PROPERTIES OF COMBINATION USING SUBFACTORIAL

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Abstract

In this paper it is an attempt to derive some combination formulas using the ideas of subfactorial

Using this concept I have derived and proved the results of combination which and been already proved.

Keywords:

Combination, permutation, factorial , subfactorial

Introduction

The idea of this journal was highlighted on solving a mathematical puzzle where I firstly came to know about the notation subfactorial. I already derived the properties of permutation and in this paper I decided to solve the properties of combination using subfactorial. Here in this paper I solved the properties of combinations

Definition of subfactorial

!n is the number of dearrangement of n objects. That is the number of permutation of n objects in order that no object stands in its original position

Relation between factorial and subfactorial

n!=!(n+1)

we know that

$$n! = \begin{cases} 1 & n = 0 \\ n(n-1)! & n > 0 \end{cases}$$

$$!n = \begin{cases} 1 & n = 0 \\ (n-1)(n-1)! & n > 0 \end{cases}$$

Result 1

!(n+1)= n !n

!n=(n-1)!(n-1)

=(n-1)(n-2)(n-3)!(n-3)

Example:

!5=4!4

=4*3!3

=4*3*2!2

=4*3*2*1!1

=4*3*2*1*1

=24

Result 2

$$\boldsymbol{nc_r} = \frac{!\,(\boldsymbol{n}+\mathbf{1})}{!\,(\boldsymbol{n}-\boldsymbol{r}+\mathbf{1})!\,(\boldsymbol{r}+\mathbf{1})}$$

Result 3

$\mathrm{n}c_0 = 1$

Proof:

$$\mathrm{n}c_0 = \frac{!(n+1)}{!(n-0+1)!(0+1)}$$

$$= \frac{!(n+1)}{!(n+1)!1}$$

$=1$

Example

$$5c_0 = \frac{!(6)}{!(6)!(0+1)}$$

$$= \frac{1}{!1}$$

$=1$

Result 4

$nc_n = 1$

Proof:

$$nc_n = \frac{!\,(n+1)}{!\,(n-n+1)!\,(n+1)}$$

$$=\frac{!(n+1)}{!(1)!(n+1)}$$

$$=1$$

Example

$$5c_5 = \frac{!\,(5+1)}{!\,(5-5+1)!\,(5+1)}$$

$$= \frac{!6}{!1!6}$$

$$=1$$

Result 5

$$nc_r = \frac{np_r}{!\,(r+1)}$$

Proof:

$$nc_r = \frac{!\,(n+1)}{!\,(n-r+1)!\,(r+1)}$$

$$=\frac{!(n+1)}{!(n-r+1)} * \frac{1}{!(r+1)}$$

$$=\frac{np_r}{!(r+1)}$$

Example:

$$5c_3 = \frac{!\,(5+1)}{!\,(5-3+1)!\,(3+1)}$$

$$=\frac{!6}{!3!4}$$

$$=\frac{!6}{!3} * \frac{1}{!4}$$

$$= \frac{5p_3}{!(3+1)}$$

Result 6

$$nc_r + nc_{r-1} = (n+1)c_r$$

Proof:

$$nc_r + nc_{r-1} = \frac{!\,(n+1)}{!\,(n-r+1)!\,(r+1)} + \frac{!\,(n+1)}{!\,(n-r+1+1)!\,(r-1+1)}$$

$$=!\,(n+1)\left[\frac{1}{!(n-r+1)!(r+1)}+\frac{1}{(n-r+1)!(n-r+1)!(r)}\right]$$

$$=\,!(\text{n+1})\left[\frac{1}{!(n-r+1)r!(r)}+\frac{1}{(n-r+1)!(n-r+1)!(r)}\right]$$

$$=\quad !(\text{n+1})\frac{n-r+1+r}{(n-r+1)!(n-r+1)r!(r)}$$

$$=\,!(\text{n+1})\frac{n+1}{(n-r+1)!(n-r+1)r!(r)}$$

$$=\frac{!(n+2)}{!(n-r+2)!(r+1)}$$

$$=\text{n+1}c_r$$

Result 7

$$nc_r=\frac{n}{r}(n-1)c_{r-1}$$

Proof:

$$\frac{n}{r}(n-1)c_{r-1}=\frac{n}{r}\frac{!\,(n-1+1)}{!\,(n-1-r+1+1)!\,(r-1+1)}$$

$$=\frac{n}{r}\frac{!(n)}{!(n-r+1)!(r)}$$

$$=\frac{!(n+1)}{!(n-r+1)!(r+1)}$$

$$=\text{n}c_r$$

Result 8

$$nc_r=\frac{n(n-1)}{r(r-1)}(n-2)c_{r-2}$$

Proof:

$$\frac{n(n-1)}{r(r-1)}(n-2)c_{r-2}=\frac{n(n-1)}{r(r-1)}\frac{!\,(n-2+1)}{!\,(n-2-r+2+1)!\,(r-2+1)}$$

$$=\frac{n(n-1)}{r(r-1)}\frac{!(n-1)}{!(n-r+1)!(r-1)}$$

$$=\frac{n!n}{r!r!(n-r+1)}$$

$$=\frac{!(n+1)}{!(n-r+1)!(r+1)}$$

$$=\text{n}c_r$$

Conclusion

In this paper I have derived some properties of combination using the idea called subfactorial. And in future I decided to solve few more formulas using this fact subfactorial

References:

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