

# PROPERTIES OF COMBINATION USING SUBFACTORIAL

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## Abstract

In this paper it is an attempt to derive some combination formulas using the ideas of subfactorial

Using this concept I have derived and proved the results of combination which and been already proved.

## Keywords:

Combination, permutation, factorial , subfactorial

## Introduction

The idea of this journal was highlighted on solving a mathematical puzzle where I firstly came to know about the notation subfactorial. I already derived the properties of permutation and in this paper I decided to solve the properties of combination using subfactorial. Here in this paper I solved the properties of combinations

## Definition of subfactorial

$!n$  is the number of dearrangement of  $n$  objects. That is the number of permutation of  $n$  objects in order that no object stands in its original position

## Relation between factorial and subfactorial

$$n! = !n + 1$$

we know that

$$n! = 1 \quad n = 0$$

$$= n(n-1)! \quad n > 0$$

$$!n = 1 \quad n = 0$$

$$= (n-1)(n-1)! \quad n > 0$$

**Result 1**

$$!(n+1) = n !n$$

$$!n = (n-1)!(n-1)$$

$$= (n-1)(n-2)(n-3)!(n-3)$$

**Example:**

$$!5 = 4!4$$

$$= 4*3!3$$

$$= 4*3*2!2$$

$$= 4*3*2*1!1$$

$$= 4*3*2*1*1$$

$$= 24$$

**Result 2**

$$nC_r = \frac{!(n+1)}{!(n-r+1)!(r+1)}$$

**Result 3**

$$nC_0 = 1$$

**Proof:**

$$nC_0 = \frac{!(n+1)}{!(n-0+1)!(0+1)}$$

$$= \frac{!(n+1)}{!(n+1)!1}$$

$$= 1$$

**Example**

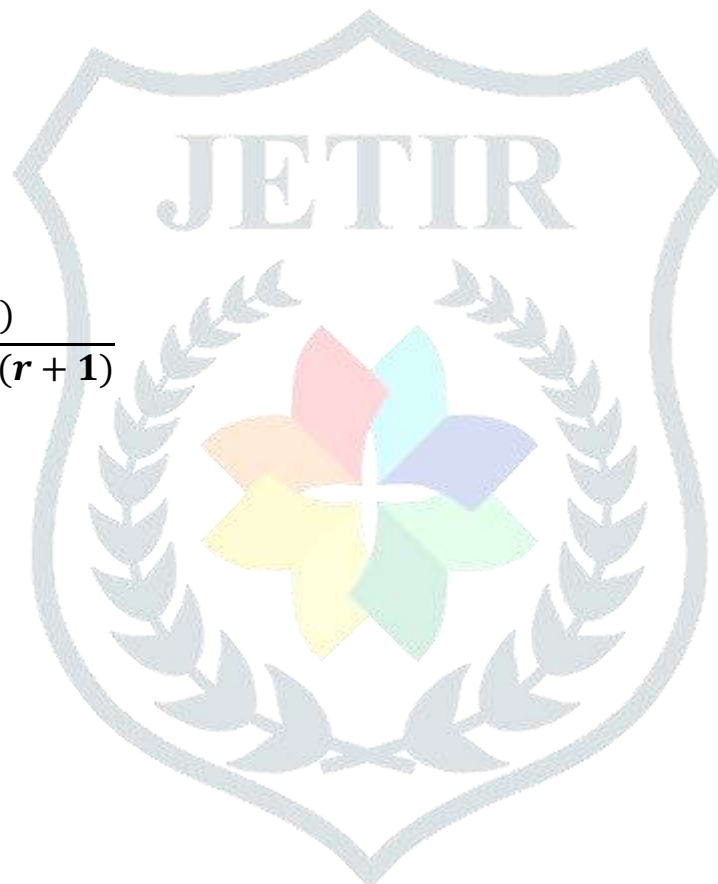
$$5C_0 = \frac{!(6)}{!(6)!(0+1)}$$

$$= \frac{1}{!1}$$

$$= 1$$

**Result 4**

$$nC_n = 1$$

**Proof:**

$$\begin{aligned}
 nC_n &= \frac{!(n+1)}{!(n-n+1)!(n+1)} \\
 &= \frac{!(n+1)}{!(1)!(n+1)} \\
 &= 1
 \end{aligned}$$

**Example**

$$\begin{aligned}
 5C_5 &= \frac{!(5+1)}{!(5-5+1)!(5+1)} \\
 &= \frac{!6}{!1!6} \\
 &= 1
 \end{aligned}$$

**Result 5**

$$nC_r = \frac{np_r}{!(r+1)}$$

**Proof:**

$$\begin{aligned}
 nC_r &= \frac{!(n+1)}{!(n-r+1)!(r+1)} \\
 &= \frac{!(n+1)}{!(n-r+1)} * \frac{1}{!(r+1)} \\
 &= \frac{np_r}{!(r+1)}
 \end{aligned}$$

**Example:**

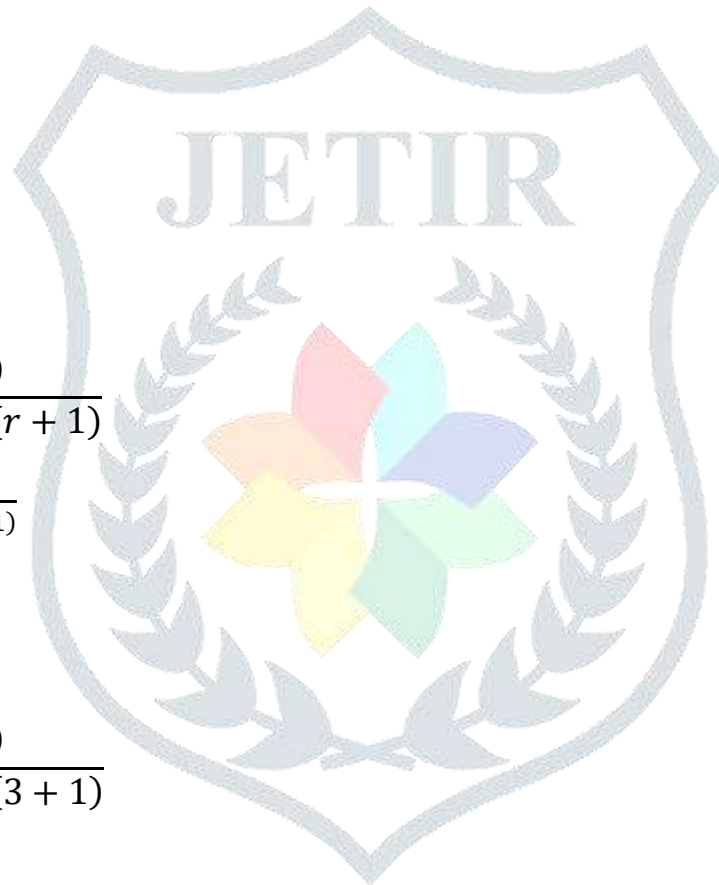
$$\begin{aligned}
 5C_3 &= \frac{!(5+1)}{!(5-3+1)!(3+1)} \\
 &= \frac{!6}{!3!4} \\
 &= \frac{!6}{!3} * \frac{1}{!4} \\
 &= \frac{5p_3}{!(3+1)}
 \end{aligned}$$

**Result 6**

$$nC_r + nC_{r-1} = (n+1)C_r$$

**Proof:**

$$nC_r + nC_{r-1} = \frac{!(n+1)}{!(n-r+1)!(r+1)} + \frac{!(n+1)}{!(n-r+1+1)!(r-1+1)}$$



$$\begin{aligned}
&=!(n+1)\left[\frac{1}{!(n-r+1)!(r+1)}+\frac{1}{(n-r+1)!(n-r+1)!(r)}\right] \\
&=!(n+1)\left[\frac{1}{!(n-r+1)r!(r)}+\frac{1}{(n-r+1)!(n-r+1)!(r)}\right] \\
&=!(n+1)\frac{n-r+1+r}{(n-r+1)!(n-r+1)r!(r)} \\
&=!(n+1)\frac{n+1}{(n-r+1)!(n-r+1)r!(r)} \\
&=\frac{!(n+2)}{!(n-r+2)!(r+1)} \\
&=n+1c_r
\end{aligned}$$

**Result 7**

$$nC_r = \frac{n}{r}(n-1)c_{r-1}$$

**Proof:**

$$\begin{aligned}
\frac{n}{r}(n-1)c_{r-1} &= \frac{n}{r} \frac{!(n-1+1)}{!(n-1-r+1+1)!(r-1+1)} \\
&= \frac{n}{r} \frac{!(n)}{!(n-r+1)!(r)} \\
&= \frac{!(n+1)}{!(n-r+1)!(r+1)} \\
&= nC_r
\end{aligned}$$

**Result 8**

$$nC_r = \frac{n(n-1)}{r(r-1)}(n-2)c_{r-2}$$

**Proof:**

$$\begin{aligned}
\frac{n(n-1)}{r(r-1)}(n-2)c_{r-2} &= \frac{n(n-1)}{r(r-1)} \frac{!(n-2+1)}{!(n-2-r+2+1)!(r-2+1)} \\
&= \frac{n(n-1)}{r(r-1)} \frac{!(n-1)}{!(n-r+1)!(r-1)} \\
&= \frac{n!n}{r!r!(n-r+1)} \\
&= \frac{!(n+1)}{!(n-r+1)!(r+1)} \\
&= nC_r
\end{aligned}$$

**Conclusion**

In this paper I have derived some properties of combination using the idea called subfactorial. And in future I decided to solve few more formulas using this fact subfactorial

References:

Gunasundari G “properties of permutation using subfactorial” Vidhyawarta international multilingual research journal, April 2018, issue 48, vol-01 ISSN 2319 9318

