INVENTORY MODEL FOR DETERIORATING ITEMS WITH PARETO LIFETIME AND CREDIT POLICY

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ABSTRACT

This article reviews an inventory model for deteriorating items having two component mixture of Pareto distribution where the supplier offers a delay in payments. Here it is assumed that demand is quadratic. Shortages are allowed which is completely backlogged. To optimize the model, empirical investigations have been carried out and sensitivity analysis occurred to evaluate the result of parameters on assessment variables and the entire cost of these models.

Keywords: Pareto distribution, Trade credit, Quadratic demand, Shortage

1.INTRODUCTION

Trade credit is an important economic phenomenon. The most prevailing practice is that the supplier may offer a credit period to the retailer to settle his account within the fixed stipulated settlement period. Thus delay in the payment offered by the supplier is a kind of price discount, since paying later indirectly reduces the purchase cost and encourages the retailers to increase their order quantity. Most of the researchers while developing EOQ models for a retailer when the supplier offers a permissible delay in payments assumed that the selling price is same as the purchase cost. Aggarwal and Jaggi (1995) and Chung (2000) lot-size model when units in inventory are subject to constant rate of deterioration and the supplier offers credit period M for settling the accounts for the purchase quantity. Chung and Huang (2007) developed a retailer's replenishment model to reflect the real-life situations by assuming that the retailers also adopt the trade credit policy to stimulate his/her customer demand.

Deterioration plays an important role in many inventory systems. Deterioration is defined as decay or damage in the quantity of the inventory, in some substances like foods, drugs, pharmaceuticals and radioactive substances. When the items of the commodity are kept in stock as an inventory for fulfilling the future demand there may be the deterioration of items which takes place in the inventory system. For items like agricultural products, chemicals etc., the life time of the commodity is random and follows a Pareto distribution. Very little work has been reported for deteriorating items with Pareto decay having stock dependent production rate and linear demand. Srinivas Rao and Begum (2007), Srinivas Rao and Eswara Rao (2011) have studied inventory models having Pareto decay distribution.

This paper investigates an inventory model with Quadratic demand and Pareto decay distribution. The Pareto distribution is capable of characterizing the lifetime of the lifetime of commodities which have a minimum period to start deteriorating and the rate of deterioration is inversely proportional to time. This model is illustrated with number of numerical investigations.

2. Assumptions and Notations:

A. The demand rate is assumed to be $D(t) = a + bt + ct^2$;the annual demand is a function of time where a>0,b>0 and c>0

B. The lead time is zero

C. Shortages allowed which is completely backlogged

D. Life time of the commodity is random and follows a pareto distribution having probability density function of the form

$$f(t)=\frac{\alpha\beta^{\alpha}}{t^{\alpha-1}},t\geq\beta,\beta\geq 0$$

and $F(t)=1-\left(\frac{\beta}{t}\right)^{\alpha}$

The instantaneous rate of deterioration is

$$\theta(t)=\frac{f(t)}{1-F(t)}=\frac{\alpha}{t},\alpha>0,t>\beta$$

E. T : is the length of the cycle

F. A: Ordering cost of the cycle

G. h : Inventory holding cost per unit per unit time

H. s :Shortages cost per unit per unit time

I. M: Permissible delay in settling the accounts

J. I_c, I_e : Interest charge and Interest earn respectively where $I_c \geq I_e$

K. P: Purchase cost per unit per unit time

L. TC_1, TC_2 : Total average cost per unit time for $M \leq t_1$ and $M > t_1$ respectively.

3.Mathematical Formulation of the Model:

Let I(t) be the inventory level at time "t" ($0 \leq t \leq T$).The differential equations governing the system in the cycle time [0,T] are

$$\frac{dI(t)}{dt}+\theta(t)I(t)=-(a+bt+ct^{2}),0\leq t\leq t_{1} \qquad [1]$$

$$\frac{dI(t)}{dt}=-(a+bt+ct^{2}),t_{1}\leq t\leq T \qquad [2]$$

With I(t)=0 at $t=t_1$

Solving equation (1) and (2) respectively, we have

$$I(t)=\frac{a}{\alpha+1}(t_{1}-t)+\frac{b}{\alpha+2}(t_{1}^{2}-t^{2})+\frac{c}{\alpha+3}(t_{1}^{3}-t^{3}) \qquad [3]$$

And for shortage

$$I(t)=a(t_{1}-t)+\frac{b}{2}(t_{1}^{2}-t^{2})+\frac{c}{3}(t_{1}^{3}-t^{3}) \qquad [4]$$

Holding cost during the cycle is

$$HC=h\int_{0}^{t_{1}}I(t)dt$$

$$HC=\frac{h}{2}t_1^{\,2}\left[\frac{a}{\alpha+1}+\frac{4b}{3(\alpha+2)}t_1+\frac{3c}{2(\alpha+3)}t_1^{\,2}\right] \qquad [5]$$

Average number of deteriorated items $= I(0)-\int_0^{t_1}(a+bt+ct^2)dt$

$$=-\alpha t_1\left(\frac{a}{\alpha+1}+\frac{bt_1}{2(\alpha+2)}+\frac{ct_1^{\,2}}{3(\alpha+3)}\right) \qquad [6]$$

Hence the expected deterioration cost per unit time is

$$D.C=P\left[-\alpha t_1\left\{\frac{a}{\alpha+1}+\frac{bt_1}{2(\alpha+2)}+\frac{ct_1^{\,2}}{3(\alpha+3)}\right\}\right] \qquad [7]$$

Over the interval (t_1,T) expected shortage cost per unit per unit time is

$$SC=s\int_{t_1}^{T}\left[a(t_1-t)+\frac{b}{2}(t_1^{\,2}-t^2)+\frac{c}{3}(t_1^{\,3}-t^3)\right]dt \qquad [8]$$

CASE-I($M\le t_1$)

For $M\le t_1$, the buyer has stock on hand beyond M and so he can use the sale revenue to earn interest at an annual rate I_e up to t_1. The interest earned denoted by IE_1, is therefore

$$IE_1=PI_e\int_0^{t_1}\left[a(t_1-t)+\frac{b}{2}(t_1^{\,2}-t^2)+\frac{c}{3}(t_1^{\,3}-t^3)\right]dt$$

$$=PI_e\left[\frac{at_1^{\,2}}{2}+\frac{bt_1^{\,3}}{3}+\frac{ct_1^{\,4}}{4}\right] \qquad [9]$$

However beyond the fixed credit period M, the unsold stock is assumed to be financed with annual charge I_C and the interest charge, denoted by IC_1, is given by

$$IC_1=PI_C\int_M^{t_1}(a+bt+ct^2)dt$$

$$=PI_C\left[a(t_1-M)+\frac{b}{2}(t_1^{\,2}-M^2)+\frac{c}{3}(t_1^{\,3}-M^3)\right] \qquad [10]$$

Therefore the total average cost in this case comes out to be

$$TC_1(t_1,T)=\frac{A+HC+DC+SC+IC_1-IE_1}{T} \qquad [11]$$

Substituting the equations of HC, DC, SC, IC_1 and IE_1 in equation (11), we have

$$=\frac{A}{T}+\frac{ht_1^{\,2}}{T}\left[\frac{a}{2(\alpha+1)}+\frac{2bt_1}{3(\alpha+2)}+\frac{3ct_1^{\,2}}{4(\alpha+3)}\right]-\alpha Pt_1\left[\frac{a}{\alpha+1}+\frac{bt_1}{2(\alpha+2)}+\frac{ct_1^{\,2}}{3(\alpha+3)}\right]$$

$$+\frac{s}{T}\left[a(t_1T-\frac{T^2}{2}-\frac{t_1^{\,2}}{2})+\frac{b}{2}(t_1^{\,2}T-\frac{T^3}{3}-\frac{2t_1^{\,2}}{3})+\frac{c}{3}(t_1^{\,3}T-\frac{T^4}{4}-\frac{3t_1^{\,4}}{4})\right]$$

$$+\frac{PI_C}{T}\left[a(t_1-M)+\frac{b}{2}(t_1^{\,2}-M^2)+\frac{c}{3}(t_1^{\,3}-M^3)\right]-\frac{PI_e t_1^{\,2}}{T}\left[\frac{a}{2}+\frac{bt_1}{3}+\frac{ct_1^{\,2}}{4}\right] \qquad [12]$$

The optimal values of t_1 and T which minimize total average cost per unit per time, can be obtained by solving following equations

$$\frac{\partial TC_1(t_1,T)}{\partial t_1}=0 \quad \text{and} \quad \frac{\partial TC_1(t_1,T)}{\partial T}=0$$

Provided they satisfy the sufficient conditions

$$\frac{\partial^2 TC_1(t_1,T)}{\partial t_1^{\,2}}>0,\frac{\partial^2 TC_1(t_1,T)}{\partial T^2}>0$$

And

$$\left[\left\{\frac{\partial^2 TC_1(t_1,T)}{\partial t_1^{\,2}}\right\}\left\{\frac{\partial TC_1(t_1,T)}{\partial T^2}\right\}-\left\{\frac{\partial^2 TC_1(t_1,T)}{\partial t_1 \partial T}\right\}\right]>0$$

CASE-II ($M>t_1$)

Since $M>t_1$, the retailer pays no interest but earns interest at an annual rate I_e during the period (0,M).But [0,T], the retailer sells product at selling price P/unit and deposits the revenue into interest earning account at the rate of I_e/year. In the period [T, M], the retailer deposits only the total revenue into an account that earns I_e/year. Hence, interest earned per time unit is

$$IE_2=PI_e\left[\int_0^{t_1}\left\{a(t_1-t)+\frac{b}{2}(t_1^{\,2}-t^2)+\frac{c}{3}(t_1^{\,3}-t^3)\right\}dt+\int_0^{t_1}\left\{a(M-t_1)+\frac{b}{2}(M^2-t_1^{\,2})+\frac{c}{3}(M^3-t_1^{\,3})\right\}dt\right]$$

$$=PI_e\left[t_1^{\,2}\left(\frac{a}{2}+\frac{bt_1}{3}+\frac{ct_1^{\,2}}{4}\right)+t_1\left\{a(M-t_1)+\frac{b}{2}(M^2-t_1^{\,2})+\frac{c}{3}(M^3-t_1^{\,3})\right\}\right]$$

Here $IC_2=0$

Total average cost per unit time is

$$TC_2(t_1,T)=\frac{A+HC+DC+SC+IC_2-IE_2}{T}$$

$$=\frac{A}{T}+\frac{ht_1^{\,2}}{T}\left[\frac{a}{2(\alpha+1)}+\frac{2bt_1}{3(\alpha+2)}+\frac{3ct_1^{\,2}}{4(\alpha+3)}\right]-\alpha Pt_1\left[\frac{a}{\alpha+1}+\frac{bt_1}{2(\alpha+2)}+\frac{ct_1^{\,2}}{3(\alpha+3)}\right]$$

$$+\frac{s}{T}\left[a(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2})+\frac{b}{2}(t_1^2T-\frac{T^3}{3}-\frac{2t_1^2}{3})+\frac{c}{3}(t_1^3T-\frac{T^4}{4}-\frac{3t_1^4}{4})\right]$$

$$-\frac{PI_e}{T}\left[t_1^2\left(\frac{a}{2}+\frac{bt_1}{3}+\frac{ct_1^2}{4}\right)+t_1\left\{a(M-t_1)+\frac{b}{2}(M^2-t_1^2)+\frac{c}{3}(M^3-t_1^3)\right\}\right] \quad [13]$$

The optimal values of t_1 and T which minimize total average cost per unit per time, can be obtained by solving following equations

$$\frac{\partial TC_2(t_1,T)}{\partial t_1}=0 \quad \text{and} \quad \frac{\partial TC_2(t_1,T)}{\partial T}=0$$

Provided they satisfy the sufficient conditions

$$\frac{\partial^2 TC_2(t_1,T)}{\partial t_1^2}>0, \frac{\partial^2 TC_2(t_1,T)}{\partial T^2}>0$$

and

$$\left[\left\{\frac{\partial^2 TC_2(t_1,T)}{\partial t_1^2}\right\}\left\{\frac{\partial TC_2(t_1,T)}{\partial T^2}\right\}-\left\{\frac{\partial^2 TC_2(t_1,T)}{\partial t_1 \partial T}\right\}\right]>0$$

4.Numerical Investigations:

Illustration-I:($M \le t_1$)

To illustrate the inventory model we consider the following hypothetical values:
A=200,a=5,b=8,c=2,h=5,p=15,α=0.5,s=10,I_C=0.15,I_e=0.11

Based on above data and using the software Mathematica5.1, we calculate the time t_1=17.698, optimal values of cycle length T=13.0669, optimal values of total average cost TC_1=102.104

Illustration-II: ($M>t_1$)

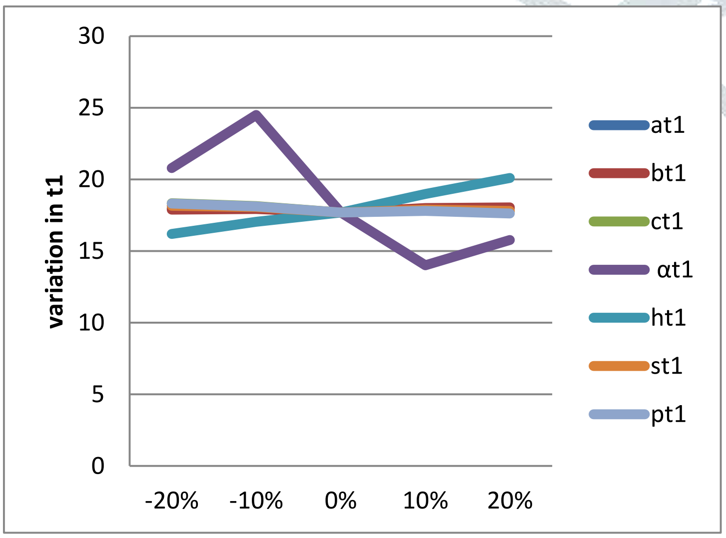
To illustrate the inventory model we consider the following hypothetical values:
A=200,a=5,b=8,c=2,h=5,p=15,α=0.5,s=10,I_e=0.11

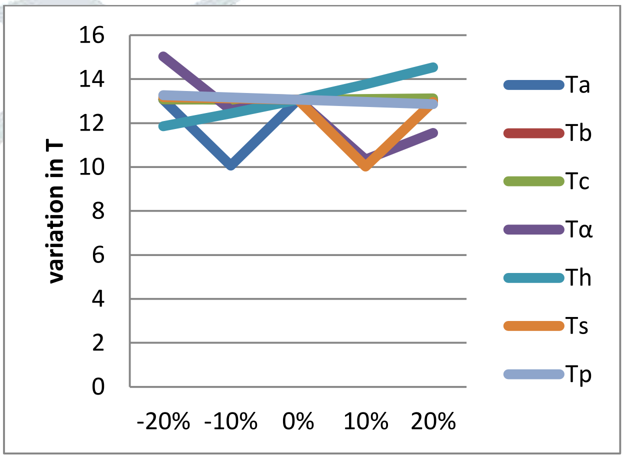
Based on above data and using the software Mathematica5.1, we calculate the time t_1=17.828, optimal values of cycle length T=21.277, optimal values of total average cost TC_2=746.122

5. Sensitivity Analysis:

The sensitivity analysis is carried to explore the effect of changes in model parameters and total average cost of the optimal policies, by varying each parameter (-20%,-10%, 0%, 10%,20%) at a time for the model under study for section 3.The results are presented in Table-1,2.The relationship between the parameters ,optimal cycle length T and optimal total average cost TC_1,TC_2 is shown in figure

TABLE-I(CASE-I)						
Sensitivity analysis of the model ($M \le t_1$)						
Variation Parameters	**Optimal Policies**	**Change in parameters**				
		-20%	**-10%**	**0%**	**10%**	**20%**
a	t_1	17.975	17.971	17.698	17.964	17.961
	T	13.07	13.068	13.0669	13.065	13.0633
	$TC_1(t_1,T)$	102.216	102.185	102.104	102.98	102.87
b	t_1	17.8924	17.9296	17.698	18.0075	18.0481
	T	13.074	13.0699	13.0669	13.0649	13.0641
	$TC_1(t_1,T)$	102.179	102.182	102.104	103.121	103.110
c	t_1	18.3338	18.1322	17.698	17.8369	17.7354
	T	13.0731	13.069	13.0669	13.0892	13.1266
	$TC_1(t_1,T)$	103.219	103.124	102.104	102.215	102.116
α	t_1	20.7855	24.4819	17.698	13.9975	15.7612
	T	15.0251	12.6045	13.0669	10.3373	11.543
	$TC_1(t_1,T)$	103.619	105.514	102.104	98.331	99.431
h	t_1	16.198	17.0431	17.698	18.983	20.1005
	T	11.8598	12.4353	13.0669	13.7616	14.5281
	$TC_1(t_1,T)$	101.819	102.741	102.104	103.412	105.212
s	t_1	18.1565	18.0607	17.698	17.8782	17.7911
	T	13.1851	13.125	13.0669	13.016	12.956
	$TC_1(t_1,T)$	103.825	103.729	102.104	102.419	102.359
P	t_1	18.316	18.11	17.698	17.797	17.628
	T	13.274	13.1702	13.0669	12.964	12.863
	$TC_1(t_1,T)$	103.185	103.026	102.104	102.241	102.112





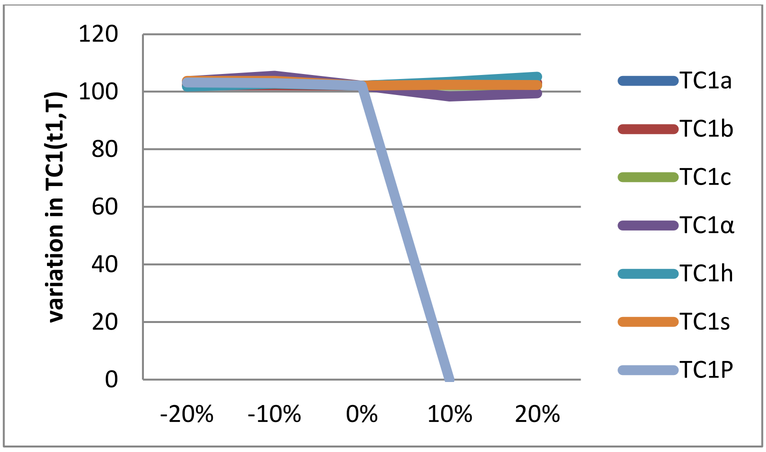
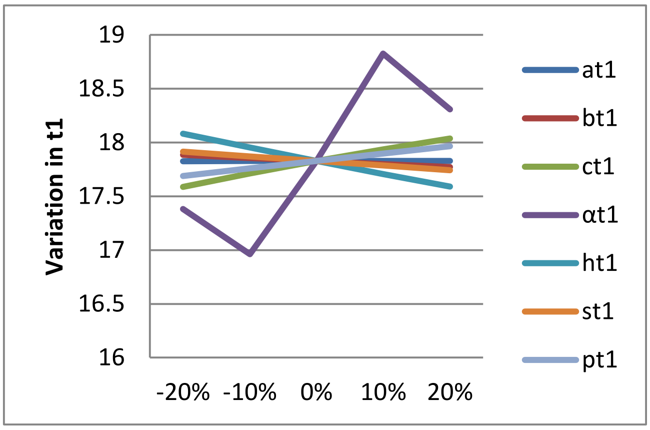
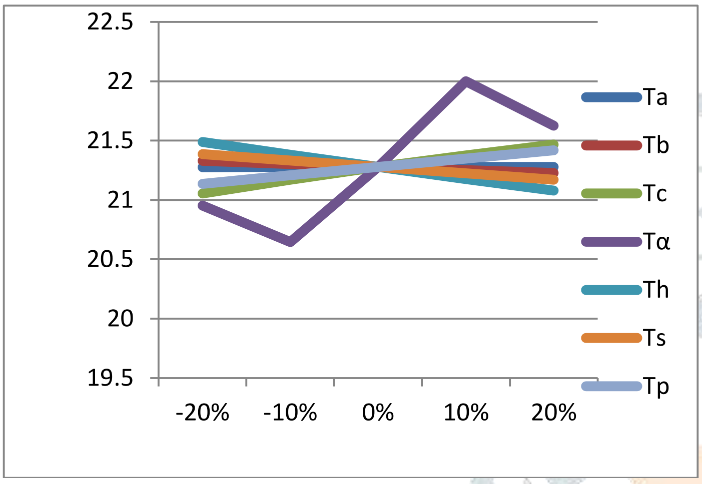


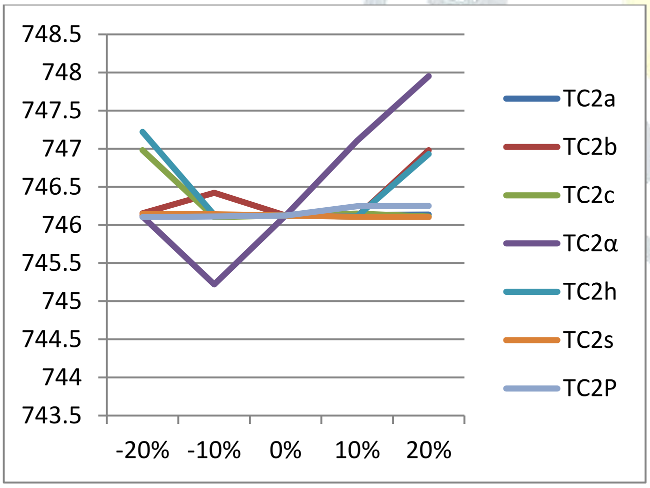
TABLE-II(CASE-II)

Sensitivity analysis of the model (M>t1)

Variation Parameters	Optimal Policies	Change in parameters				
		-20%	-10%	0%	10%	20%
a	t_1	17.8271	17.8277	17.828	17.829	17.8297
	T	21.2767	21.277	21.2773	21.2776	21.2779
	$TC_2(t_1,T)$	746.110	746.115	746.122	746.131	746.135
b	t_1	17.8856	17.8569	17.828	17.8001	17.772
	T	21.3292	21.3032	21.2773	21.2516	21.2261
	$TC_2(t_1,T)$	746.151	746.421	746.122	746.112	746.98
c	t_1	17.587	17.7122	17.828	17.9364	18.0371
	T	21.0559	21.171	21.2773	21.376	21.4677
	$TC_2(t_1,T)$	746.94	746.102	746.122	746.149	747.110
α	t_1	17.3818	16.9631	17.828	18.8256	18.3076
	T	20.9515	20.6442	21.2773	21.9988	21.625
	$TC_2(t_1,T)$	746.109	745.219	746.122	747.101	747.95
h	t_1	18.082	17.9532	17.828	17.7074	17.5901
	T	21.4876	21.3809	21.2773	21.1768	21.0792
	$TC_2(t_1,T)$	747.219	746.132	746.122	746.111	746.95
s	t_1	17.9143	17.8712	17.828	17.7859	17.7438
	T	21.387	21.3319	21.2773	21.2232	21.1696
	$TC_2(t_1,T)$	746.145	746.132	746.122	746.107	746.102
P	t_1	17.6877	17.7591	17.828	17.8978	17.9672
	T	21.1367	21.2071	21.2773	21.3475	21.4176
	$P(t_1,T)$	746.101	746.109	746.122	746.243	746.251







Results and Discussion:

The two numerical investigations for case-I $M \le t_1$ and case-II $M > t_1$ are considered to study the effect of changes of the system parameters a, b, c, α, h, s and p on the total average cost. Whenever one parameter is charging by some percentage all other parameters are kept at their original values. Investigation has been done for positive and negative charges of these seven parameters. The results obtained are discussed in Table-I (case-I) and Table-II(case-II).A sensitivity analysis has also been conducted to reflect the effects of varied system inputs on the system performance. It is evident that the parameter α is comparatively less sensitive then other parameters in table-I where as a, b, s, p are less sensitive then other parameters. Further in Table-I ,b and h is more sensitive for positive charges, but the parameter c, α and s are more sensitive for negative charges then the positive charges. Where as in Table-II parameter α is more sensitive for positive charges than negative charges. Further it can examine various policy scenarios that are associated with different levels of economic inputs**.**

CONCLUSION:

The model developed in this paper assumes demand of a product to be quadratic with respect to time and follows Pareto life time deterioration with trade credit .Here shortages allowed which is completely backlogged. In many practical situations the procurement is done from different sources. For the first time, the utility of Pareto distribution in inventory models is done because of its close reality to the practical situations. The solution procedure of the model demonstrated through numerical illustration. The sensitivity analysis of the model reveals that the life time distribution parameters and demand function parameters have significant influence on optimal operating policies.

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