An Extenstion of Fermat's Last Theorem in five dimensional Euclidean space

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Abstract

The Fermat last theorem states that there is no integer triple (a,b,c) such that $a^n + b^n = c^n \, for$ n >2. And in an extension of Fermat's last theorem, they shown that $a^n + b^n + c^n = d^n$ is true for n=2,3 and in the extention of an extension of fermat's last theorem proved that $a^n + b^n + c^n + d^n = e^n$ for n=2 and now in this paper it is an attempt to show that $a^n + b^n + c^n + d^n + e^n = f^n$ for n=2

Key words: integer quadruple, integer quintuples, integer sextuples, five dimension Euclidean space, Fermat last theorem

Introduction

Pierre Fermat (1601-1665) wrote a comment by the side while reading a book of Pythagoras triple that there is no integer triple(a,b,c), for which $a^n + b^n = c^n \, for \, n > 2$ this result known as Fermat's last theorem and unsolved till 1995 when Andrew wiles in a 110-page paper was able to provide a proof [2]

Then in an extension of Fermat's last theorem it is an attempt to extend the Fermat's last theorem to integer quadruple and proved the result for $a^n + b^n + c^n = d^n$ for n=2,3

And in an extension of an extension of fermat's last theorem proved for integer quintuples $a^n + b^n + c^n + d^n = e^n \, for \, n = 2$.

Now in this paper it is an attempt to solve to solve for integer sextuples

$a^n + b^n + c^n + d^n + e^n = f^n \, for \, n = 2$

Preliminary results

We present few results on integer sextuple

Result 1

If (a,b,c,d,e,f) is an integer sextuples, then multiple of any integer n with this integer sextuples is again an integer sextuples (na,nb,nc,nd,ne)

Proof

$$(na)^2 + (nb)^2 + (nc)^2 + (nd)^2 + (ne)^2 = n^2a^2 + n^2b^2 + n^2c^2 + n^2d^2 + n^2e^2$$

$$= n^2(a^2 + b^2 + c^2 + d^2 + e^2)$$

$$= n^2(f^2)$$

Result 2

For any sextuples (a,b,c,d,e,f), if a is even and b,c,d,e are odd then f cannot be an odd

Proof

Let

a =2p, b=2q+1, c=2r+1, d=2s+1 e=2t+1

for p,q,r,s,t∈ z

if $a^2 + b^2 + c^2 + d^2 + e^2 =$

$(2p)^2 + (2q+1)^2 + (2r+1)^2 + (2s+1)^2 + (2t+1)^2$

$= 4p^2 + 4q^2 + 1 + 4q + 4r^2 + 1 + 4r + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t$

$= 2(2p^2 + 2q^2 + 2r^2 + 2s^2 + 2t^2 + 2q + 2r + 2s + 2t + 2)$

Which is an even number

Result 3

For any sextuples (a,b,c,d,e,f), if a,b are even and c,d,e are odd then f must be an odd number

Proof :

Let a=2p, b=2q, c=2r+1, d=2s+1, e=2t+1

$a^2 + b^2 + c^2 + d^2 + e^2 = 4p^2 + 4q^2 + (2r+1)^2 + (2s+1)^2 + (2t+1)^2$

$= 4p^2 + 4q^2 + 4r^2 + 1 + 4r + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t$

$f^2 = 2(2p^2 + 2q^2 + 2r^2 + 2r^2 + 2t^2 + 2r + 2s + 2t) + 3$

Which is an odd number

Result 4

For any sextuples (a,b,c,d,e,f),if a,b,c are even and d, e, are odd then f must be an even

Proof

Let a=2p, b=2q, c=2r, d=2s+1, e= 2t+1

$a^2 + b^2 + c^2 + d^2 + e^2 = (2p)^2 + (2q)^2 + (2r)^2 + (2s+1)^2 + (2t+1)^2$

$= 4p^2 + 4q^2 + 4r^2 + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t$

$f^2 = 4p^2 + 4q^2 + 4r^2 + 4s^2 + 4s + 4t + 2$

$= 2(2p^2 + 2q^2 + 2r^2 + 2s^2 + 2s + 2t + 1)$

An even number

Result: 5

For any sextuples (a, b, c, d, e, f),if a, b, c, d are even and e is odd then f must be odd

Proof

Let a=2p, b=2q, c=2r, d=2s, e= 2t+1

$a^2+b^2+c^2+d^2+e^2=4p^2+4q^2+4r^2+4s^2+4t^2+4t+1$

$f^2 = an\ odd\ number$

Result 6

For any sextuples (a, b, c, d, e, f), if a, b, c, d, e are even then f must be even

Proof

Let a=2p b=2q c=2r, d=2s, e= 2t

$$a^2+b^2+c^2+d^2+e^2=4p^2+4q^2+4r^2+4s^2+4t^2$$

$$=2(2p^2+2q^2+2r^2+2s^2+2t^2)$$

$$f^2 = an\ even\ number$$

Result 7

For any sextuples (a, b, c, d, e, f), if a, b, c, d, e are odd then f must be odd

Proof:

a =2p+1, b=2q+1, c=2r+1, d=2s+1 e=2t+1

for p, q, r, s, t ∈ z

if $a^2+b^2+c^2+d^2+e^2=$

$(2p+1)^2+(2q+1)^2+(2r+1)^2+(2s+1)^2+(2t+1)^2$

$$=4p^2+4p+1+4q^2+1+4q+4r^2+1+4r+4s^2+1+4s+4t^2+1+4t$$

$=2(2p^2+2q^2+2r^2+2s^2+2t^2+2p+2q+2r+2s+2t)+5$

Which is an odd number

Main result

Integer sextuples and the five dimensional Euclidean

Here we relate the integer sextuples (a,b,c,d,e,f) to points on five dimensional Euclidean space and a solution is obtain for the equation $x^2+y^2+z^2+u^2+v^2=1$ from we get the general solution for integer sextuples is $a^2+b^2+c^2+d^2+e^2=f^2$

Let (a, b, c, d, e, f), be integer sextuples for which $a^2+b^2+c^2+d^2+e^2=f^2$,

Divide by f^2, we get $\left(\frac{a}{f}\right)^2+\left(\frac{b}{f}\right)^2+\left(\frac{c}{f}\right)^2+\left(\frac{d}{f}\right)^2+\left(\frac{e}{f}\right)^2=1$

$\left(\frac{a}{f},\frac{b}{f},\frac{c}{f},\frac{d}{f}\right)$ *is the* Solution of the equation $x^2+y^2+z^2+u^2+v^2=1$

Here $x^2+y^2+z^2+u^2+v^2=1$ is an 5 dimensional Euclidean space whose coordinates (x, y, z, u, v) are rational number. Notice that it has 10 coordinates' points $(\pm1,0,0,0,0),(0,\pm1,0,0,0),(0,0,\pm1,0,0),(0,0,0,\pm1,0)$ *and* $(0,0,0,0,\pm1)$

Suppose we consider a vector b and the line L going through the point $((-1,0,0,0,0)$ having b as its direction. The line L is given by the vector equation

$$L: r=-i+tb$$

Where b=$b_1i+b_2j+b_3k+b_4l+b_5m$ the Cartesian equation of L is given by

$\frac{x+1}{b_1}=\frac{y}{b_2}=\frac{z}{b_3}=\frac{u}{b_4}=\frac{v}{b_5}$ To find the intersection of space and L, we have to solve

$x^2+y^2+z^2+u^2+v^2=1$ and $\frac{x+1}{b_1}=\frac{y}{b_2}=\frac{z}{b_3}=\frac{u}{b_4}=\frac{v}{b_5}$

From the above equation we get

Y=$\frac{b_2(x+1)}{b_1}$, z=$\frac{b_3(x+1)}{b_1}$, $u=\frac{b_4(x+1)}{b_1}$, $v=\frac{b_5(x+1)}{b_1}$

Now substitute these values in Euclidean space equation we have

$$x^2+\left(\frac{b_2(x+1)}{b_1}\right)^2+\left(\frac{b_3(x+1)}{b_1}\right)^2+\left(\frac{b_4(x+1)}{b_1}\right)^2+\left(\frac{b_5(x+1)}{b_1}\right)^2=1$$

$$b_1{}^2x^2+b_2{}^2(x+1)^2+b_3{}^2(x+1)^2+b_4{}^2(x+1)^2+b_5{}^2(x+1)^2=b_1{}^2$$

$$\left(b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2\right)x^2+2\left(b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2\right)x+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2=b_1{}^2$$

$$\left(b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2\right)(x^2+2x+1)=2b_1{}^2(x+1)$$

$$\left(b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2\right)(x+1)^2=2b_1{}^2(x+1)$$

$$\left(b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2\right)(x+1)=2b_1{}^2$$

$$x=\frac{b_1{}^2-b_2{}^2-b_3{}^2-b_4{}^2-b_5{}^2}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2}$$

Similarly the value of y , z and w are

Y=$\frac{2b_1b_2}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2}$,

$$z=\frac{2b_1b_3}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2},\; u=\frac{2b_1b_4}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2},$$

$$v=\frac{2b_1b_5}{b_1{}^2+b_2{}^2+b_3{}^2+b_4{}^2+b_5{}^2}$$

Thus every point (x, y, z, u, v) on the 5 dimensional Euclidean space

$x^2+y^2+z^2+u^2+v^2=1$

Is $\left(\frac{{b_1}^2-{b_2}^2-{b_3}^2-{b_4}^2-{b_5}^2}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2}, \frac{2b_1b_2}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2}, \frac{2b_1b_3}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2}, \frac{2b_1b_4}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2}, \frac{2b_1b_5}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2}\right)$

Now we give a result which obtain the sextuples (a, b, c, d, e, f)

Theorem

For any integer $a = p^2 - q^2 - r^2 - s^2 - t^2,\ b = 2pq, c = 2pr, d = 2ps, e = 2pt\ for\ p, q, r, s, t \in z$ then $f = p^2 + q^2 + r^2 + s^2 + t^2$, Will satisfies the equation $a^2 + b^2 + c^2 + d^2 + e^2 = f^2$

Proof

Now $a^2 + b^2 + c^2 + d^2 + e^2 = (p^2 - q^2 - r^2 - s^2 - t^2)^2 + (2pq)^2 + (2pr)^2 + (2ps)^2 + (2pt)^2$

$$=\left(p^2 - (q^2 + r^2 + s^2 + t^2)\right)^2 + (2pq)^2 + (2pr)^2 + (2ps)^2 + (2pt)^2$$

$$=p^4 - 2(q^2 + r^2 + s^2 + t^2)p^2 + (q^2 + r^2 + s^2 + t^2)^2 + 4p^2q^2 + 4p^2r^2 + 4p^2s^2 + 4p^2t^2$$

$$=p^4 + 2(q^2 + r^2 + s^2 + t^2)p^2 + (q^2 + r^2 + s^2 + t^2)^2$$

$$=(p^2 + q^2 + r^2 + s^2 + t^2)^2 = f^2$$

Conclusion

In this paper i extend the Fermat's last theorem and an attempt made to produce the result for $a^n + b^n + c^n + d^n + e^n = f^n$ for n=2, thus we called an extension of Fermat's last theorem in five dimensional Euclidean space

References

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