A STUDY OF LINEAR AND NON LINEAR DIFFUSION EQUATIONS ARISING IN FLUID FLOW THROUGH POROUS MEDIA BY HOMOTOPY PERTURBATION METHOD

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ABSTRACT:

In this present paper, The approximate solution of imbibition phenomenon governed by linear and non-linear partial differential equation is discussed. When water is injected in the injection well for recovering the reaming oil after primary oil recovery process, it comes to contact with the native oil and at that time the imbibition phenomenon occurs due to different viscosity. The Saturation of injected water is calculated by homotopy perturbation method for Linear and Non linear differential equation of Imbibition phenomena under assumption that Saturation is decomposed in to saturation of different levels. The obtained results as compared with previous works are highly accurate. For the mathematical modeling, we consider the homogeneous porous medium and homotopy preturbation method has been used to solve the partial differential equation governed by it. The graphical representation of the solution is given by Met lab and physically interpreted.

KEY WORDS:

Imbibition Phenomenon, Linear equation, Non-linear diffusion equation, Homotopy Perturbation method.

INTRODUCTION:

This paper discusses mathematically phenomenon of imbibition in double phase flow of two immiscible fluids in a homogeneous porous media with capillary pressure. It is well known that when a porous medium of length (L), field with some fluid (N) is brought in to contact with another fluid (I) preferentially wets the medium, it is observe that there is a spontaneous flow of the wetting fluid into the medium and a counter of the resident fluid from the medium. The phenomenon is called Imbibition and has been discussed by many authors from different points.[8][10]

One of the most important process in oil recovery is the spontaneous imbibition which is driven by capillary force. Such spontaneous imbibition may occur in the form of co-current imbibition or counter current imbibition. The direction of flow is the main difference between these two crucial mechanisms for imbibition. In co-current imbibition, the wetting and non-wetting phases flow in the same direction with the non-wetting phase being pushed out ahead of the wetting phase. In counter current imbibition, the wetting and non-wetting phases flow in the opposite directions.[10]
In the present paper, we have discussed the imbibition phenomenon arising in the flow of two immiscible fluid flows through homogeneous porous media with the effect of capillary pressure and obtained an approximate solution of the nonlinear differential system governing imbibition phenomena through Homotopy perturbation method.

FORMULATION OF THE PROBLEM :

Assuming that Darcy's Law, equation of continuity, imbibition condition $V_i = -V_n$ [9], are valid in investigated phenomenon for both fluids I and N respectively. The combination of these equation and

condition yields a partial differential equation of imbibition of two immiscible fluids in homogeneous porous media.
By substituting values of fictitious relative permeability,

$$K_i = S_i^{\,3}, K_n = S_n = (1 - \alpha S_i) \text{ where } \alpha = 1.11 \tag{1.1}$$

$$P_c = \beta(S_i^{-1} - C) \tag{1.2}$$

In the equation of imbibition phenomenon, we get

$$P\frac{\partial S_i}{\partial t} - \frac{1}{S_i}\frac{\partial}{\partial X}\left[\frac{K(1-\alpha S_i)S_i^{\,3}}{(1-\alpha S_i+mS_i^{\,3})}\frac{\beta}{S_i^{\,2}}\frac{\partial S_i}{\partial X}\right] = 0 \tag{1.3}$$

Where P = porosity of medium , K is permeability of medium, s_i and s_n are saturation of fluids I and N respectively, P_c is capillary pressure , β is capillary pressure coefficient, $m = \frac{s_n}{s_i}$.

Since from experimental results of Egenev's [2], we may consider $mS_i^{\,3}$ small enough to be neglected from (1.3). The equation (1.3) reduce to

$$P\frac{\partial S_i}{\partial t} - \frac{K\beta}{S_i}\frac{\partial}{\partial t}\left[S_i\frac{\partial S_i}{\partial t}\right] = 0 \tag{1.4}$$

Selecting new variables

$$\mathrm{X} = \frac{x}{L} \quad \text{and} \quad T = \frac{K\beta}{S_i P L^2}t$$

Equation (1.4) becomes

$$\frac{\partial S_i}{\partial T} = \frac{\partial}{\partial X}\left[D(S_i)\frac{\partial S_i}{\partial X}\right][2] \tag{1.5}$$

Where $D(S_i) = \alpha$

Equation (1.5) is a linear diffusion equation for the saturation of an injected fluid.

HOMOTOPY PERTURBATION METHOD (HPM):

To explain this method let us consider the following function:

$$A(u) - f(r) = 0, \qquad r \in \Omega \tag{2.1}$$

With boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad \mathrm{r} \subset \tau \tag{2.2}$$

Where A is a general differential operator,B is a boundary operator, $f(r)$ is a known analytic function, and Ω the boundary of the domain.

The operator A can be generally divided into linear and nonlinear parts, say L(u) and N(u). so Eq (2.1) can be written as

$$L(u) + N(u) - f(r) = 0 \tag{2.3}$$

He's constructed a homotopy $v(r,p): \Omega \times [0,1] \to R$ which satisfies :

$$H(v,p) = (1-p)[L(v) - L(v_0)] + p[A(v) - f(r)] = 0 \tag{2.4}$$

Where $r \in \Omega$, $p \epsilon [0,1]$ that is called homotopy parameter, and v_0 is an initial approximation of (2.1) which satisfies the boundary conditions from equation (2.4) we will have,

$$H(v,0) = L(v) - L(v_0) = 0$$

(2.5)

And

$$H(v,1) = [A(v) - f(r)] = 0 \tag{2.6}$$

In topology, $L(v) - L(v_0)$ is called deformation, and $[A(v) - f(r)]$ is called Homotopic. The embedding parameter p monotonically increases from zero to unit as the trivial problem $H(v,0) = 0$ in (2.5) is continuously deforms the original problem in (2.6), $H(v,1) = 0$. The embedding parameter $p \epsilon$ $[0,1]$ can be considered as an expanding parameter. Apply the perturbation technique due to the fact that $0 \le p \le 1$, can be considered as a small parameter, the solution of (2.1) and (2.2) can be assumed as a series in p,

As follows,

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \cdots \ldots \ldots \ldots \ldots \tag{2.7}$$

Setting $p = 1$ yields in the approximate solution of equation (2.7)

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots \ldots \ldots \ldots \tag{2.8}$$

Equation (2.8) is the solution of equation (2.1) obtained by Homotopy perturbation method [4].

(I) SOLUTION OF LINEAR EQUATION BY HPM :

The linear equation (1.5) which we have got,

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \quad ; 0 \le x \le 1 \ , t \ge 0 \tag{2.9}$$

with initial condition

$$u(x,0) = \sin(2\pi . x) \tag{2.10}$$

and boundary condition is

$$u(0,t) = u(1,t) = 0 \tag{2.11}$$

To solve this problem we use Homotopy perturbation method.

Homotopy $v(r,p)$: $\Omega \times [0,1] \rightarrow R$ for equation (1.5) is define as

$$(1-p)[\frac{\partial v}{\partial t} - \frac{\partial u_0}{\partial t}] + p\left[\frac{\partial v}{\partial t} - \alpha \frac{\partial^2 v}{\partial x^2}\right] = 0 \tag{2.12}$$

$$\frac{\partial v}{\partial t} - \alpha p \frac{\partial^2 v}{\partial x^2} = 0$$

$$\frac{\partial (v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \ldots\ldots)}{\partial t} - \alpha p \frac{\partial^2 (v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \ldots\ldots)}{\partial x^2} = 0 \tag{2.13}$$

For zeroth order of p :

$$\frac{\partial v_0}{\partial t} = 0 \tag{2.14}$$

Then $v_0(x,t) = \sin(2\pi.x)$

For first order of p :

$$\frac{\partial v_1}{\partial t} - \alpha \frac{\partial^2 v_0}{\partial x^2} = 0 \tag{2.15}$$

$$\frac{\partial v_1}{\partial t} + 4\,\mu^2 \alpha \sin(2\pi.x) = 0 \tag{2.16}$$

$$v_1\,(x,t) = sin(2\pi.x) - 4\,\pi^2 \alpha \sin(2\pi.x)\,t \tag{2.17}$$

For second order of :

$$\frac{\partial v_2}{\partial t} - \alpha \frac{\partial^2 v_1}{\partial x^2} = 0 \tag{2.18}$$

$$v_2\,(x,t) = sin(2\pi.x) - 4\,\pi^2 \alpha \sin(2\pi.x)\,t + 8\,\pi^4 \alpha^2 \sin(2\pi.x) t^2 \tag{2.19}$$

Using equation (2.12) for other order of , we can obtain the following results:

$$v(x,t) = \sin(2\pi.x)[1 - (4\,\pi^2 \alpha.t) + \frac{1}{2}\,(4\,\pi^2 \alpha.t)^2 - \ldots\ldots\ldots\ldots..] \tag{2.20}$$

It is obvious that $v(x,t)$ converges to the exact solution as increasing order of :

$$v(x,t) = \sin(2\pi.x).\exp(-4\,\pi^2 \alpha.t\,) \tag{2.21}$$

Equation (2.21) is the Homotopy solution of equation (2.9), the infinite series converges to exact solution of the linear diffusion equation.

(II) SOLUTION OF NON – LINEAR EQUATIONS BY HPM :

The nonlinear diffusion equation is the prominent example of porous medium equation.

Take $D(S_i) = S_i$ in equation (1.5) , we get

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(D(u)\frac{\partial u}{\partial x} \right) \tag{3.1}$$

Boundary conditions are given by

$$u(0,t) = 0 \quad \text{and} \quad u\left(\frac{\pi}{2}, t\right) = 1 \tag{3.2}$$

Initial condition is given by

$$u(x,0) = \sin x \tag{3.3}$$

Homotopy $v(r,p)$: $\Omega \times [0,1] \to R$ for equation (3.1) is define as

$$\mathrm{H(v,p)} = (1-p)\left[\frac{\partial v}{\partial t} - \frac{\partial v_o}{\partial t}\right] + p\left[\frac{\partial v}{\partial t} - v\frac{\partial^2 v}{\partial x^2} + \left(\frac{\partial v}{\partial x}\right)^2\right]$$

$$= (1-p)\left[\frac{\partial v_o}{\partial t} + p\frac{\partial v_1}{\partial t} + p^2\frac{\partial v_2}{\partial t} + p^3\frac{\partial v_3}{\partial t} + p^4\frac{\partial v_3}{\partial t} + \cdots - \frac{\partial v_o}{\partial t}\right]$$
$$+ p\left(\begin{array}{c} \left[\frac{\partial v_o}{\partial t} + p\frac{\partial v_1}{\partial t} + p^2\frac{\partial v_2}{\partial t} + p^3\frac{\partial v_3}{\partial t} + p^4\frac{\partial v_4}{\partial t} + \cdots \ldots\right] \\ -(v_0 + pv_1 + p^2 v_2 + p^3 v_3 + p^4 v_4 + \cdots) \\ \left(\frac{\partial^2 v_0}{\partial x^2} + p\frac{\partial^2 v_1}{\partial x^2} + p^2\frac{\partial^2 v_2}{\partial x^2} + p^3\frac{\partial^2 v_3}{\partial x^2} + p^4\frac{\partial^2 v_4}{\partial x^2} + \cdots\right) \\ + \left(\frac{\partial v_o}{\partial x} + p\frac{\partial v_1}{\partial x} + p^2\frac{\partial v_2}{\partial x} + p^3\frac{\partial v_3}{\partial x} + p^4\frac{\partial v_4}{\partial x} + \cdots \ldots\right)^2 \end{array}\right) \tag{3.4}$$

Comparing powers of P ,

$$p^0 : \frac{\partial v_0}{\partial t} - \frac{\partial v_0}{\partial t} = 0$$

$$p^1 : \frac{\partial v_1}{\partial t} + \frac{\partial v_0}{\partial t} - v_0\left(\frac{\partial^2 v_0}{\partial x^2}\right) + \left(\frac{\partial v_0}{\partial x}\right)^2 = 0$$

$$p^2 : \frac{\partial v_2}{\partial t} - v_0\left(\frac{\partial^2 v_1}{\partial x^2}\right) - v_1\left(\frac{\partial^2 v_0}{\partial x^2}\right) + 2\left(\frac{\partial v_0}{\partial x}\right)\left(\frac{\partial v_1}{\partial x}\right) = 0$$

$$p^3 : \frac{\partial v_3}{\partial t} - v_0\left(\frac{\partial^2 v_2}{\partial x^2}\right) - v_1\left(\frac{\partial^2 v_1}{\partial x^2}\right) - v_2\left(\frac{\partial^2 v_0}{\partial x^2}\right) + \left(\frac{\partial v_1}{\partial x}\right)^2 + 2\left(\frac{\partial v_0}{\partial x}\right)\left(\frac{\partial v_2}{\partial x}\right) = 0$$

$$p^4 : \frac{\partial v_4}{\partial t} - v_0\left(\frac{\partial^2 v_3}{\partial x^2}\right) - v_1\left(\frac{\partial^2 v_2}{\partial x^2}\right) - v_2\left(\frac{\partial^2 v_1}{\partial x^2}\right) + v_3\left(\frac{\partial^2 v_0}{\partial x^2}\right) + 2\left(\frac{\partial v_0}{\partial x}\right)\left(\frac{\partial v_3}{\partial x}\right) + 2\left(\frac{\partial v_1}{\partial x}\right)\left(\frac{\partial v_2}{\partial x}\right) = 0$$

.... (3.5)

Solving all above partial differential equation we get,

$v_0 = \sin x$

$v_1 = -t$

$v_2 = \frac{t^2}{2!} \sin x$

$v_3 = \frac{-t^3}{3!}$

$v_4 = \frac{t^4}{4!} \sin x$

....

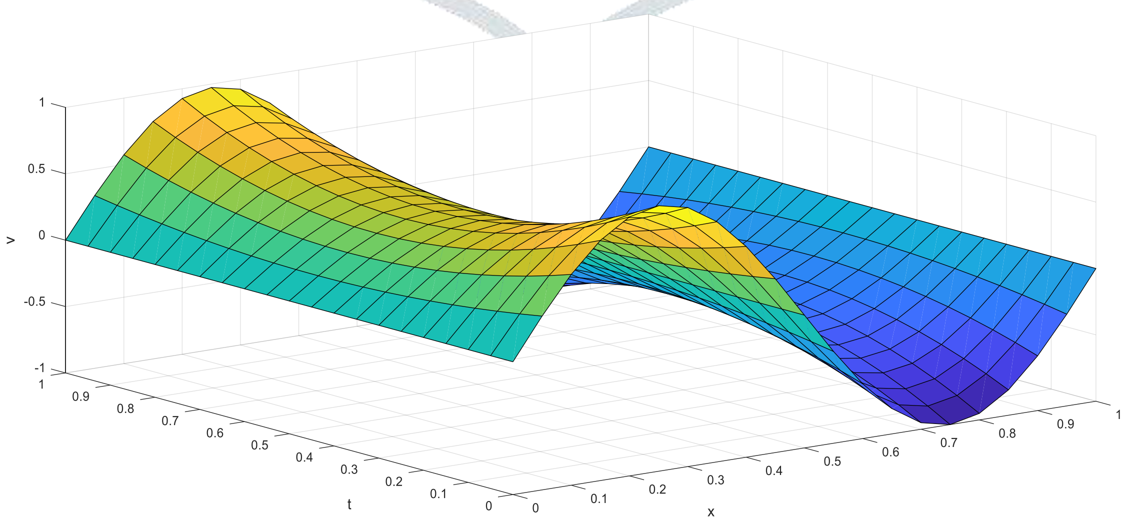
Solution of equation (3.1) can be written as ,

$u = v_0 + v_1 + v_2 + \cdots \ldots \ldots \ldots \ldots \ldots$

$$u(x,t) = \sin x - t + \frac{t^2}{2!} \sin x - \frac{t^3}{3!} + \frac{t^4}{4!} \sin x + \ldots \ldots \ldots \ldots \quad (3.6)$$

Equation (3.6) is the Homotopy solution of equation (3.1), the infinite series converges to exact solution of the non-linear diffusion equation given by (3.1)[6]

GRAPH : I

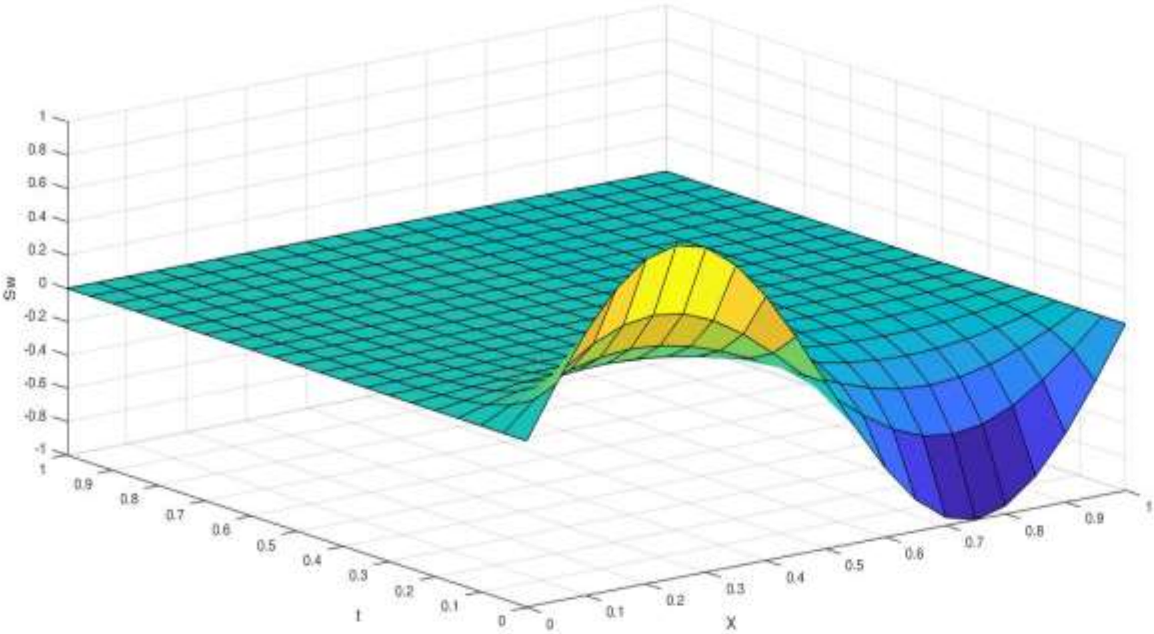


Graphs represent the Homotopy perturbation method solution $v(x,t)$ for $\alpha = 0.05$ and$\alpha = 0.1$. Respectively for $0 \le x \le 1$ and $0 \le t \le 0.5$

TABLE - I

X/T	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0	0	0	0	0	0	0	0	0	0
0.05	0.30886552	0.253588778	0.208205	0.170943	0.14035	0.115232	0.094609	0.077677	0.063776	0.063776	0.042991
0.1	0.587527526	0.482379475	0.396049	0.32517	0.266975	0.219195	0.145545	0.147758	0.121315	0.121315	0.081778
0.15	0.808736061	0.663998978	0.545165	0.447598	0.367493	0.301724	0.247725	0.203391	0.16699	0.16699	0.112567
0.2	0.950859461	0.780686976	0.64097	0.526257	0.432075	0.354748	0.291259	0.239134	0.196337	0.196337	0.13235
0.25	0.999999683	0.82103272	0.674095	0.553454	0.454404	0.373081	0.306312	0.251492	0.206483	0.206483	0.139189
0.3	0.951351376	0.781090855	0.641301	0.52653	0.432298	0.354931	0.29141	0.239257	0.196438	0.196438	0.132418
0.35	0.809671788	0.664767241	0.545796	0.448116	0.367918	0.302073	0.248012	0.203626	0.167184	0.167184	0.112698
0.4	0.588815562	0.483436995	0.396918	0.325883	0.26756	0.219676	0.180361	0.148082	0.121581	0.121581	0.081957
0.45	0.31037991	0.254832142	0.209226	0.171781	0.141038	0.115797	0.095073	0.078058	0.064088	0.064088	0.043202
0.5	0.001592653	0.001307621	0.001074	0.000881	0.000724	0.000594	0.000488	0.000401	0.000329	0.000329	0.000222
0.55	-0.30735035	-0.25234477	-0.20718	-0.1701	-0.13966	-0.11467	-0.09415	-0.0773	-0.06346	-0.06346	-0.04278
0.6	-0.586238	-0.48132073	-0.39518	-0.32446	-0.26639	-0.21871	-0.17957	-0.14743	-0.12105	-0.12105	-0.0816
0.65	-0.80779828	-0.66322903	-0.54453	-0.44708	-0.36707	-0.30137	-0.24744	-0.20315	-0.1668	-0.1668	-0.11244
0.7	-0.95036513	-0.78028112	-0.64064	-0.52598	-0.43185	-0.35456	-0.29111	-0.23901	-0.19623	-0.19623	-0.13228
0.75	-0.99999715	-0.82103064	-0.67409	-0.55345	-0.4544	-0.37308	-0.30631	-0.25149	-0.20648	-0.20648	-0.13919
0.8	-0.95184088	-0.78149275	-0.64163	-0.5268	-0.43252	-0.35511	-0.29156	-0.23938	-0.19654	-0.19654	-0.13249
0.85	-0.81060546	-0.66553382	-0.54643	-0.44863	-0.36834	-0.30242	-0.2483	-0.20386	-0.16738	-0.16738	-0.11283
0.9	-0.5901021	-0.48449329	-0.39778	-0.32659	-0.26814	-0.22016	-0.18076	-0.14841	-0.12185	-0.12185	-0.08214
0.95	-0.31189351	-0.25607486	-0.21025	-0.17262	-0.14173	-0.11636	-0.09554	-0.07844	-0.0644	-0.0644	-0.04341
1	-0.0031853	-0.00261524	-0.00215	-0.00176	-0.00145	-0.00119	-0.00098	-0.0008	-0.00066	-0.00066	-0.00044

GRAPH : II



Graphs represent the Homotopy perturbation method solution $v(x,t)$ for $\alpha = 0.1$ and$\alpha = 0.1$. Respectively for $0 \le x \le 1$ and $0 \le t \le 0.5$

TABLE – II

X/T	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0	0	0	0	0	0	0	0	0	0
0.05	0.30886552	0.20820475	0.140349813	0.094609129	0.063775555	0.115231825	0.028979885	0.0195352	0.013168584	0.008876878	0.005983861
0.1	0.587527526	0.396049458	0.26697502	0.179966567	0.121314591	0.219195296	0.055125869	0.037160081	0.025049431	0.0168857	0.011382568
0.15	0.808736061	0.545165059	0.367493125	0.247725334	0.166990447	0.301723975	0.075881173	0.051151131	0.03448073	0.023243293	0.015668191
0.2	0.950859461	0.640969754	0.432074605	0.291259398	0.196336548	0.354747501	0.089216167	0.060140186	0.040540208	0.027327958	0.018421644
0.25	0.999999683	0.67409494	0.454404133	0.306311624	0.206483181	0.373080779	0.093826841	0.063248219	0.042635318	0.028740261	0.019373671
0.3	0.951351376	0.641301352	0.432298134	0.291410077	0.196438121	0.354931025	0.089262322	0.060171299	0.040561181	0.027342096	0.018431174
0.35	0.809671788	0.545795829	0.367918323	0.248011959	0.16718366	0.302073077	0.07596897	0.051210314	0.034520625	0.023270186	0.01568632
0.4	0.588815562	0.396917717	0.26756031	0.180361108	0.121580549	0.219675838	0.055246721	0.037241547	0.025104346	0.016922718	0.011407522
0.45	0.31037991	0.209225593	0.141037958	0.095073004	0.064088251	0.115796815	0.029121976	0.019630983	0.01323315	0.008920402	0.0060132
0.5	0.001592653	0.0010736	0.000723708	0.000487848	0.000328856	0.000594188	0.000149434	0.000100732	6.79033E-05	4.57733E-05	3.08555E-05
0.55	-0.30735035	-0.20718338	-0.139661312	-0.094145014	-0.0634627	-0.11466654	-0.028837721	-0.019439368	-0.013103984	-0.008833332	-0.005954506
0.6	-0.586238	-0.395180194	-0.266389054	-0.17957157	-0.12104833	-0.2187142	-0.055004877	-0.037078521	-0.024994451	-0.016848638	-0.011357586
0.65	-0.80779828	-0.544532907	-0.367066994	-0.247438082	-0.16679681	-0.30137411	-0.075793185	-0.051091818	-0.034440747	-0.023216341	-0.015650023
0.7	-0.95036513	-0.640636531	-0.431849981	-0.291107979	-0.19623448	-0.35456308	-0.089169786	-0.060108921	-0.040519132	-0.027313751	-0.018412067
0.75	-0.99999715	-0.67409323	-0.45440298	-0.306310847	-0.20648266	-0.37307983	-0.093826603	-0.063248058	-0.042635209	-0.028740188	-0.019373622
0.8	-0.95184088	-0.641631324	-0.432520566	-0.291560018	-0.19653919	-0.35511365	-0.089308251	-0.060202259	-0.040582051	-0.027356164	-0.018440658
0.85	-0.81060546	-0.546425214	-0.368342589	-0.248297954	-0.16737645	-0.30242141	-0.076056574	-0.051269368	-0.034560432	-0.02329702	-0.015704408
0.9	-0.5901021	-0.397784969	-0.26814492	-0.180755191	-0.1218462	-0.22015582	-0.055367434	-0.037322919	-0.025159199	-0.016959694	-0.011432447
0.95	-0.31189351	-0.210245905	-0.141725746	-0.095536638	-0.06440078	-0.11636151	-0.029263992	-0.019726715	-0.013297683	-0.008963904	-0.006042524
1	-0.0031853	-0.002147197	-0.001447415	-0.000975695	-0.00065771	-0.00118838	-0.000298867	-0.000201465	-0.000135806	-9.15464E-05	-6.1711E-05

GRAPH : III

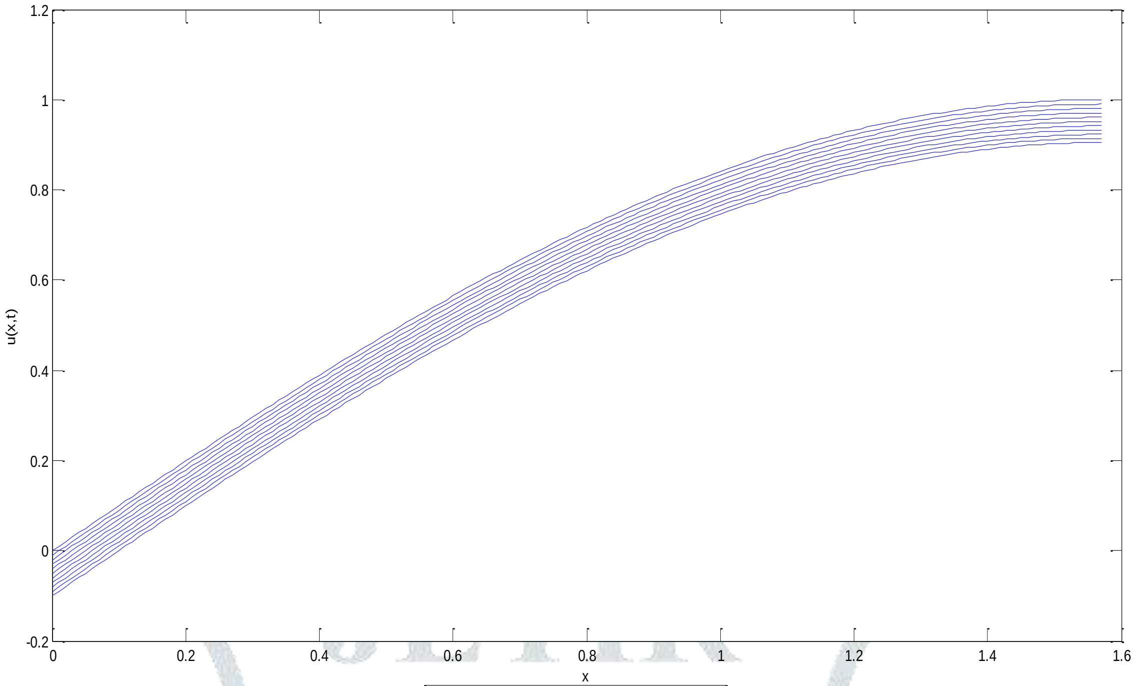


Figure - I - Stauration for different value of time t

CONCUSION:

Homotopy Perturbation Method is applied to solve linear and nonlinear partial differential equation.The results for the saturation of water are shown in Graphs. It can be seen that the saturation of water fluctuate with time and space in Graph – I and Graph – II and after certain time of period water is fully saturated. The saturation is decrease constantly with time and space in graph – III so it is obvious that water is little saturated with time and space. Furthermore, the exact solution can easily be obtained by using HPM. The results show that HPM is a powerful tool for obtaining exact solution of linear and nonlinear equations. The computations associated in this work were performed by using Matlab.

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