Properties Of Permutation Using Double Factorial

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Abstract

In this paper contains to calculation some permutation formulae using the techniques of double factorial. Applying this concept I have calculated and proved the results of permutation which had been already calculated.

Keywords : Permutation, Factorial, Double factorial.

Introduction

The properties of permutation is already proved using some technique of factorial and sub factorial. In same concept I decided to verify and proved the properties of permutation using this double factorial.

1. Basic definitions

1.1 Definition of Permutation

A permutation is an arrangement of all or part of a set of objects, with regard to the order of the arrangement.

Computing the number of permutations of n objects taken r at a time is

$$np_r = n(n-1)(n-2)\ldots\ldots(n-r+1)$$
$$= \frac{n!}{(n-r)!}$$

1.2 Definition of factorial

Factorial says to multiply all whole numbers from the chosen number down to 1. The symbol is "! " The formula is

$$n! = n(n-1)(n-2)\ldots\ldots\times 3\times 2\times 1$$

1.3 Definition of double factorial

The double factorial or semi factorial of a number n(denoted by $n!!$) is the product of all the integers from 1 up to n that have the same parity (odd or even) as n. Formula is

For n is even, then

$$n!! = n(n-2)(n-4)\ldots\ldots\times 6\times 4\times 2$$

For n is odd , then

$$n!! = n(n-2)(n-4)\ldots\ldots\times 5\times 3\times 1$$

Example:

$$10!! = 10\times 8\times 6\times 4\times 2$$

$$= 3840$$

$$9!! = 9 \times 7 \times 5 \times 3 \times 1$$

$$= 945$$

Note : As (n!)! and not n!!

2. Relation between factorial and double factorial

For even n=2x, x≥ 0 then

$$n!! = 2^x\, x!$$

For odd n=2x-1 , x≥ 1 then

$$n!! = \frac{(2x)!}{2^x\, x!}$$

Example :

For n=10=2(5)

$$10!! = 2^5\, 5!$$

$$= 32 \times 120$$

$$= 3840$$

For n=9=2(5)-1

$$9!! = \frac{10!}{2^5\, 5!}$$

$$= \frac{3628800}{3840}$$

$$= 945$$

2.1 Useful theorem

For any non negative integer n, then $\frac{n!}{n!!} = (n-1)!!$ (or) $n! = (n-1)!! \times n!!$

Proof

i. If n is odd

$$\frac{n!}{n!!} = \frac{n(n-1)(n-2)\ldots\ldots \times 3 \times 2 \times 1}{n(n-2)(n-4)\ldots\ldots \times 5 \times 3 \times 1}$$

$$= (n-1)(n-3)\ldots\ldots \times 4 \times 2$$

$$= (n-1)!!$$

ii. If n is even

$$\frac{n!}{n!!} = \frac{n(n-1)(n-2)\ldots\ldots \times 3 \times 2 \times 1}{n(n-2)(n-4)\ldots\ldots \times 6 \times 4 \times 2}$$

$$= (n-1)(n-3)\ldots\ldots \times 3 \times 1$$

$$= (n-1)!!$$

Combining this cases for any non negative integer n

$$\frac{n!}{n!!} = (n-1)!!$$

$$\Leftrightarrow n! = (n-1)!! \times n!!$$

Example :

For n= 6

$$n! = 6!$$
$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 720$$
$$(n-1)!! \times n!! = 5!! \times 6!!$$
$$= 5 \times 3 \times 1 \times 6 \times 4 \times 2$$
$$= 720$$

Result :1

$0!! = 1$ and $(-1)!! = 1$

Proof

w.k.t $n!! = 2^x\, x!$

$$0!! = 2^0\, 0!$$
$$= 1$$

Result:2

$$n!! = n(n-2)!!$$
$$(n+1)!! = (n+1)(n-1)!!$$

Example:

$$6!! = 6 \times 4!!$$
$$= 6 \times 4 \times 2!!$$
$$= 6 \times 4 \times 2$$
$$= 48$$

Result:3

$$np_r = \frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}$$

Proof:

$$np_r = \frac{n!}{(n-r)!}$$

Since, $n! = (n-1)!! \times n!!$

Therefore,

$$np_r = \frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}$$

Example:

$$6p_3 = \frac{6!! \times 5!!}{2!! \times 3!!}$$
$$= \frac{6 \times 4 \times 2 \times 5 \times 3 \times 1}{2 \times 3 \times 1}$$
$$= 120$$

Result:4

$np_n = n!! \times (n-1)!!$

Proof:

w.k.t

$$np_r = \frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}$$

$$np_n = \frac{n!! \times (n-1)!!}{(n-n-1)!! \times (n-n)!!}$$

$$= \frac{n!! \times (n-1)!!}{(-1)!! \times (0)!!}$$

$$= n!! \times (n-1)!!$$

Example:

$$6p_6 = 6!! \times 5!!$$

$$= 6 \times 4 \times 2 \times 5 \times 3 \times 1$$

$$= 720$$

Result :5

$np_1 = n$

Proof:

$$np_r = \frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}$$

$$np_1 = \frac{n!! \times (n-1)!!}{(n-1-1)!! \times (n-1)!!}$$

$$= \frac{n!!}{(n-2)!!}$$

$$= \frac{n(n-2)!!}{(n-2)!!}$$

$$= n$$

Example:

$$6p_1 = \frac{6!! \times 5!!}{4!! \times 5!!}$$

$$= \frac{6 \times 4 \times 2}{4 \times 2}$$

$$= 6$$

Result:6

$np_{n-1} = np_n$

Proof:

w.k.t

$$np_n = n!! \times (n-1)!! \qquad \text{-----------------------}(1)$$

$$np_{n-1} = \frac{n!! \times (n-1)!!}{(n-n+1-1)!! \times (n-n-1)!!}$$

$$nPn = \frac{n!! \times (n-1)!!}{0!! \times (-1)!!}$$

$np_{n-1} = n!! \times (n-1)!!$ ----------------------(2)

From and (1) and (2)

$np_{n-1} = np_n$

Example:

When n=5,

$np_{n-1} = np_n$

$np_{n-1} = 5p_4$

$= 5!! \times 4!!$

$= 5 \times 3 \times 1 \times 4 \times 2$

$= 120$

$np_n = 5p_5$

$= 5!! \times 4!!$

$= 5 \times 3 \times 1 \times 4 \times 2$

$= 120$

Result:7

$np_r = n\,(n-1)p_{r-1}$

Proof:

RHS= $n\,(n-1)p_{r-1}$

$$= \frac{n\ (n-2)!! \times (n-1)!!}{(n-1-r+1-1)!! \times (n-1-r+1)!!}$$

$$= \frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}$$

$= np_r$

=LHS

Example:

LHS $= 6p_3$

$$= \frac{6!! \times 5!!}{2!! \times 3!!}$$

$$= \frac{6 \times 4 \times 2 \times 5 \times 3 \times 1}{2 \times 3 \times 1}$$

$= 120$

RHS = $6\ 5p_2$

$$= 6 \times \frac{4!! \times 5!!}{2!! \times 3!!}$$

$$= \frac{6 \times 4 \times 2 \times 5 \times 3 \times 1}{2 \times 3 \times 1}$$
$$= 120$$

Result:8

$$np_0 = 1$$

Proof:

w.k.t

$$np_0 = \frac{n!! \times (n-1)!!}{(n-1)!! \times (n)!!}$$
$$= 1$$

Example:

$$5p_0 = 1$$

Result:9

$$\frac{np_r}{np_{r-1}} = n - r + 1$$

proof

$$\frac{np_r}{np_{r-1}} = \frac{\frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}}{\frac{n!! \times (n-1)!!}{(n-r+1-1)!! \times (n-r+1)!!}}$$
$$= \frac{(n-r)!! \times (n-r+1)!!}{(n-r-1)!! \times (n-r)!!}$$
$$= \frac{(n-r+1)(n-r+1)!!}{(n-r-1)!!}$$
$$= n - r + 1$$

Example:

$$\frac{5p_3}{5p_2} = \frac{4!! \times 5!!}{2!! \times 1!!} \times \frac{2!! \times 3!!}{4!! \times 5!!}$$
$$= 3$$
$$= 5 - 3 + 1$$

Result:10

$$(n-1)p_r + r\,(n-1)p_{r-1} = np_r$$

Proof:

$$\text{LHS} = (n-1)p_r + r\,(n-1)p_{r-1}$$
$$= \frac{(n-2)!!\,(n-1)!!}{(n-r-2)!!\,(n-r-1)!!} + \frac{r(n-2)!!\,(n-1)!!}{(n-r-1)!!\,(n-r)!!}$$
$$= \frac{(n-r)(n-2)!!\,(n-1)!!}{(n-r)(n-r-2)!!\,(n-r-1)!!} + \frac{r(n-2)!!\,(n-1)!!}{(n-r-1)!!\,(n-r)!!}$$
$$= \frac{(n-r)(n-2)!!\,(n-1)!!}{(n-r)!!\,(n-r-1)!!} + \frac{r(n-2)!!\,(n-1)!!}{(n-r-1)!!\,(n-r)!!}$$

$$= \frac{(n-2)!!\,(n-1)!!}{(n-r)!!\,(n-r-1)!!}(n-r+r)$$
$$= \frac{(n-2)!!\,(n-1)!!}{(n-r)!!\,(n-r-1)!!}(n)$$
$$= \frac{(n)!!\,(n-1)!!}{(n-r)!!\,(n-r-1)!!}$$
$$= np_r$$
=RHS

Example:

When n=5,

$$5p_3 = \frac{4!!\ \times 5!!}{2!!\ \times 1!!}$$
$$= \frac{4\times 2\times 5\times 3\times 1}{2\times 1}$$
$$= 60$$
$$4p_3 + 3\ 4p_2 = \frac{4!!\ \times 3!!}{0!!\ \times 1!!} + 3\ \frac{4!!\ \times 3!!}{2!!\ \times 1!!}$$
$$= \frac{4\times 2\times 3\times 1}{1} + \frac{3\times 4\times 2\times 3\times 1}{2\times 1}$$
$$= 24 + 36$$
$$= 60$$

Conclusion

In this paper I have calculated and proved some results with example, properties of permutation applying the technique of double factorial and I extend my paper to solve the same concept to find out the properties of combinations.

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