THE RELATIONSHIP STRONGLY REGULAR GRAPH AND CATEGORIES ASSOCIATED WITH COLOURED N-GRAPHS

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ABSTRACT

In traditional studies of graph theory, the graphs allow only one edge to be incident to any two vertices, not necessarily distinct, and the graph morphisms must map edges to edges and vertices to vertices while preserving incidence. The most common category considered in (undirected) graph theory is a category where graphs are defined as having at most one edge incident to any two vertices and at most one loop incident to any vertex. Graph coloring is one of the early areas of graph theory. The problem of coloring a map so that adjacent regions get different colors translates into a graph coloring problem in the following way: Given a map with regions, we form a graph *G* by representing each region with a vertex and putting an edge between two vertices if the corresponding regions are adjacent on the map. There is a coloring of the map such that neighboring regions get different colors if and only if there is an assignment of labels to the vertices of *G* such that vertices which are joined by an edge are assigned different labels.

Keywords: Graph theory, N-Graph**,** Colored Graph

INTRODUCTION

Graph theory is one of the branches of modern mathematics having experienced a most impressive development in recent years. In the beginning, Graph theory was only a collection of recreational or challenging problems like Euler tours or the four colouring of a map, with no clear connection among them, or among techniques used to attach them. The aim was to get a “yes” or “no” answer to simple existence questions. Under the impulse of Game Theory, Management Sciences and Transportation Network Theory, the main concern shifted to the maximum size of entities attached to a graph.

A graph:

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices, and the links that connect the vertices are called edges.

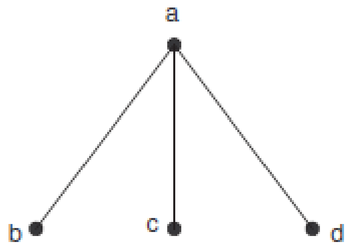

Figure 1 A an example of a graph

Applications of Graph Theory

Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in physical, biological and social systems. Many problems of practical interest can be represented by graphs.

Graph-theoretic methods, in various forms, have proven particularly useful in Linguistics, since natural language often lends itself well to discrete structure. Traditionally, syntax and compositional semantics follow tree-based structures, whose expressive power lies in the Principle of Compositionality, modeled in a hierarchical graph. Within lexical semantics, especially as applied to computers, modeling word meaning is easier when a given word is understood in terms of related words; semantic networks are therefore important in computational linguistics. Still other methods in phonology (e.g. Optimality Theory, which uses lattice graphs) and morphology (e.g. finite-state morphology, using finite-state transducers) are common in the analysis of language as a graph. Indeed, the usefulness of this area of mathematics to linguistics has borne organizations such as Text Graphs, as well as various 'Net' projects, such as Word Net, Verb Net and others.

In Mathematics, graphs are useful in geometry and certain parts of topology, e.g. Knot Theory. Algebraic graph theory has close links with group theory. A graph structure can be extended by assigning a weight to each edge of the graph. Graphs with weights or weighted graphs are used to represent structures in which pair wise connections have some numerical values. For example, if a graph represents a road network, the weights could represent the length of each road. A digraph with weighted edges in the context of graph theory is called a network. Network analysis has many practical applications, for example, to model and analyze traffic networks. The field of mathematics plays a vital role in various fields. One of the important areas in mathematics is graph theory which is used in structural models. This structural arrangements of various objects or technologies lead to new inventions and modifications in the existing environment for enhancement in those fields. Graph theoretical concepts are widely used to study and model various applications, in different areas. They include, study of molecules, construction of bonds in chemistry and the study of atoms. Similarly, graph theory is used in sociology for example to measure actors' prestige or to explore diffusion mechanisms.

Representation of a graph

It is customary to represent a graph by a diagram and refer to the diagram itself as the graph. Each point is represented by a small dot and each line is represented by a line segment joining the two points with which the line is incident. Thus a diagram of graph depicts the incidence relation holding between its points and lines. In drawing a graph it is immaterial whether the lines are drawn straight or curved, long or short and what is important is the incidence relation between its points and lines.

N-Graph

A basic construction in the theory of n-graphs is fusion. Consider two points *u* and *v* in an *n*-graph *G* (or in two n-graphs G_1 and G_2 and let G^* be the n-graph obtained by

(1) removing *u*, *v* and all edges connecting them

(2) Re-connecting the 'free' edges (previously incident to one of *u* or *v*) of like color.

Then G^* is said to be obtained from *G* by fusion on *u* and *v*. If there are m ≥ 1 edges connecting *u* and *v* then the graph removed in step (1) is called an m-dipole and the fusion is called removing a dipole. The inverse operation is called adding a dipole. If *J* denotes the set of colors of a dipole *D* of an n-graph *G* and if *u* and *v* lie in the same residue of type [*n*] -.*J*, then *D* is called degenerate. Otherwise *D* is non-degenerate.
Call two n-graphs equivalent if one can be obtained from the other by a sequence of adding or removing non-degenerate dipoles. Figure. 1.9 shows three equivalent 3-graphs. First dipole d_l is added and then dipole d_2 is removed. Removing a non-degenerate dipole in *G* corresponds in *G* to removing a ball and identifying two hemispheres on the boundary in the natural way. This makes the 'if' part of the following theorem reasonable. What is surprising is that the converse is also true [4].

Colored Graph Properties

Definition 1. *A colored graph is a graph in which each vertex is assigned a color. A properly colored graph is a colored graph whose color assignments conform to the coloring rules applied to the graph. The chromatic number of a graph G, denoted _(G), is the least number of distinct colors with which G can be properly colored.*

The concept of the chromatic number of a graph is one of the most interesting in all of graph theory. While there is no general rule defining a graph's chromatic number, we instead place an upper bound on the chromatic number of a graph based on the graph's maximum vertex degree. That is, we say that for a graph G with maximum vertex degree Δ, $X(G) \leq f(\Delta)$ where $f(\Delta)$ is some function of the maximum vertex degree.

The remainder of this paper deals with the problem of finding a suitable upper bound for the chromatic number of any graph in each of three sets of coloring rules. We begin with the simplest set of rules, regular coloring.

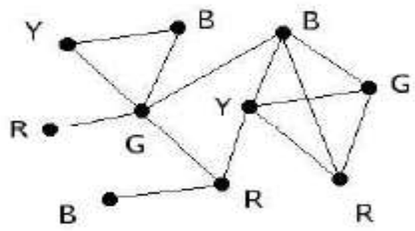

Figure 2 properly colored graph of chromatic number 4

Regular Coloring

As stated above, regular coloring is a rule for coloring graphs which states that no two adjacent vertices may have the same color. See Figure 3 for an example. In the figure, graph *G* is properly colored by regular coloring rules, while *G'* is not, as it contains two adjacent vertices that are both colored with color *R*.

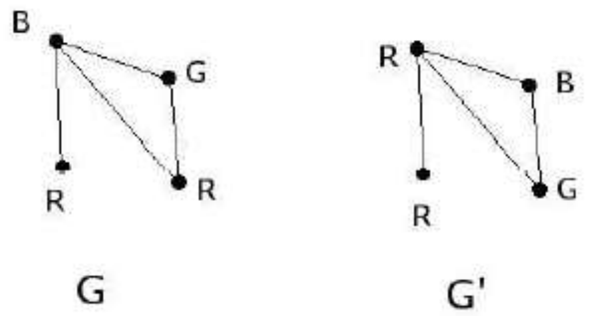

Figure 3 Two colored graphs G is properly colored, G' is not.

Given this coloring rule, it becomes apparent why we may safely ignore disconnected graphs in our exploration of graph coloring. As the coloring rules deal with vertices that are adjacent, the colors on the vertices of each disjoint part of a disconnected graph have no bearing whatsoever on the colors of the vertices on any other disjoint part. Thus, we may treat the each of the disjoint parts of the graph as if they were individual, connected graphs.

Relationship between Colorings in Some Regular Graphs

The concept of range coloring of order *k* was first presented by Lozano et al. (2009). In this paper, we shows that if a regular graph *G* admits an equitable range coloring *c* of order Δ with $(\Delta + 1)$ colors then there is an equitable total coloring of G - with the same set of colors - that extends c. We also show that there are infinite graphs satisfying this theorem. Such graphs are called Harmonics. We generate Harmonic Graphs which are Cartesian products of cycles and their complements. These graphs are regular and they admit an equitable total coloring under the above conditions.

Concepts:

A graph $G(V, E)$ is k-regular if all its vertices have the same degree k. A complete graph on n vertices is a $(n - 1)$-regular graph; and it is denoted Kn. Let M be a matching in a graph G; a vertex v of G is said *M-saturated* by G if some edge of M is incident to v; otherwise v is *M-unsaturated*. A matching that saturates all vertices of G is called a perfect matching.

Cartesian product of Graphs

The Cartesian product $G \Delta H$ of two graphs G and H with vertex sets $V(G)$ and $V(H)$ is the graph such that its vertex set is $V(G) \Delta H(G)$. Any two vertices (g, h) and (g', h') of the $G\Delta H$ are adjacent whenever $g = g'$ and h is adjacent to h' in H or $h = h'$ and g is adjacent with g' in G. This definition can be found in Kemnitz & Marangio (2003), Seoud et al. (1997) or Zmazek & Zerovnik (2002).

Conclusion

The equitable coloring is valuable because it can be used in task allocation in general, in order to guarantee the balance in the distribution of tasks. A colouring of a strongly regular graph is an allocation of colours (or treatments) to the vertices of the graph. Such a colouring is balanced if every pair of distinct colours occurs equally often on the ends of an edge. When the graph is the complete regular multipartite graph a balanced colouring is just a balanced incomplete-block design, or 2-design. strongly regular graphs may be generalized to arbitrary association schemes. When the association scheme is a collection of circular blocks then a colouring is balanced if the design in blocks is a 2-design and there is undirectional neighbour balance at all distances around the circle.

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