OPTIMAL SOLUTION OF A TRIANGULAR INTUITIONISTIC FUZZY TRANSPORTATION PROBLEM BY USING NEW RANKING FUNCTION

R.Santhi[1],E. Kungumaraj[2]
[1]Department of Mathematics, NGM College, Pollachi, Tamilnadu 642001, India
[2]Department of Mathematics, Sri Krishna Arts & Science College, Coimbatore, Tamilnadu, India

Abstract:

This paper presents the method to derive the minimum transportation cost of certain products when the supply and demand are symbolized as intuitionistic fuzzy number. By using the new ranking method, a new procedure is deciphered to find the minimum transportation cost for Intuitionistic fuzzy transportation problem. In addition to this, numerical illustration is also provided for the clear understanding.

1. Introduction:

The transportation problem is one of the sub-classes of Linear Programming Problems in which the objective is to transport various quantities of a single homogeneous commodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum. To achieve this objective we must know the amount and location of available supplies and the quantities demanded. And also the costs that result from transporting one unit of commodity from various origins to various destinations should also be known.

Intuitionistic fuzzy set is a tool in modelling real life problems like sale analysis, new product marketing, financial services, negotiation process, psychological investigations etc.,. Also it has greater influence in solving transportation problem to find the optimal solution in which the cost, supply and demand are Intuitionistic fuzzy numbers. The concept of intuitionistic fuzzy sets was presented by Atanassov which was established to be extremely useful to deal with vagueness. The intuitionistic fuzzy set separates the degree of membership with degree of non-membership of an element in the set. This is the main advantage of the intuitionistic fuzzy set.

Paul et al. [7] proposed a new method for solving transportation problem using triangular intuitionistic fuzzy number. Gani et al. [10] introduced revised distribution method for solving intuitionistic fuzzy transportation problem. Geetharamani et al., [2] introduced an innovative method to solve fuzzy transportation problem via. Robust ranking. SenthilKumar et al., [13] gave systematic approach for solving mixed intuitionistic fuzzy transportation problem. Gani et al., [7] proposed a new method for solving intuitionistic fuzzy transportation problem. Hussain et al., [5] discussed algorithmic approach for solving intuitionistic fuzzy transportation problem. Recently in 2014, Pramila and Uthra [11] have discussed the intuitionistic fuzzy transportation problem by using accuracy function to defuzzy the Intuitionistic fuzzy number. In 2017, S.K Bharati[1] introduced the new ranking method of

Intuitionistic fuzzy number. Based on this ranking method, in this paper, we discussed the new method to obtain the optimal transportation cost of Triangular Intuitionistic fuzzy transportation problem

2. Preliminaries

2.1 Fuzzy Set:

Let X be anonemptyset. A fuzzy set $\bar{A}$ of X is defined as $\bar{A} = \{(x, \mu_{\bar{A}}(x)); x \in X\}$ where $\mu_{\bar{A}}(x)$ is called the membership function which maps each element of X to a value between 0 and 1.

2.2 Fuzzy Number:

A fuzzy number $\bar{A}$ is a convex normalized fuzzy set on the real line $\mathbb{R}$ such that:

(i) There exist at least one $x \in \mathbb{R}$ with$\mu_{\bar{A}}(x) = 1$;

(ii) $\mu_{\bar{A}}(x)$is piecewise continuous.

2. 3 Triangular Fuzzy Number:

A fuzzy number $\bar{A}$ is denoted by 3 – tuples (a_1, a_2, a_3)wherea_1, a_2 *and* a_3 are real numbers and $a_1 \leq a_2 \leq a_3$ with the membership function defined as

$$\mu_{\bar{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

2. 4 Intuitionistic Fuzzy Set :

Let X be a nonempty set. An intuitionistic fuzzy set $\bar{A}^I$ of X is defined as

$\bar{A} = \{(x, \mu_{\bar{A}^I}(x), \gamma_{\bar{A}^I}(x)); x \in X\}$ where $\mu_{\bar{A}^I}(x)$ and $\gamma_{\bar{A}^I}(x)$ are membership and nonmembership functions such that $\mu_{\bar{A}^I}(x), \gamma_{\bar{A}^I}(x) : X \rightarrow [0, 1]$ and

$0 \leq \mu_{\bar{A}^I}(x) + \gamma_{\bar{A}^I}(x) \leq 1$ for all x ∈ X.

2. 5 Intuitionistic Fuzzy Number:

An intuitionistic fuzzy subset $\bar{A}^I = \{(x, \mu_{\bar{A}^I}(x), \gamma_{\bar{A}^I}(x)); x \in A\}$ of the real line $\mathbb{R}$is called an intuitionistic fuzzy number (IFN) if the following conditions hold:

(i) There exists $x \in \mathbb{R}$such that $\mu_{\bar{A}^I}(x) = 1$ and $\gamma_{\bar{A}^I}(x) = 0$.

(ii) $\mu_{\bar{A}^I}(x)$is a continuous function from $\mathbb{R} \rightarrow [0, 1]$ such that $0 \leq \mu_{\bar{A}^I}(x) + \gamma_{\bar{A}^I}(x) \leq 1$for all $x \in X$.

2. 6 Triangular Intuitionistic Fuzzy Number: A triangular Intuitionistic fuzzy Number$\bar{A}^I$ is denoted by $\bar{A}^I = (a_1, a_2, a_3;\ a_1', a_2, a_3')$where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ with the following membership function $\mu_{\bar{A}^I}(x)$ and non-membership function $\gamma_{\bar{A}^I}(x)$

$$\mu_{\bar{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & for\ a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2} & for\ a_2\le x\le a_3\\ 0 & otherwise\end{cases}$$

$$\gamma_{\bar{A}^I}(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1{}'} & for\ a_1{}'\le x\le a_2\\ \dfrac{x-a_2}{a_3{}'-a_2} & for\ a_2\le x\le a_3{}'\\ 1 & otherwise\end{cases}$$

2. 7 Operations on Triangular Intuitionistic Fuzzy Numbers:

Let a = $(a_1, a_2, a_3;\ a_1', a_2, a_3')$and b = $(b_1, b_2, b_3;\ b_1', b_2, b_3')$be two triangular fuzzy numbers then the arithmetic operations on a andb as follows.

(i) Addition: a + b = $(a_1+b_1, a_2+b_2, a_3+b_3; a_1'+b_1', a_2+b_2, a_3'+b_3')$
(ii) Subtraction: a − b = $(a_1-b_3, a_2-b_2, a_3-b_1; a_1'-b_3', a_2-b_2, a_3'-b_1')$

(iii)Multiplication:a.b = $(a_1.b_1, a_2.b_2, a_3.b_3; a_1'.b_1', a_2.b_2, a_3'.b_3')$

(iv) Scalar Multiplication:k$(a_1, a_2, a_3;\ a_1', a_2, a_3') = (ka_1, ka_2, ka_3; ka_1', ka_2, ka_3')\ if\ k>0$

k$(a_1, a_2, a_3;\ a_1', a_2, a_3') = (ka_3, ka_2, ka_1; ka_3', ka_2, ka_1')\ if\ k<0$

2. 8 Defuzzification:

Let $\bar{A}^I = (a_1, a_2, a_3)(b_1, b_2, b_3)$ be triangular Intuitionistic fuzzy number. The ranking of a TIFN is given by

$$\Re(\bar{A}^I) = \left(\frac{a_3-a_1}{b_3-b_1}\right)\left(D^S(\bar{A}^I, \overline{O^I})\right)$$

Where $D^S(\bar{A}^I, \overline{O^I}) = \frac{(a_1+a_3+4a_2+b_1+b_3)}{8}$

3. Intuitionistic Fuzzy balanced Transportation problem:

Consider a transportation problem with m intuitionistic fuzzy(IF) origins and n IF destinations. Let c_{ij}(i=1,2,…,m, j=1,2,…,n) be the cost of transporting one unit of theproduct from ith origin to jth destination. Let $\bar{a_i}^I$ (i = 1, 2, …, m) be the quantity of commodity available at IF origin i. Let $\bar{b_j}^I$(j= 1, 2, …, n) is quantity of commodity needed of IF destination j. Let x_{ij}(i=1,2,…,m, j=1,2,…,n) is quantity transported from ith IF origin to jth destination.

Mathematical Model of Intuitionistic Fuzzy transportation is

Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} * x_{ij}$

Subject to$\sum_{j=1}^{n} x_{ij} = a_i$, i= 1, 2, . . . , m,

$\sum_{i=1}^{m} x_{ij} = b_j$, j = 1, 2, . . . , n,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Here x_{ij} is a non-negative triangular Intuitionistic fuzzy number, where
n= total number of sources
m= total number of destinations
a_i= the fuzzy availability of the product at i^{th} source
b_j= the fuzzy demand of the product at j^{th} destination
c_{ij}= the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th}destination
x_{ij}= the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost

$$\sum_{i=1}^{m} a_i = \text{total fuzzy availablity of the product}$$

$$\sum_{j=1}^{n} b_j = \text{total fuzzy demand of the product}$$

$\sum_{i=1}^{p} \sum_{j=1}^{q} c_{ij} * x_{ij}$ = total fuzzy transportation cost

4. New procedure for obtain the optimal solution

Step 1: In the IFTP defuzzify the quantities of the problem and if any of the values are not integers, round off into integers.
Step 2: Select the minimum odd cost from all cost in the matrix. Suppose all the costs are even, multiply each column by 1/2.
Step 3: Subtract selected least odd cost only from odd cost in the matrix. Now there will be at least one zero and remaining all cost become even.
Step 4: Allocate minimum of supply / demand at the place of zeros.
Step 5: After the allotment, multiply each column by 1/2.
Step 6: Again select minimum odd cost in the remaining column except zeros in that column.
Step 7: Go to step 3 and repeat step 4 and 5 till optimal solution are obtained.
Step 8: Finally total minimum cost is calculated as sum of the product of the cost and corresponding allocated value of supply/demand.

5. Numerical Example

Consider an Intuitionistic fuzzy transportation problem whose cost, supply and demand values are triangular intuitionistic fuzzy number

	I	II	III	IV	Supply
A	(14, 16, 18 ; 12, 16, 20)	(0, 1, 2 ; -1, 1, 3)	(7,8, 9 ; 6, 8, 10)	(11, 13, 15 ; 10, 13, 16)	(4, 8, 12; 2, 8, 14)
B	(8, 11, 14 ; 7, 11, 15)	(3, 4, 5 ; 2, 4, 6)	(5, 7, 9 ; 4, 7, 10)	(8, 10, 12 ; 6, 10, 14)	(10, 12, 14; 8, 12, 16)

C	(6, 8, 10 ; 5, 8, 11)	(13, 15, 17 ; 12, 15, 18)	(7, 9,11 ; 6, 9, 12)	(1, 2, 3 ; 0, 2, 4)	(14, 16, 18; 12, 16, 20)
D	(5, 6, 7 ; 4, 6, 8)	(11, 12, 13 ; 10, 12, 14)	(3, 5, 7 ; 1, 5, 9)	(12, 14, 16 ; 11, 14, 17)	(16, 20, 24; 12, 20, 28)
Demand	(4, 8, 12; 2, 8, 14)	(4, 8, 12; 2, 8, 14)	(4, 8, 12; 2, 8, 14)	(4, 8, 12; 2, 8, 14)	

By using the new ranking function the above IFTN can be converted into the crisp values

$$x_{11} = (14,16,18; 12,16,20)$$

By the given ranking function

$$\Re(\bar{A}^I) = \left(\frac{a_3 - a_1}{b_3 - b_1}\right)\left(D^S(\bar{A}^I, \overline{O^I})\right)$$

Where $D^S(\bar{A}^I, \overline{O^I}) = \frac{(a_1+a_3+4a_2+b_1+b_3)}{8}$

$$\Rightarrow D^S(\bar{A}^I, \overline{O^I}) = \frac{14 + 18 + 4(16) + 12 + 20}{8} = \frac{128}{8} = 16$$

$$and \left(\frac{a_3 - a_1}{b_3 - b_1}\right) = \left(\frac{18 - 14}{20 - 12}\right) = \frac{4}{8} = \frac{1}{2}$$

$$\therefore \Re(\bar{A}^I) = \left(\frac{a_3 - a_1}{b_3 - b_1}\right)\left(D^S(\bar{A}^I, \bar{O}^I)\right) = \frac{1}{2}(16) = 8$$

Similarly by using the given ranking function we formulate the crisp table which is given below

	D_1	D_2	D_3	D_4	Supply
A	8	1	4	8	4
B	8	2	4	4	6
C	4	10	6	1	8
D	2	6	2	8	10
Demand	4	5	12	7	

Since the minimum odd cost in the odd matrix is 1, subtract 1 from all the odd costs and allocate minimum of supply or demand to the cell where there are zero cost then delete the row or column.

	D_1	D_2	D_3	D_4	Supply
A	8	0(4)	4	8	**4**
B	8	2	4	4	**6**
C	4	10	6	0(7)	**8**
D	2	6	2	8	**10**
Demand	**4**	**5**	**12**	**7**	

Now all the cost is even, hence multiply all the cost by ½ and subtract the minimum odd cost from all the odd cost.

	D_1	D_2	D_3	Supply
$\boldsymbol{O_2}$	4	1	2	**6**
$\boldsymbol{O_3}$	2	5	3	**1**
$\boldsymbol{O_4}$	1	3	1	**10**
Demand	**4**	**5**	**12**	

	D_1	D_2	D_3	Supply

O_2	4	0	2	**6**
O_3	2	4	2	**1**
O_4	0	3	0	**10**
Demand	**4**	**5**	**12**	

Proceeding like this, we get

	D_1	D_2	D_3	D_4	Supply
A	8	1(4)	4	8	4
B	8	2(1)	4(5)	4	6
C	4	10	6(1)	1(7)	8
D	2(4)	6	2(6)	8	10
Demand	4	5	12	7	

The minimum transportation cost associated with this solution is

Z = (4x1)+(2x1)+(4x5)+(6x1)+(1x7)+(2x4)+(2x6)

= 4+2+20+6+7+8+12

= 59

6. Conclusion:

In this paper, the transportation costs are considered as imprecise numbers by intuitionistic fuzzy numbers which are more realistic and general in nature. More over Intuitionistic fuzzy transportation problem of triangular numbers has been transformed into crisp transportation problem using the new ranking method of Intuitionistic fuzzy numbers. Numerical example shows that, this method is easy to solve the intuitionistic fuzzy transportation problems and to obtain the optimal (minimum) cost of the transportation problem compared with existing methods.

References:

[1]. S.K. Bharati, Ranking method of Intuitionistic Fuzzy Numbers, Global Journal of Pure and Applied Mathematics, ISSN 0973-1768 Volume 13, N0 9(2017), 4595-4608

[2]. G.Geetharamani and S.Devi, An innovative method for solving fuzzy transportation problem, *Indian Journal of Applied Research*, 4 (2014) 399-402.

[3]. D.Ghosh and A.Pal, Use of fuzzy relational maps and intuitionistic fuzzy sets to analyze health problem of agricultural labourers, *Annals of Pure and Applied Mathematics*, 5(1) (2013) 1-10.

[4]. M.A.Hakim, An alternative method to find initial basic feasible solution of a transportation problem, *Annals of Pure and Applied Mathematics*, 1(2) (2012) 203-209.

[5]. R.J.Hussain and P.Senthil Kumar, Algorithm approach for solving intuitionistic fuzzy transportation problem, *Applied Mathematical Sciences*, 6 (80) (2012) 3981- 3989.

[6]. P.Jayaraman and R.Jahirhussian, Fuzzy optimal transportation problems by improved zero suffix method via robust rank techniques, *International Journal of Fuzzy Mathematics and Systems*, 3(4) (2013) 303-311.

[7]. R.John Paul Antony, S.JohnsonSavarimuthu and T.Pathinathan, Method for solving the transportation problem using triangular intuitionistic fuzzy number, *International Journal of Computing Algorithm*, 3 (2014) 590-605.

[8].A.NagoorGani and S.Abbas, Solving intuitionistic fuzzy transportation problem using zero suffix algorithm, *IJMSEA*, 6(III) (2012) 73-82.

[9]. A.NagoorGani and S.Abbas, A new method for solving intuitionistic fuzzy transportation problem, *Applied Mathematical Sciences*, 7(28) (2013) 1357 – 1365.

[10]. A.NagoorGani and S.Abbas, Revised distribution method for intuitionistic fuzzy transportation problem, *International Journal of Fuzzy Mathematical Archive*, 2 (2014) 96-103.

[11]. K. Pramila and G. Uthra, Optimal solution of an Intuitionistic fuzzy transportation problem, Annals of Pure and Applied Mathematics, Vol.8, No.2, (2014), 67-73

[12]. R. Santhi and E. Kungumaraj, Maximization Transportation Problem by Using Intuitionistic Fuzzy Triangular Number, International Journal of Trend in Research and Development, Volume 4(5), ISSN: 2394-9333

[13]. P.Senthil Kumar and R.JahirHussain, A systematic approach for solving mixed intuitionistic fuzzy transportation problems, *International Journal of Pure and Applied Mathematics*, 92(2) (2014) 181-190.

[14]. P.Senthil Kumar and R.JahirHussain, A method for solving balanced intuitionistic fuzzy assignment problem, *International Journal of Engineering Research and Applications*, 4(3) (2014) 897-903.