospheric fluid in atmospheric aerosols are discussed. This paper deals with the effects of electric field on velocity profiles and skin friction (at both lower and upper surface at boundaries) of atmospheric aerosols. It concludes that the velocity and skin friction coefficient increases under the influence of electric field in the presence of couple stress but in the absence of couple stress the skin friction coefficient decreases. The above mathematical model is the representative of effects of electric field and couple stress on the dynamic behavior of aerosols in the atmosphere.

References


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