Modified Polytropic $f(T)$ gravity model

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Abstract: In this paper, we reconstruct the $f(T)$ gravity model with the polytropic gas dark energy. We also find the equation of state of the $f(T)$ gravity model in the context of polytropic gas to explain the acceleration of the universe.

Keywords: *Dark energy, Polytropic gas*, $f(T)$ gravity.

I. INTRODUCTION

Many Cosmological experiments and observations such as Type 1a Supernovae [1]-[3], Cosmic Microwave Background Radiation [4], Large Scale Structure [5], [6], Wilkinson Microwave Anisotropy Probe [7], Sloan Digital Sky Survey [8], etc. indicates that our universe expands under an accelerated expansion. In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [9], [10]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field. About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy [11]. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models to explain the cosmic acceleration of the universe [12]-[14]. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [15]. An interesting alternative to General Relativity is the so-called $f(T)$ gravity, which has received considerable attention as a possible explanation of the late time acceleration of the universe [16]-[18]. It is based on the old idea of the "teleparallel" equivalence of the General Relativity [19]. : In this paper, we reconstruct the $f(T)$ gravity model with the polytropic gas dark energy. We also find the equation of state of the $f(T)$ gravity model in the context of polytropic gas to explain the acceleration of the universe.

II. THE BASIC EQUATION OF THE $f(T)$ GRAVITY MODEL

The action of the $f(T)$ gravity is given by [17], [18]

$$S = \frac{1}{2k^2}\int d^4 x\sqrt{-g}[f(T) + L_m] \quad (1)$$

Where $k^2 = 8\pi G$, T is the torsion scalar, $f(T)$ is the general differentiable function of the torsion T and L_m is the Lagrangian density of the matter inside the universe

The modified Friedmann equation in the case of $f(T)$ gravity for the spatially flat FRW universe are given by

$$H^2 = \frac{1}{3}(\rho_m + \rho_T) \quad (2)$$

$$2\dot{H} + 3H^2 = -(p_m + p_T) \quad (3)$$

Where

$$\rho_T = \frac{1}{2}(2Tf_T - f - T) \quad (4)$$

$$p_T = -\frac{1}{2}\left[-8\dot{H}f_{TT} + \left(2T - 4\dot{H}\right)f_T - f + 4\dot{H} - T\right] \quad (5)$$

$$T = -6H^2 \quad (6)$$

Here $H = \frac{\dot{a}}{a}$ is the Hubble parameter, ρ_m and p_m are the energy density and pressure of the matter inside the universe, ρ_T and p_T are the torsion contributions to the energy density and pressure. Also f_T and f_{TT} denotes one and two times derivatives with respect to the torsion scalar T .

The energy conservation laws are

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0 \quad (7)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0 \quad (8)$$

Using (4) & (5), the equation of state (EOS) due to torsion contribution is defined by

$$\omega_T = -1 + \frac{4\dot{H}(2Tf_{TT} + f_T - 1)}{2Tf_T - f - T} \quad (9)$$

The scale factor $a(t)$ is represented by [21]

$$a(t) = a_o(t_s - t)^{-h}, t \le t_s, h > 0 \quad (10)$$

From (5) & (9) one can write

$$H = \frac{h}{t_s - t} \quad (11)$$

$\dot{H} = \frac{H^2}{h} = \frac{h}{(t_s - t)^2}$ (12)

$T = -6\left(\frac{h}{t_s - t}\right)^2$ (13)

From the equations (10) & (11), the scale factor $a(t)$ can be rewritten as

$a(t) = a_o\left(\frac{H}{h}\right)^h$ (14)

From the equation (12), we see that $\dot{H} > 0$ which represent a supper accelerated FRW universe with a big rip singularity at $t = t_s$. Also from the equation (13), we see that

When $t = t_s$, $T = -\infty$ (15)

III. POLYTROPIC $f(T)$ GRAVITY MODEL

The equation of state (EOS) of the polytropic gas is given by [20]

$p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}}$ (16)

Wherep_Λ,ρ_Λ,k,and n are the pressure, energy density, polytropic constant and polytropic index respectively.

Using the EOS (16) into the conservation equation (7) and integrating we get

$\rho_\Lambda = \left[-k + Ba^{3/n}\right]^{-n}$ (17)

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe [13]

From (14) and (17) one can obtain

$\rho_\Lambda = \left[-K + \alpha(-6H^2)^{\frac{3h}{2n}}\right]^{-n}$ (18)

Where $\alpha = Ba_0^{\frac{3}{n}}(-6H^2)^{\frac{-3h}{2n}}$ (19)

Equating (4) and (18) we get

$(-12H^2 f_T - f + 6H^2) - \left[-K + \alpha(-6H^2)^{\frac{3h}{2n}}\right]^{-n} = 0$ (20)

Using the solution of (20) into (9), the EOS of parameter of torsion contribution is given by [20]

$\omega_T = -1 - \frac{1}{\frac{K}{B}\left[a_0\left(\frac{H}{h}\right)^h\right]^{\frac{-3}{n}} - 1}$, h>0 (21)

Equation (12) can be rewritten as

$\frac{\dot{H}}{H} = \frac{H}{h}$ (22)

Using (22) in (21) we get

$\omega_T = -1 - \frac{1}{\frac{K}{B}\left[a_0\left(\frac{\dot{H}}{H}\right)^h\right]^{\frac{-3}{n}} - 1}$, h>0 (23)

When $\frac{K}{B}\left[a_0\left(\frac{\dot{H}}{H}\right)^h\right]^{\frac{-3}{n}} > 1$ then from (23) we see that $\omega_T < -1$ which represents a phantom like accelerated universe.

IV CONCLUSION

We see from above discussion that the equation of state of the $f(T)$ gravity model in the context of polytropic gas represent a phantom like accelerated universe.

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