

Properties Of Permutation Using Double Factorial

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Abstract

In this paper contains to calculation some permutation formulae using the techniques of double factorial. Applying this concept I have calculated and proved the results of permutation which had been already calculated.

Keywords : Permutation, Factorial, Double factorial.

Introduction

The properties of permutation is already proved using some technique of factorial and sub factorial. In same concept I decided to verify and proved the properties of permutation using this double factorial.

1. Basic definitions

1.1 Definition of Permutation

A permutation is an arrangement of all or part of a set of objects, with regard to the order of the arrangement.

Computing the number of permutations of n objects taken r at a time is

$$\begin{aligned} nP_r &= n(n-1)(n-2) \dots \dots (n-r+1) \\ &= \frac{n!}{(n-r)!} \end{aligned}$$

1.2 Definition of factorial

Factorial says to multiply all whole numbers from the chosen number down to 1. The symbol is "!" The formula is

$$n! = n(n-1)(n-2) \dots \dots \times 3 \times 2 \times 1$$

1.3 Definition of double factorial

The double factorial or semi factorial of a number n (denoted by $n!!$) is the product of all the integers from 1 up to n that have the same parity (odd or even) as n . Formula is

For n is even, then

$$n!! = n(n-2)(n-4) \dots \dots \times 6 \times 4 \times 2$$

For n is odd, then

$$n!! = n(n-2)(n-4) \dots \dots \times 5 \times 3 \times 1$$

Example:

$$10!! = 10 \times 8 \times 6 \times 4 \times 2$$

$$= 3840$$

$$9!! = 9 \times 7 \times 5 \times 3 \times 1$$

$$= 945$$

Note : As $(n!)!$ and not $n!!$

2. Relation between factorial and double factorial

For even $n=2x$, $x \geq 0$ then

$$n!! = 2^x x!$$

For odd $n=2x-1$, $x \geq 1$ then

$$n!! = \frac{(2x)!}{2^x x!}$$

Example :

For $n=10=2(5)$

$$10!! = 2^5 5!$$

$$= 32 \times 120$$

$$= 3840$$

For $n=9=2(5)-1$

$$9!! = \frac{10!}{2^5 5!}$$

$$= \frac{3628800}{3840}$$

$$= 945$$

2.1 Useful theorem

For any non negative integer n , then $\frac{n!}{n!!} = (n-1)!!$ (or) $n! = (n-1)!! \times n!!$

Proof

i. If n is odd

$$\frac{n!}{n!!} = \frac{n(n-1)(n-2) \dots \times 3 \times 2 \times 1}{n(n-2)(n-4) \dots \times 5 \times 3 \times 1}$$

$$= (n-1)(n-3) \dots \times 4 \times 2$$

$$= (n-1)!!$$

ii. If n is even

$$\frac{n!}{n!!} = \frac{n(n-1)(n-2) \dots \times 3 \times 2 \times 1}{n(n-2)(n-4) \dots \times 6 \times 4 \times 2}$$

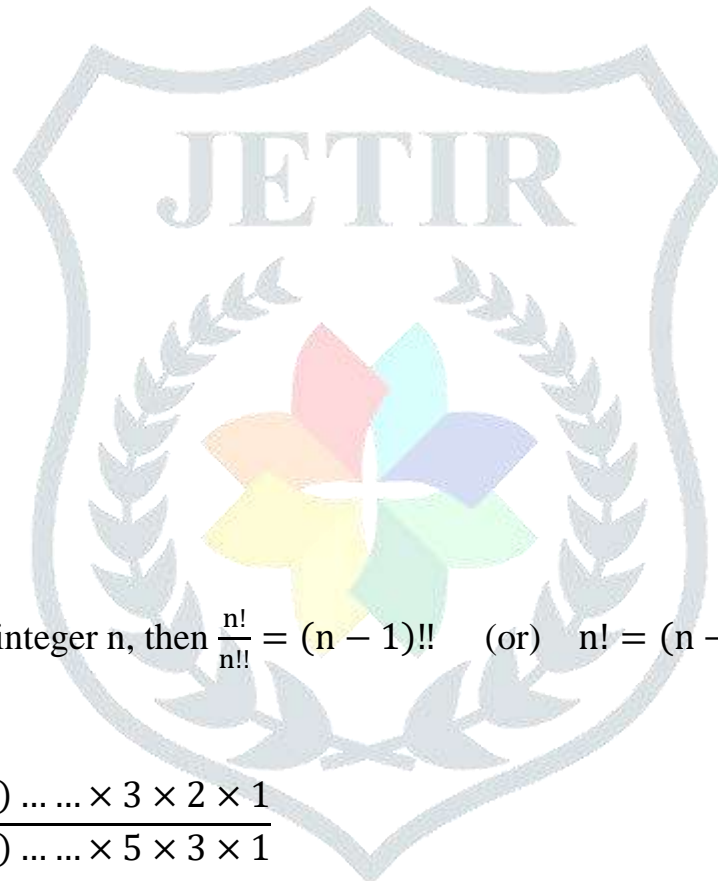
$$= (n-1)(n-3) \dots \times 3 \times 1$$

$$= (n-1)!!$$

Combining this cases for any non negative integer n

$$\frac{n!}{n!!} = (n-1)!!$$

$$\Leftrightarrow n! = (n-1)!! \times n!!$$



Example :For $n=6$

$$\begin{aligned}n! &= 6! \\ &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= 720\end{aligned}$$

$$\begin{aligned}(n-1)!! \times n!! &= 5!! \times 6!! \\ &= 5 \times 3 \times 1 \times 6 \times 4 \times 2 \\ &= 720\end{aligned}$$

Result :1

$$0!! = 1 \text{ and } (-1)!! = 1$$

Proof

$$\begin{aligned}\text{w.k.t } n!! &= 2^x x! \\ 0!! &= 2^0 0! \\ &= 1\end{aligned}$$

Result:2

$$\begin{aligned}n!! &= n(n-2)!! \\ (n+1)!! &= (n+1)(n-1)!!\end{aligned}$$

Example:

$$\begin{aligned}6!! &= 6 \times 4!! \\ &= 6 \times 4 \times 2!! \\ &= 6 \times 4 \times 2 \\ &= 48\end{aligned}$$

Result:3

$$n p_r = \frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}$$

Proof:

$$n p_r = \frac{n!}{(n-r)!}$$

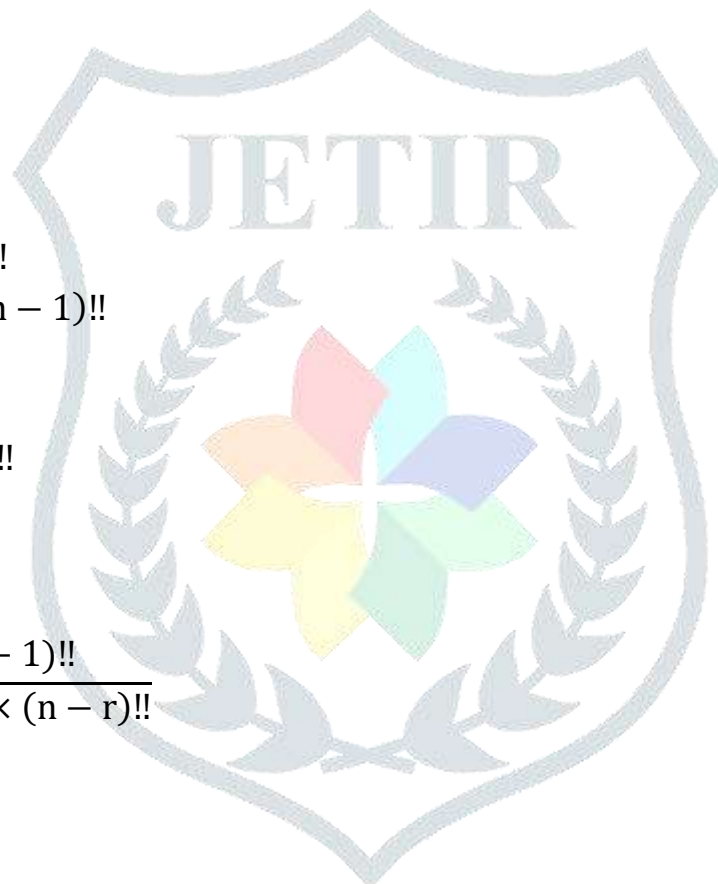
Since, $n! = (n-1)!! \times n!!$

Therefore,

$$n p_r = \frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}$$

Example:

$$\begin{aligned}6 p_3 &= \frac{6!! \times 5!!}{2!! \times 3!!} \\ &= \frac{6 \times 4 \times 2 \times 5 \times 3 \times 1}{2 \times 3 \times 1} \\ &= 120\end{aligned}$$



Result:4

$$n p_n = n!! \times (n - 1)!!$$

Proof:

w.k.t

$$\begin{aligned} n p_r &= \frac{n!! \times (n - 1)!!}{(n - r - 1)!! \times (n - r)!!} \\ n p_n &= \frac{n!! \times (n - 1)!!}{(n - n - 1)!! \times (n - n)!!} \\ &= \frac{n!! \times (n - 1)!!}{(-1)!! \times (0)!!} \\ &= n!! \times (n - 1)!! \end{aligned}$$

Example:

$$\begin{aligned} 6 p_6 &= 6!! \times 5!! \\ &= 6 \times 4 \times 2 \times 5 \times 3 \times 1 \\ &= 720 \end{aligned}$$

Result :5

$$n p_1 = n$$

Proof:

$$\begin{aligned} n p_r &= \frac{n!! \times (n - 1)!!}{(n - r - 1)!! \times (n - r)!!} \\ n p_1 &= \frac{n!! \times (n - 1)!!}{(n - 1 - 1)!! \times (n - 1)!!} \\ &= \frac{n!!}{(n - 2)!!} \\ &= \frac{n(n - 2)!!}{(n - 2)!!} \\ &= n \end{aligned}$$

Example:

$$\begin{aligned} 6 p_1 &= \frac{6!! \times 5!!}{4!! \times 5!!} \\ &= \frac{6 \times 4 \times 2}{4 \times 2} \\ &= 6 \end{aligned}$$

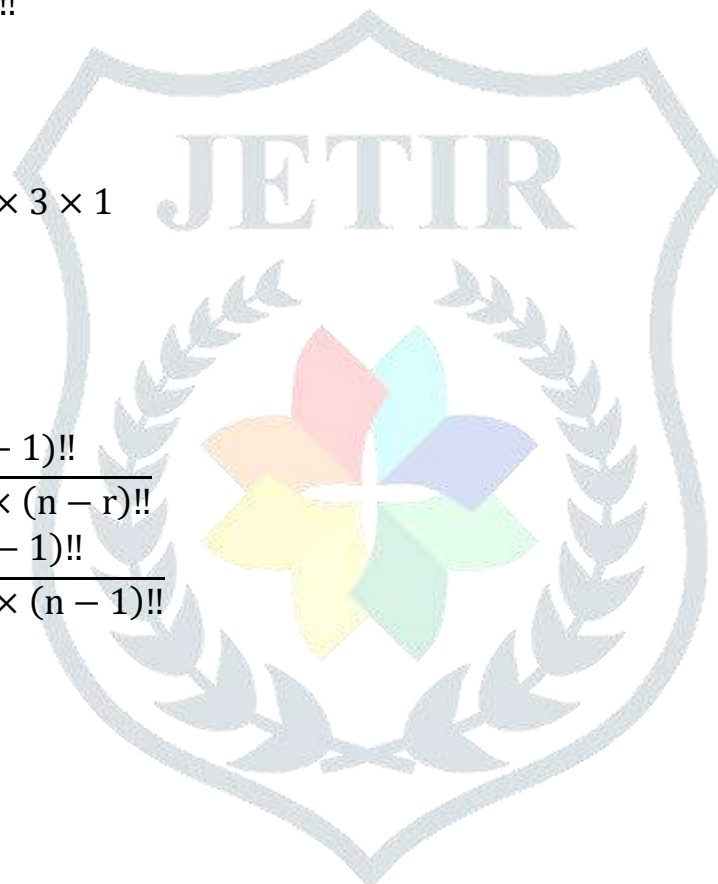
Result:6

$$n p_{n-1} = n p_n$$

Proof:

w.k.t

$$n p_n = n!! \times (n - 1)!! \quad \text{-----(1)}$$



$$n p_{n-1} = \frac{n!! \times (n-1)!!}{(n-n+1-1)!! \times (n-n-1)!!}$$

$$n P_n = \frac{n!! \times (n-1)!!}{0!! \times (-1)!!}$$

$$n p_{n-1} = n!! \times (n-1)!! \text{ -----(2)}$$

From and (1) and (2)

$$n p_{n-1} = n p_n$$

Example:

When $n=5$,

$$n p_{n-1} = n p_n$$

$$n p_{n-1} = 5 p_4$$

$$= 5!! \times 4!!$$

$$= 5 \times 3 \times 1 \times 4 \times 2$$

$$= 120$$

$$n p_n = 5 p_5$$

$$= 5!! \times 4!!$$

$$= 5 \times 3 \times 1 \times 4 \times 2$$

$$= 120$$

Result:7

$$n p_r = n (n-1) p_{r-1}$$

Proof:

$$\text{RHS} = n (n-1) p_{r-1}$$

$$= \frac{n (n-2)!! \times (n-1)!!}{(n-1-r+1-1)!! \times (n-1-r+1)!!}$$

$$= \frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}$$

$$= n p_r$$

$$= \text{LHS}$$

Example:

$$\text{LHS} = 6 p_3$$

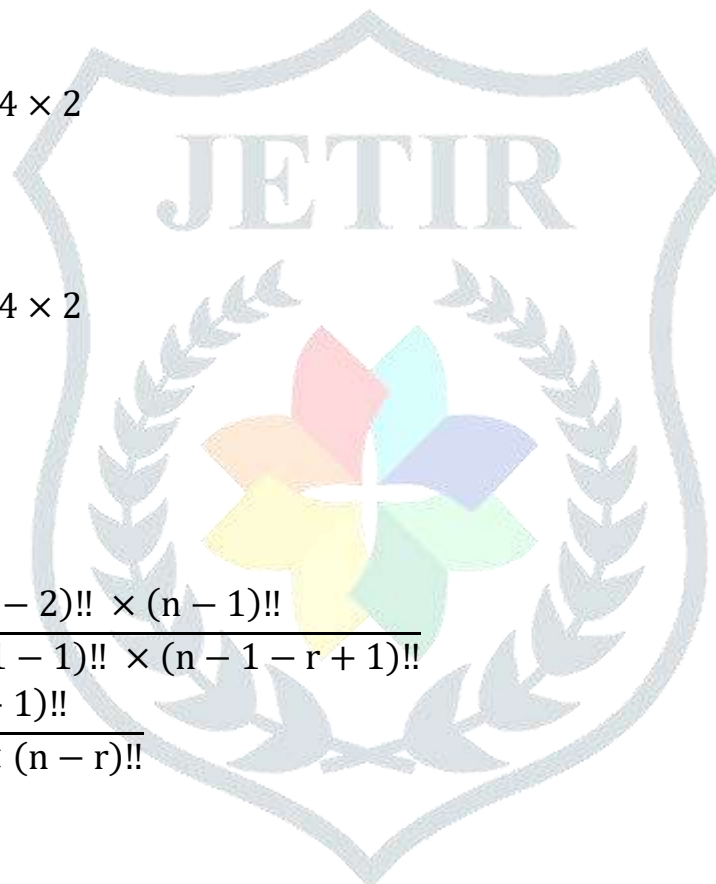
$$= \frac{6!! \times 5!!}{2!! \times 3!!}$$

$$= \frac{6 \times 4 \times 2 \times 5 \times 3 \times 1}{2 \times 3 \times 1}$$

$$= 120$$

$$\text{RHS} = 6 \cdot 5 p_2$$

$$= 6 \times \frac{4!! \times 5!!}{2!! \times 3!!}$$



$$= \frac{6 \times 4 \times 2 \times 5 \times 3 \times 1}{2 \times 3 \times 1}$$

$$= 120$$

Result:8

$$np_0 = 1$$

Proof:

w.k.t

$$np_0 = \frac{n!! \times (n-1)!!}{(n-1)!! \times (n)!!}$$

$$= 1$$

Example:

$$5p_0 = 1$$

Result:9

$$\frac{np_r}{np_{r-1}} = n - r + 1$$

proof

$$\frac{np_r}{np_{r-1}} = \frac{\frac{n!! \times (n-1)!!}{(n-r-1)!! \times (n-r)!!}}{\frac{n!! \times (n-1)!!}{(n-r+1-1)!! \times (n-r+1)!!}}$$

$$= \frac{(n-r)!! \times (n-r+1)!!}{(n-r-1)!! \times (n-r)!!}$$

$$= \frac{(n-r+1)(n-r+1)!!}{(n-r-1)!!}$$

$$= n - r + 1$$

Example:

$$\frac{5p_3}{5p_2} = \frac{4!! \times 5!!}{2!! \times 1!!} \times \frac{2!! \times 3!!}{4!! \times 5!!}$$

$$= 3$$

$$= 5 - 3 + 1$$

Result:10

$$(n-1)p_r + r(n-1)p_{r-1} = np_r$$

Proof:

$$\text{LHS} = (n-1)p_r + r(n-1)p_{r-1}$$

$$= \frac{(n-2)!!(n-1)!!}{(n-r-2)!!(n-r-1)!!} + \frac{r(n-2)!!(n-1)!!}{(n-r-1)!!(n-r)!!}$$

$$= \frac{(n-r)(n-2)!!(n-1)!!}{(n-r)(n-r-2)!!(n-r-1)!!} + \frac{r(n-2)!!(n-1)!!}{(n-r-1)!!(n-r)!!}$$

$$= \frac{(n-r)(n-2)!!(n-1)!!}{(n-r)!!(n-r-1)!!} + \frac{r(n-2)!!(n-1)!!}{(n-r-1)!!(n-r)!!}$$

$$\begin{aligned}
&= \frac{(n-2)!!(n-1)!!}{(n-r)!!(n-r-1)!!} (n-r+r) \\
&= \frac{(n-2)!!(n-1)!!}{(n-r)!!(n-r-1)!!} (n) \\
&= \frac{(n)!!(n-1)!!}{(n-r)!!(n-r-1)!!} \\
&= n p_r \\
&= \text{RHS}
\end{aligned}$$

Example:

When $n=5$,

$$\begin{aligned}
5p_3 &= \frac{4!! \times 5!!}{2!! \times 1!!} \\
&= \frac{4 \times 2 \times 5 \times 3 \times 1}{2 \times 1} \\
&= 60
\end{aligned}$$

$$\begin{aligned}
4p_3 + 3 \cdot 4p_2 &= \frac{4!! \times 3!!}{0!! \times 1!!} + 3 \frac{4!! \times 3!!}{2!! \times 1!!} \\
&= \frac{4 \times 2 \times 3 \times 1}{1} + \frac{3 \times 4 \times 2 \times 3 \times 1}{2 \times 1} \\
&= 24 + 36 \\
&= 60
\end{aligned}$$

Conclusion

In this paper I have calculated and proved some results with example, properties of permutation applying the technique of double factorial and I extend my paper to solve the same concept to find out the properties of combinations.

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