DUFOUR EFFECT ON UNSTEADY FREE CONVECTIVE FLOW PAST AN INCLINED PERMEABLE MOVING SURFACE WITH ALIGNED MAGNETIC FIELD

G.Sreedhar[1], B. Rama Bhupal Reddy[2]
[1]Research Scholar, Department of Mathematics, Rayalaseema University, Kurnool, A.P., India.
[2]Professor, Department of Mathematics, K.S.R.M. College of Engineering (Autonomous), Kadapa-516003, A.P., India.

ABSTRACT:

This article focuses the effects of heat absorption and diffusion thermo on unsteady free convective flow of a viscous, incompressible and electrically conducting fluid past an inclined permeable moving plate through a porous medium with aligned magnetic field. The free stream velocity is supposed to follow the exponentially increasing small perturbation law. At the permeable surface, time-dependent wall suction is assumed to occur. The non-dimensional governing equations are solved analytically by two-term harmonic and non-harmonic functions. The velocity, temperature and concentration distributions are analyzed for different values of parameters.

Key words: Aligned magnetic field, inclined surface, Dufour effect, unsteady, porous medium.

1. INTRODUCTION:

Simultaneous heat and mass transfer flow through porous medium has many engineeering and physical applications such as drying of porous solids, geothermal resorvoirs, enriched oil revovery, thermal insulation and cooling of nuclear reactors. In the presence or absence of a porous medium, the combined thermal convection past a semi-infinite vertical plate has been studied by many authors [1-4].

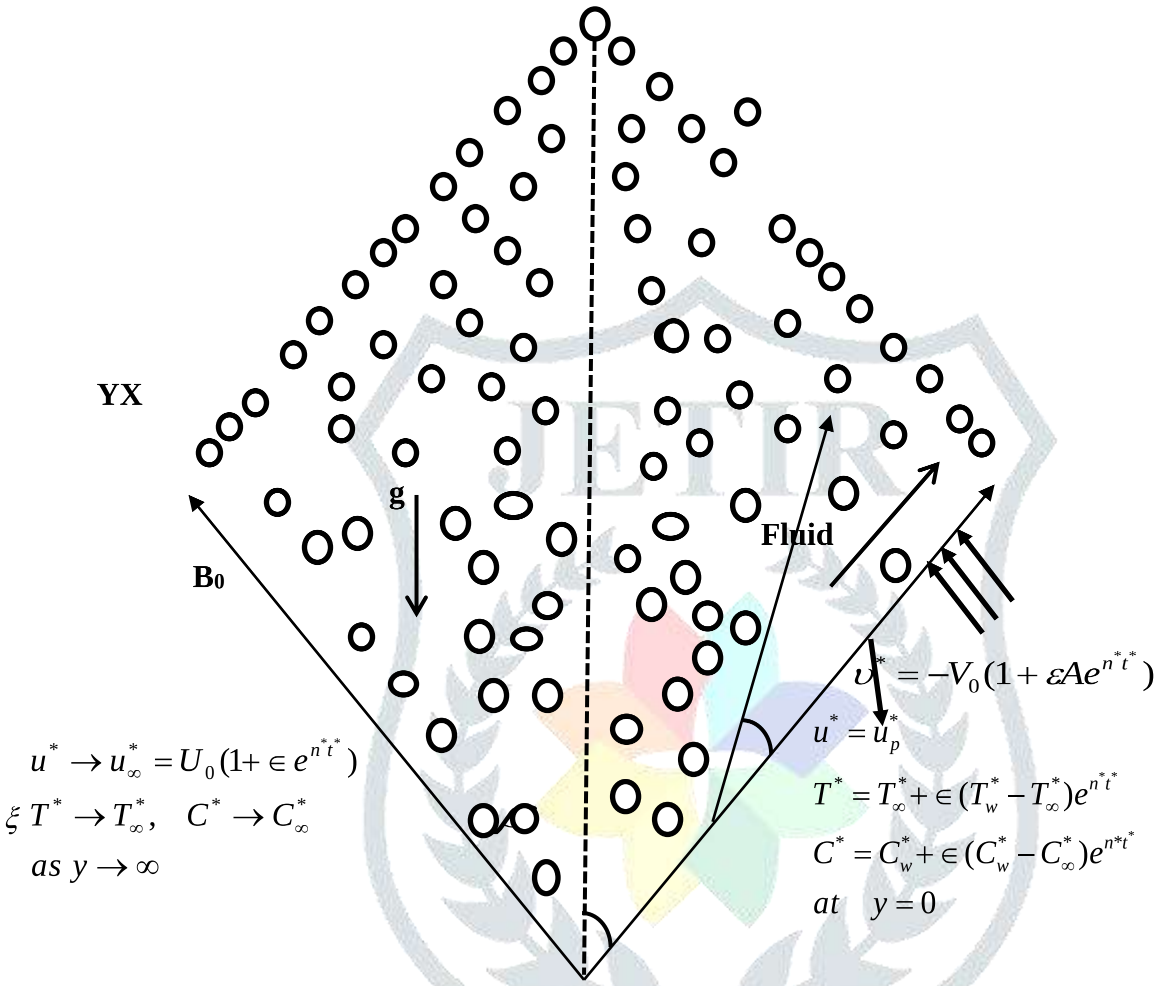
The problem of free convection and mass transfer flow of an electrically conducting fluid past an inclined heated surface under the influence of a magnetic field has attracted interest in view of its application to geophysics, astrophysics and many engineering problems, such as cooling of nuclear reactors, the boundary layer control in aerodynamics and cooling towers. In light of these applications natural convection over an inclined plate was first studied experimentally by Rich [5]. Chen et al. [6] have obtained a numerical solution for the problem of natural convection over an inclined plate with variable surface temperature. Free convection heat transfer from an isothermal plate with arbitrary inclination was investigated by Yu and Lin [7]. Chamkha [8] developed a mathematical model governing boundary layer flow past an inclined plate embedded in a porous medium with non-uniform transverse magnetic field. Manjulatha et al. [9] focused on aligned magnetic field effect of free convective steady flow of a viscous, incompressible and electrically conducting fluid past an infinite vertical porous plate with heat source and radiation absorption. Ramaprasad et al. [10] have studied the free convective heat and mass trasnfer flow past an inclined moving surface of an electrically conducting, viscous, incompressible fluid in the presence of magnetic field.Balakrishna et al. [11] analyzed the thermal radiation, heat absorption and chemical reaction effects on casson fluid flow past an infinite inclined plate embedded in porous medium.

The dufour effect have garnered considerable interest in both Newtonian and Non-Newtonian convective heat and mass trasnfer. Dufour effect is important for intermediate molecular weight gases in coupled heat and mass trasnfer binary systems. In all the above studies Dufour effect was neglected. Hence the main object of the present investigation is to study the Dufour effect on unsteady magneto hydro dynamic convective heat and mass transfer through a porous medium past an inclinedpermeable moving plate in the presence of heat absorption and aligned magnetic field.

2. MATHEMATICAL FORMULATION:

We consider unsteady two-dimensional flow of laminar, incompressible, viscous, electrically conducting and heat absorbing fluid past an inclinedpermeable moving plate embedded in a porous medium in the presence of thermal radiation and aligned magnetic field effects.The flow is assumed to be in the x-direction, which is taken along the semi-infinite inclined moving plate and y-axis normal to it. A magnetic field of uniform strength $B_0$is introduced normal to the direction of the flow. The permeable plate moves with a constant velocity in the direction of fluid flow, and the free stream velocity follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and the concentration at the wall as well as the suction velocity are exponentially varying with time. The governing equations for this investigation are based on the balances of mass, linear momentum, energy and concentration species.

The governing equations can be written in cartesian frame based on the above assumptions as:



Physical Model

$$\frac{\partial \upsilon^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + \upsilon^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu \frac{\partial^2 u^*}{\partial y^2} + g(\cos\beta)\,\beta_T(T - T_\infty) + g(\cos\beta)\beta_c(c - c_\infty) - \nu \frac{u^*}{K^*} - \frac{\sigma}{\rho} B_0^2 (Sin^2 \xi)\, u^* \tag{2}$$

$$\frac{\partial T}{\partial t^*} + \upsilon^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} - \frac{Q_0}{\rho c_p}(T - T_\infty) + \frac{DK_T}{c_s c_p}\frac{\partial^2 c}{\partial y^{*2}} \tag{3}$$

$$\frac{\partial c}{\partial t^*} + \upsilon^* \frac{\partial c}{\partial y^*} = D \frac{\partial^2 c}{\partial y^{*2}} \tag{4}$$

The magnetic and viscous dissipations are neglected in this study. The third and fourth terms on the RHS of the momentum equation (2) denote the thermal and concentration buoyancy effects, respectively. Also, the second and last terms of the energy equation (3) reperesents the heat absorption and diffusion thermo effects.

The apporpriate boundary conditions for the velocity, temperature and concentration fields are

$$u^* = u_p^*, T = T_w + \varepsilon(T_w - T_\infty)e^{n^* t^*}, c = c_w + \varepsilon(c_w - c_\infty)e^{n^* t^*} \ at \quad y^* = 0 \tag{5}$$

$$u^* \to U_\infty^* = U_0(1+\varepsilon e^{n^* t^*}), T \to T_\infty, \quad c \to c_\infty \ as \quad y^* \to \infty \tag{6}$$

It is clear from equation (1) that the suction velocity at the plate surface is a function of time only. Assuming that it takes the following exponential form:

$$\upsilon^* = -V_0(1+\varepsilon A e^{n^* t^*}) \tag{7}$$

where A is a real positive constant, ε and εA are small less than unity, and $V_0$is a scale suction velocity which has non-zero positive constant. Outside the boundary layer, equation (2) gives

$$-\frac{1}{\rho} = \frac{\partial U_\infty^*}{\partial t^*} + \frac{\upsilon}{K^*}U_\infty^* + \frac{\sigma}{\rho}B_0^2 Sin^2 \xi U_\infty^* \tag{8}$$

It is convenient to employ the following dimensionless variables:

$$u = \frac{u^*}{U_0}, \quad \upsilon = \frac{\upsilon^*}{V_0}, \quad y = \frac{V_0 y^*}{\upsilon}, \quad U_\infty = \frac{U_\infty^*}{U_0}, \quad U_p = \frac{u_p^*}{U_0}, \quad t = \frac{t^* V_0^2}{\upsilon},$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{c - c_\infty}{c_w - c_\infty}, \quad n = \frac{n^* \upsilon}{V_0^2}, \quad k = \frac{K^* V_0^2}{\upsilon^2}, \quad \Pr = \frac{\upsilon \rho c_p}{k} = \frac{\upsilon}{\alpha},$$
$$Sc = \frac{\upsilon}{D}, \quad M = \frac{\sigma \upsilon B_0^2}{\rho V_0^2}, \quad Gr = \frac{\upsilon \beta_T g(T_w - T_\infty)}{U_0 V_0^2}, \quad Gm = \frac{\upsilon \beta_c g(c_w - c_\infty)}{U_0 V_0^2},$$
$$Q = \frac{\upsilon Q_0}{\rho c_p V_0^2}, \quad Du = \frac{D K_T}{\upsilon c_s c_p}\frac{(c_w - c_\infty)}{(T_w - T_\infty)} \tag{9}$$

In view of equations (7)-(9), equations (2)-(4) reduce to the following dimensionless form:

$$\frac{\partial u}{\partial t} - (1+\varepsilon A e^{nt})\frac{\partial u}{\partial y} = \frac{\partial U_\infty}{\partial t} + \frac{\partial^2 u}{\partial y^2} + Gr_1\theta + Gm_1\phi + N(U_\infty - u) \tag{10}$$

$$\frac{\partial \theta}{\partial t} - (1+\varepsilon A e^{nt})\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - Q\theta + Du\frac{\partial^2 \phi}{\partial y^2} \tag{11}$$

$$\frac{\partial \phi}{\partial t} - (1+\varepsilon A e^{nt})\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} \tag{12}$$

where $N = (M\ Sin^2\xi + \frac{1}{k}), Gr_1 = Gr\cos\beta, Gm_1 = Gm\cos\beta$

The dimensionless form of the boundary conditions (5) and (6) become

$$u = U_p, \quad \theta = 1 + \varepsilon e^{nt}, \quad \phi = 1 + \varepsilon e^{nt} \quad at \quad y = 0 \tag{13}$$

$$u \to U_\infty, \quad \theta \to 0, \quad \phi \to 0 \quad as \quad y \to \infty \tag{14}$$

3. SOLUTION OF THE PROBLEM:

Equations (10) – (12) represent a set of partial differential equations that cannot be solved in closed form. However, it can be reduced to a set of ordinary differential equations in dimensionless form that can be solved analytically. This can be done by representing the velocity, temperature and concentration as

$$\begin{aligned} u &= u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2) + \cdots \\ \theta &= \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2) + \cdots \\ \phi &= h_0(y) + \varepsilon e^{nt} h_1(y) + O(\varepsilon^2) + \cdots \end{aligned} \tag{15}$$

Substituting equation (15) in equations (10) – (12), equating the harmonic and non-harmonic terms, and neglecting the higher-order terms of $O(\varepsilon^2)$, one obtains the following pairs of equations for (u_0, θ_0, h_0) and (u_1, θ_1, h_1).

$$u_0'' + u_0' - Nu_0 = -Gr_1\theta_0 - Gm_1 h_0 - N \tag{16}$$

$$u_1'' + u_1' - (N+n)u_1 = -Au_0' - n - N - Gr_1\theta_1 - Gm_1 h_1 \tag{17}$$

$$\theta_0'' + \Pr\theta_0' - Q\Pr\theta_0 = -\Pr Du h_0'' \tag{18}$$

$$\theta_1'' + \Pr\theta_1' - (n+Q)\Pr\theta_1 = -\Pr A\theta_0' - \Pr Du h_1'' \tag{19}$$

$$h_0'' + Sch_0' = 0 \tag{20}$$

$$h_1'' + Sch_1' - Scnh_1 = -ScAh_0' \tag{21}$$

Where a prime denotes ordinary differentiation with respect to y.
The corresponding boundary conditions can be written as

$$\begin{aligned} &u_0 = U_p, \quad u_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 1, \quad h_0 = 1, \quad h_1 = 1 \quad at \quad y = 0 \\ &u_0 = 1, \quad u_1 = 1, \quad \theta_0 \to 0, \quad \theta_1 \to 0, \quad h_0 \to 0, \quad h_1 \to 0 \quad as \quad y \to \infty \end{aligned} \tag{22}$$

Solutions of equations (16) – (21) subject to equation (22) canbe shown to be

$$u_0 = C_9 e^{-m_2 y} + C_{10} e^{-Scy} + C_{11} e^{-m_4 y} + 1 \tag{23}$$

$$u_1 = C_{12} e^{-m_1 y} + C_{13} e^{-m_2 y} + C_{14} e^{-m_3 y} + C_{15} e^{-m_4 y} + C_{16} e^{-Scy} + C_{17} e^{-m_5 y} + 1 \tag{24}$$

$$\theta_0 = C_3 e^{-m_2 y} + C_4 e^{-Scy} \tag{25}$$

$$\theta_1 = C_5 e^{-m_1 y} + C_6 e^{-m_2 y} + C_7 e^{-Scy} + C_8 e^{-m_3 y} \tag{26}$$

$$h_0 = e^{-Scy} \tag{27}$$

$$h_1 = C_1 e^{-m_1 y} + C_2 e^{-Scy} \qquad (28)$$

4. RESULTS AND DISCUSSION:

Numerical evaluation of the analytical results reported in the previous section was performed and a representative set of results is reported graphically in Figures 1-9. These results are obtained to illustrate the influence of the heat absorption coefficient *Q*, Schmidt number Sc, Dufour number *Du*, Magnetic field parameter M, Prandtl number Pr, Permeability parameter k, inclined angle parameter β, aligned angle ξ, the thermal Grashof number Gr and solutal Grashof number Gm. The value of Schmidt number Sc is taken for water-vapour (Sc = 0.60). Throughout the calculations physical variables Gr = 2 and Gm = 2 are taken which correspond to a cooling problem and also A = 0.5, n = 0.1, u_p = 0.5, t = 1.0, ε = 0.2.

The effects of magnetic field parameter and Prandtl number on the velocity field are shown in figure 1. It is noticed that the velocity of the fluid decreases with an increase of magnetic parameter and Prandtl number.It is a well-established fact that the magnetic field presents a damping effect on the velocity field by creating a drag force that opposes the fluid motion, causing the velocity to decrease.

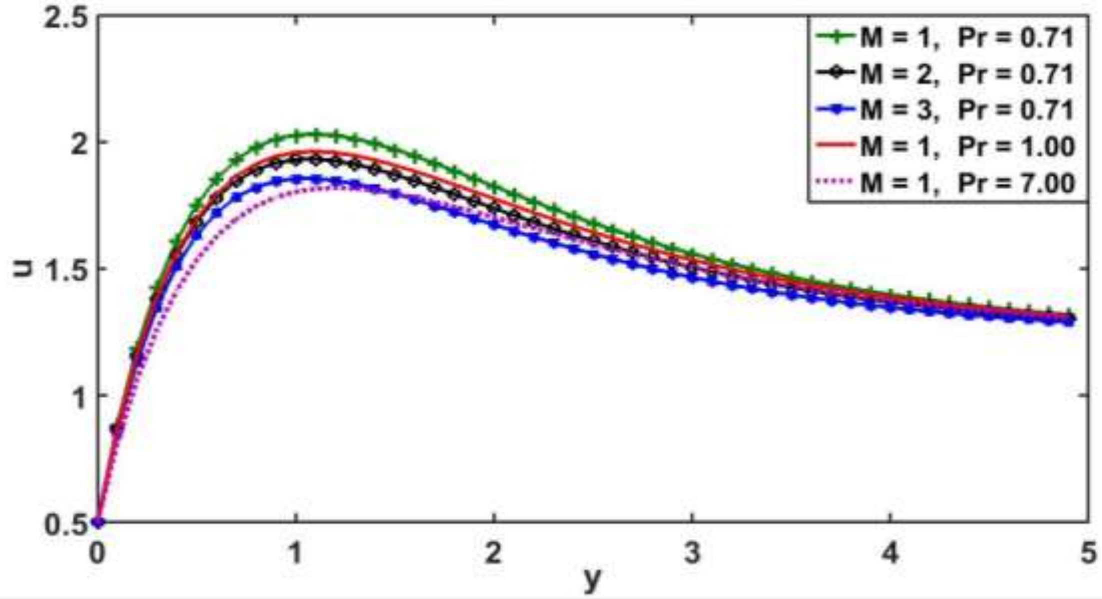


Figure1:Effects of Mand Pr on velocity profiles when k = 0.5, Du = 1.0, Q = 1.0, $\beta = \frac{\pi}{6}, \quad \xi = \frac{\pi}{6}$.

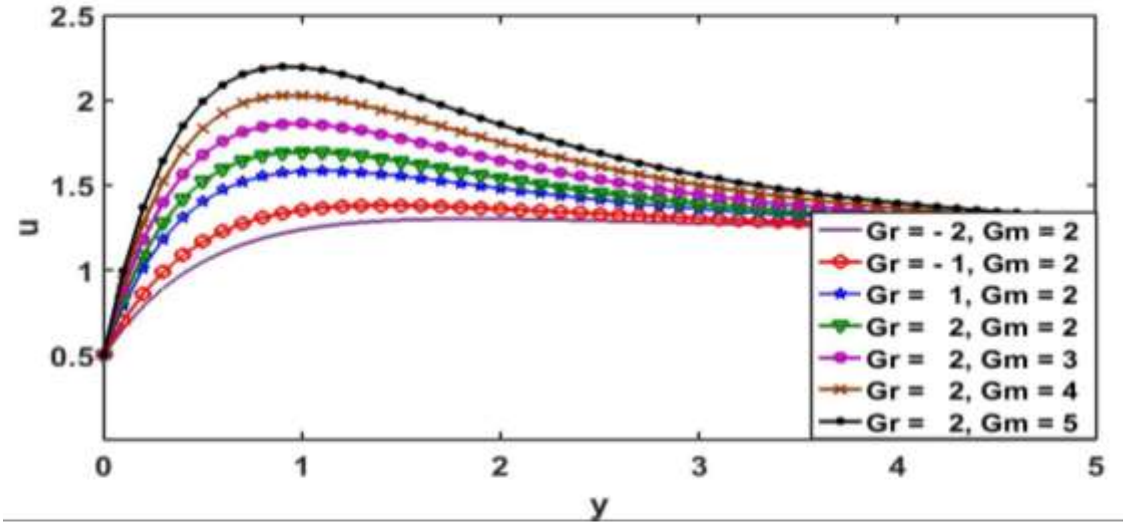


Figure 2: Effects of Grand Gm on velocity profiles when Du = 1.0, k = 0.5, M = 1.0, Pr = 0.71, $\beta = \frac{\pi}{6}$, $\xi = \frac{\pi}{6}$, Q = 1.0.

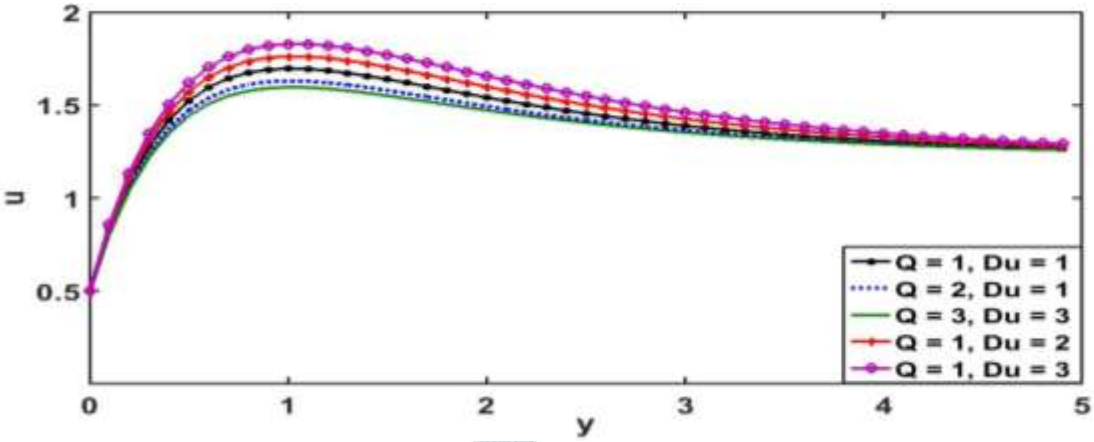


Figure3:Effects of $Q$and Du on velocity profiles when k = 0.5, M = 1.0, Pr = 0.71, $\beta = \frac{\pi}{6}$, $\xi = \frac{\pi}{6}$.

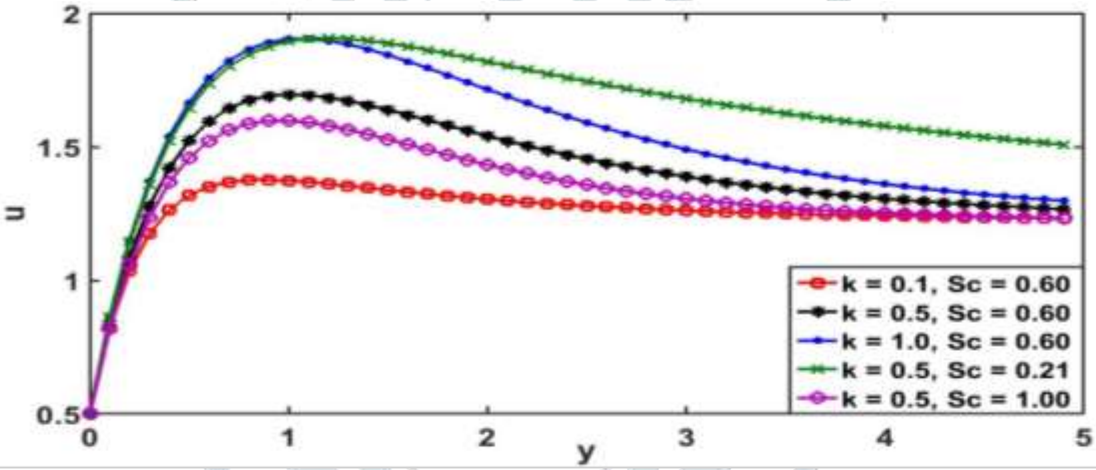


Figure 4: Effects of k and Sc on velocity profiles when Du = 1.0, M = 1.0, Pr = 0.71, Q = 1.0, $\beta = \frac{\pi}{6}$, $\xi = \frac{\pi}{6}$.

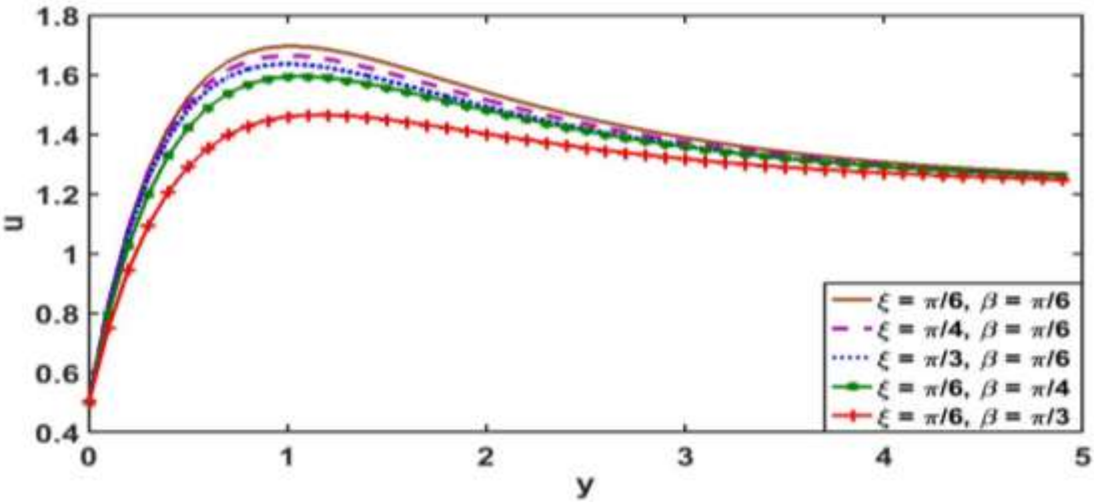


Figure 5: Effects of β and ξ on velocity profiles when Du = 1.0, M = 1.0, Pr = 0.71, Q = 1.0, k = 0.5, Sc = 0.6.

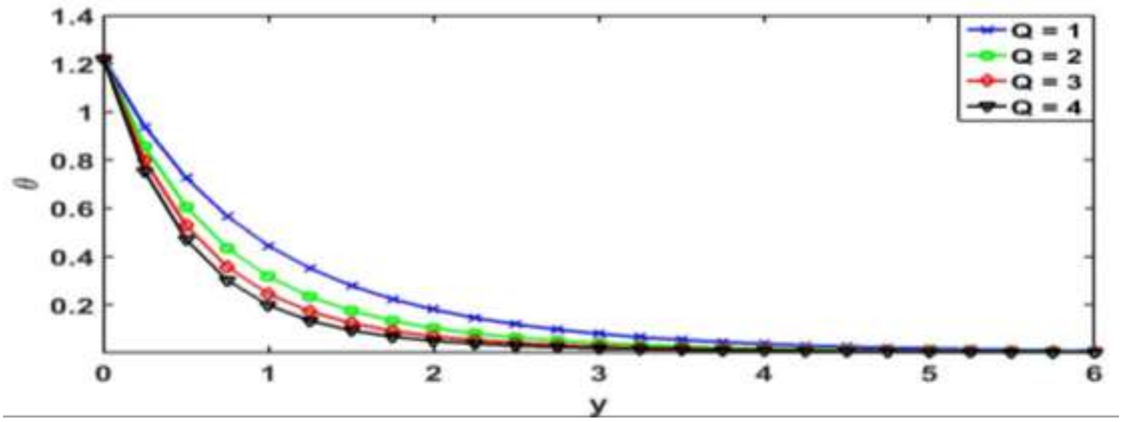


Figure6:Effects of *Q* on temperature profiles whenPr = 0.71, Du = 1.0.

The influences of the thermal and solutal Grashof number on the fluid velocity are presented in Figure 2. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermo buoyancy force. The solutal Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and peak value is more distinctive due to increases in the species buoyancy force. Also, as Gr and Gm increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

Figure 3 illustrate the influences of the heat absorption coefficient *Q* and Dufour number Du on the velocity profile respectively. Physically speaking, the presence of heat absorption (thermal sink) effects has the tendency to reduce the fluid temperature. This causes the thermal buoyancy effects to decrease resulting in a net reduction in the fluid velocity. From figure this behaviour is clearly obvious. But opposite phenomenon is noticed in the case of Dufour number. i.e., as Du increases velocity of the fluid accelerates.

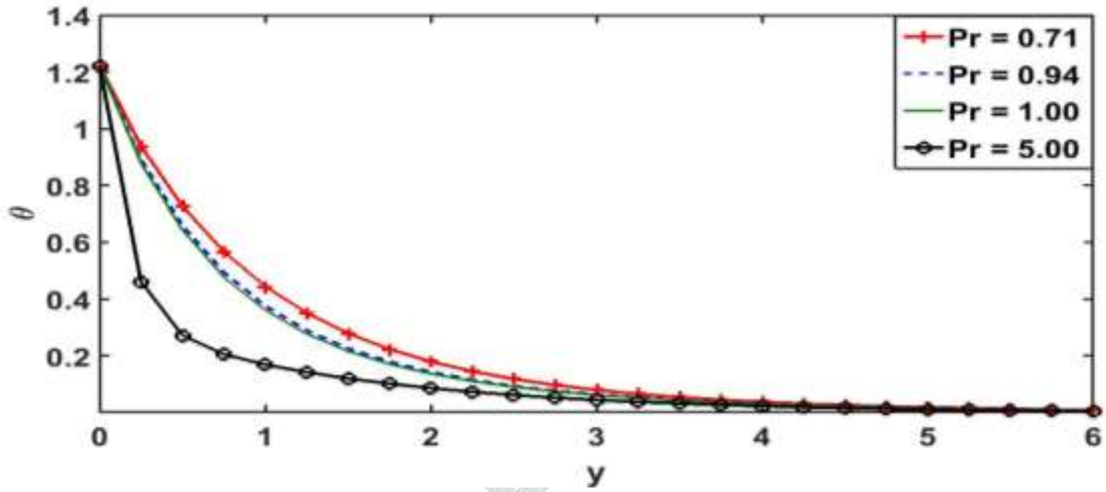


Figure7:Effects of Pr on temperature profiles when *Q* = 1.0, Du = 1.0.

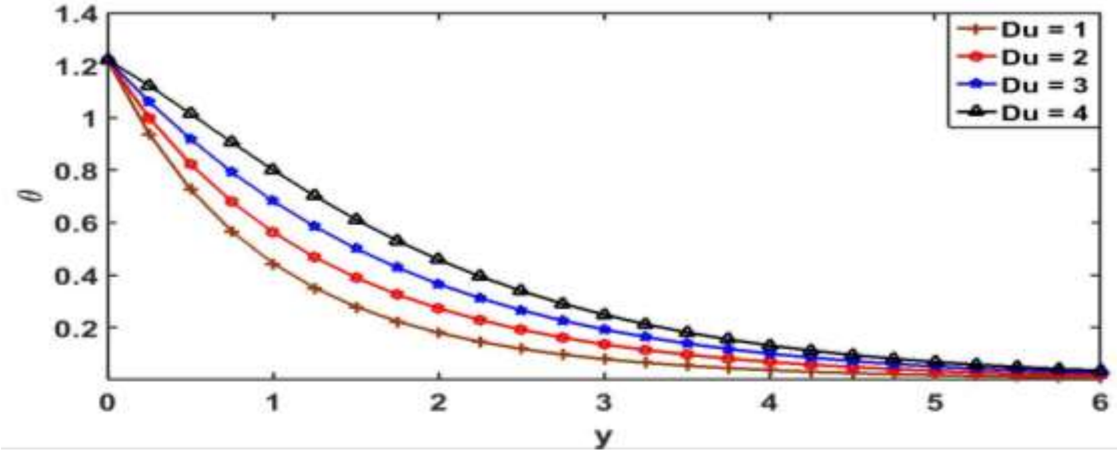


Figure 8: Effects of Du on temperature profiles when A = 0.5, n = 0.1, Q = 1.0, t = 1.0, Pr = 0.71, ε = 0.2.

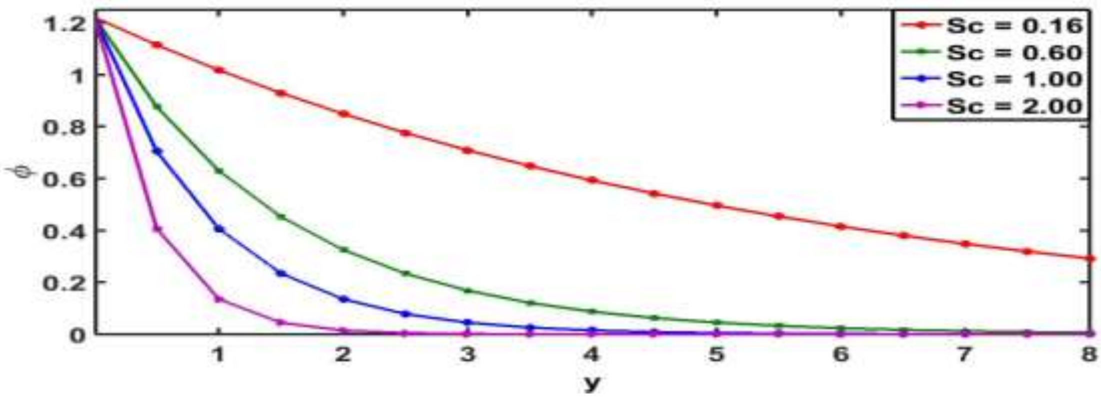


Figure9:Effects of Sc on concentration profiles

The effects of permeability parameter and Schmidt number on the velocity profile is shown in figure 4. It is noticed that the velocity increases with an increase in porosity whereas velocity decreases with an increase in Schmidt number.

The effects of aligned angle ξ and inclination of the surface β on velocity profiles are shown in figure 5. It is noticed that velocity decreases due to both aligned and inclined angles ξ and β. This may happen because an increase in aligned angle strengthens the applied magnetic field. Also the fluid has higher velocity when the surface is vertical (β = 0) than when inclined because of the fact that the buoyancy effect decreases due to gravity components ($g\cos\beta$), as the plate is inclined.

The effect of heat absorption parameter Q on the temperature field is shown in Figure 6. It is noticed that the temperature decreases with an increase in heat absorption parameter.The effect of Prandtl number Pr on the temperature profile is shown in figure 7. It is seen that the increase in the Prandtl number leads to fall in the temperature of the fluid. The reason is that lower Pr value has more uniform temperature distribution across the thermal boundary layer as compared to higher Pr value. This phenomenon occurs when the lesser values of Prandtl number are equivalent to increasing thermal conductivity. Therefore, heat is capable to diffuse away from the heated surface more quickly compare to bigger values of Prandtl number.

Figure 8 depicts the Dufour number effect on the temperature distribution. It is noticed that as the effect of diffusion thermo increases, the temperature distribution increases and goes linearly near the boundary layer.Figure 9 display the effect of Schmidt number Sc over concentration profiles. As the Schmidt number increases, the concentration of the flow field decreases.

5. CONCLUSIONS:

The governing equations for unsteady convective heat and mass transfer flow past an semi-infinite permeable moving inclined plate embedded ina porous medium with aligned magnetic field, heat absorption and Dufour effects was formulated. A perturbation technique is employed to solve the resulting coupled partial equations. It is observed that when the Schmidt number was increased, the concentration level was decreased resulting in a decreased fluid velocity.An increase in the Prandtl number and heat absorption is observed to lead to decrease in thermal boundary layer.It is found that, the velocity decreases with increasing the heat absorption coefficient, magnetic parameter and Prandtl number, where as reverse trend is seen with increasing the porous parameter, thermal and solutal Grashof numbers. The fluid velocity and temperature distribution increases with the increase of Dufour number. The effect of angle of inclination or aligned anlge is to decrease the fluid velocity.

6. REFERENCES:

[1] Ranganathan, P., Viskanta, R., Mixed convection boundary layer flow along a vertical surface in a porous medium, Numerical Heat Transfer, 7(3), 305-317, 1984.

[2] Minkowycs, W.J., Cheng, P., Chong Hyuck Chang., Mixed convection about a non isothermal cylinder and sphere in a porous medium, Numerical Heat Transfer, 8(3), 349-359, 1985.

[3] Nakayama, A., and Koyama, H., A general similarity transformation for combined free and forced convection flows with in a fluid saturated porous medium. Journal of Heat Trasnfer, 109(4), 1041-1045, 1987.

[4] Hsieh, J.C., Chen, T.S., and Armaly, B.F., Non similarity solutions for mixed convection from vertical surfaces in porous media variable surface temperature or heat flux, International Journal of Heat and Mass Transfer, 36(6), 1485-1493, 1993.

[5] Rich, B., An investigation of heat transfer from an inclined plate in free convection, Transactions of the ASME, 75, 489-499, 1953.

[6] Chen, T.S., Tien, H.C., and Armaly, B.F., Natural convection on horizontal inclined and vertical plates with variable surface temperature or heat flux, International Journal of Heat and Mass Transfer, 29(10), 1465-1478, 1986.

[7] Yu, W.S., and Lin, H.T., Free convection heat transfer from an isothermal plate with arbitrary inclination, Warme-und Stoffubertragung,23(4), 203-211, 1988.

[8] Ali.J. Chamkha, Hydromagnetic natural convection from an isothermal inclined surface adjacent to a thermally stratified porous medium, International Journal of Engineering Science, 35(10-11), 975-986, 1997.

[9] Manjulatha, V., Varma, S.V.K., and Raju, V.C.C., Effects of radiation absorption and mass transfer on the free convective flow past a vertical flat plate through a porous medium in an aligned magnetic field, Applications and Applied Mathematics An International Journal, 9(1), 75-93, 2014.

[10] Ramaprasad, J.L., Balamurugan, K.S., and Dharmaiah, G., Unsteady MHD convective heat and mass transfer flow past an inclined moving surface with heat absorption, JP Journal of Heat and Mass Transfer, 13(1), 33-51, 2016.

[11] Balakrishna, S., Rama Mohan, S., Viswanaatha Reddy, G., and S.V. K. Varma., Effects of chemical reaction on unsteady MHD casson fluid flow past a moving infinite inclined plate through poros medium, Internaional Journal of Engineering Science and Computing, 8(7), 18658-18666, 2018.