Universe dominated by dark energy in presence of polytropic gas

Prasanta Das[1], Dr.Shibu.Basak[2]
[1]Research Scholar, Department of Mathematical Sciences, Bodoland University, Kokrajhar, BTC, Assam, India.
[2] Associate Prof.and Head of Department of Mathematics, Kokrajhar Govt. College, Kokrajhar, BTC, Assam, India

Abstract: Polytropic gas has been thought as an alternative dark energy model by many cosmologists. Here we have discussed about the cases of dominance of phantom dark energy or quintessence in universe due to the presence of polytropic gas.

Keywords: ***Dark energy, Polytropic gas.***

1. INTRODUCTION :- Many Cosmological experiments and observations such as Type 1a Supernovae [1]-[3], Cosmic Microwave Background Radiation [4], Large Scale Structure [5], [6], Wilkinson Microwave Anisotropy Probe [7], Sloan Digital Sky Survey [8], etc. indicates that our universe expands under an accelerated expansion. In standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [9],[10]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field. About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy [11]. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models to explain the cosmic acceleration of the universe [12]-[14]. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [15].

2. FORMULATION OF THE PROBLEM:- The polytropic gas has been proposed as an alternative dark energy model to explain the acceleration of the universe and its equation of state (EOS) is given by [13]

$$p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}} \tag{1}$$

Where p_Λ, ρ_Λ, k, and n are the pressure, energy density, polytropic constant and polytropic index respectively.

The conservation equation for the dark energy in the FRW universe is given by

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0 \tag{2}$$

Where H is the Hubble parameter and a dot are denotes the differentiation with respect to the cosmological time.

Using the EOS (1) into the conservation equation (2) and integrating we get

$$\rho_\Lambda = \left[-k + Ba^{3/n}\right]^{-n} \tag{3}$$

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe [13]

When $k < Ba^{3/n}$, we see that $\rho_\Lambda > 0$ for any arbitrary value of n; when $k > Ba^{3/n}$, we see that $\rho_\Lambda > 0$ for even value of n.Also when $k = Ba^{3/n}$,we see that $\rho_\Lambda \to \infty$ and the polytropic gas has a finite time singularity at $a_s = \left(\frac{k}{B}\right)^{n/3}$.

Using equations (1) & (3), the EOS parameter of the polytropic gas dark energy model is obtained as

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - k} \quad (4)$$

When $k > Ba^{3/n}$, we see that $\omega_\Lambda < -1$ which corresponds to a universe dominated by phantom dark energy; when $k < Ba^{3/n}$, we see that $\omega_\Lambda > -1$ which corresponds to a quintessence like accelerated universe; also when $k = Ba^{3/n}$, we see that $\omega_\Lambda \to \infty$ which corresponds to a singularity at $a_s = \left(\frac{k}{B}\right)^{n/3}$.

Now we assume the scalar field and potential dark energy model. The energy density and pressure of the scalar field $\phi(t)$ and potential $V(\phi)$ are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (5)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (6)$$

Where $\frac{1}{2}\dot{\phi}^2$ is the kinetic energy and $V(\phi)$ is the potential energy of the scalar field ϕ

Using equations (1) &(3) into the equations (5)&(6) we can find the scalar potential and kinetic energy terms for the polytropic gas as

$$V(\phi) = \frac{\frac{B}{2}a^{3/n} - K}{\left(Ba^{3/n} - K\right)^{n+1}} \quad (7)$$

$$\dot{\phi}^2 = \frac{Ba^{3/n}}{\left(Ba^{3/n} - K\right)^{n+1}} \quad (8)$$

When $k > Ba^{3/n}$, we see that $\dot{\phi}^2 < 0$(negative kinetic energy), therefore the scalar field is a phantom field. The phantom field lead to super accelerated expansion of the universe. When $k < Ba^{3/n}$, we see that, $\dot{\phi}^2 > 0$(positive kinetic energy), therefore the scalar is a quintessence field.

Using equations (5) & (6), the EOS parameter for the scalar fields is

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} = -1 + \frac{2\dot{\phi}^2}{\dot{\phi}^2 + 2V(\phi)} \quad (9)$$

When $\dot{\phi}^2 = 0$, then equation (9) gives $\omega_\phi = -1$, when $V(\phi) = 0$, then equation (9) gives $\omega_\phi = 1$. Here $\omega_\phi = -1$and $\omega_\phi = 1$ representing the vacuum fluid and stiff fluid dominated universe respectively. When $V(\phi) > \frac{1}{2}\dot{\phi}^2$, then equation (9) gives $\omega_\phi > -1$ which corresponds a quintessence dominated universe.

Considering negative kinetic energy in the equations (5) & (6), we get as follows

$$\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (10)$$

$$p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi) \quad (11)$$

Using equations (10) & (11), the EOS of parameter for the scalar field is

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{-\frac{1}{2}\dot{\phi}^2 - V(\phi)}{-\frac{1}{2}\dot{\phi}^2 + V(\phi)} = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 - 2V(\phi)} = -1 + \frac{2\dot{\phi}^2}{\dot{\phi}^2 - 2V(\phi)} = -1 + \frac{\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 - V(\phi)} \quad (12)$$

When $\dot{\phi}^2 = 0$, then equation (12) gives $\omega_\phi = -1$; when $V(\phi) = 0$, then equation (12) gives $\omega_\phi = 1$,When $V(\phi) > \frac{1}{2}\dot{\phi}^2$, then equation (12) gives $\omega_\phi < -1$ which corresponds a phantom field. The phantom field lead to super accelerated expansion of the universe.

3. CONCLUSIONS :- Due the presence of polytropic gas in the form $p_\Lambda = k\rho_\Lambda^{1+\frac{1}{n}}$, where p_Λ, ρ_Λ , k and n are pressure, energy density, polytropic constant and polytropic index, a universe may be dominated by phantom dark energy or quintessence dark energy according as $\omega_\Lambda \left(= \frac{p_\Lambda}{\rho_\Lambda}\right) \lessgtr -1$. Also for $V(\phi) > \frac{1}{2}\dot{\phi}^2$ the universe may be dominated by phantom dark energy or quintessence dark energy according as negative kinetic energy$(-\frac{1}{2}\dot{\phi}^2)$ or positive kinetic energy$(\frac{1}{2}\dot{\phi}^2)$.

Acknowledgments

We would like to thank Dr.D.K.Mitra (HOD), Department of Mathematical Sciences, Bodoland University, Kokrajhar, BTC, Assam, India and our colleagues for valuable Suggestions.

REFERENCES

[1] S.Perlmutter, "Supernovae, Dark Energy and the accelerating Universe", *Physics Today*, vol 56, pp. 53, April 2003.
[2] A.G. Riess, et al.: "Type Ia Supernova Discoveries at z > 1 from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution", *Astrophys. J.*, vol. 607, pp. 665-687, June 2004
[3] A.G. Riess, et al., "New Hubble space telescope discoveries of type Ia supernovae at z≥ 1: narrowing constraints on the early behavior of dark energy", *Astrophys. J* .vol. 659, pp. 98–121, Apr 2007
[4] R. R. Caldwell and M. Doran, "Cosmic microwave background and supernova constraints on quintessence: concordance regions and target models," *Physical Review D*, vol. 69, no.10, Article ID103517, 2004
[5] T. Koivisto and D. F. Mota, "Dark energy anisotropic stress and large scale structure formation," *Physical Review D*, vol. 73, no. 8, Article ID 083502, 12 pages, 2006. View at Publisher · View at Google Scholar ·View at MathSciNet
[6] S. F. Daniel, R. R. Caldwell, A. Cooray, and A. Melchiorri, "Large scale structure as a probe of gravitational slip," *Physical Review D*, vol. 77, no. 10, Article ID 103513, 12 pages, 2008. View at Publisher· View at Google Scholar
[7] D. N. Spergel, et al., "First-Year Wilkinson Microwave Anisotropy Probe (WMAP)*Observations: Determination of Cosmological Parameters", *Ap J S*, vol.148, pp.135-159, Sep. 2003.
[8] M.Tegmark, et al,"Cosmological parameters from SDSS and WMAP", *Physical Review D*, vol-69, pp.103501, May 2004
[9] J.M.Overduin, F.I. Cooperstok,"Evolution of the scale factor with a variable cosmological term", Phys.Rev. D, vol-58, 043506, July 1998
[10] V.Sahni, A.Starobinsky, "The case for a positive cosmological Λ term", *Int. J. Mod.Phys. D*, vol-09, No-04, pp373-443, August 2009.
[11] S.Weinberg, "The cosmological constant" Rev. Mod. Phys, vol-61, no.-1, pp.1-23, January1989
[12] U. Mukhopadhyay, S. Ray, S.B. Dutta Choudhury, "Dark energy with polytropic equation of state", *Mod.Phys. Lett.A*, vol. 23, pp. 3187-3198, Dec 2008.
[13] K. Karami, S. Ghaffari, J. Fehri "Interacting polytropic gas model of phantom dark energy in non-flat universe", *Eur. Phys. J. C.*, vol. 64, pp.85-88, Nov.2009
[14] J. Christensen-Dalsgard, *Lecture Notes on Stellar and Structure and Evolution*, 6th edn, Aarhus University Press, Aarhus, 2004.
[15] S.Nojiri, S.D. Odintsov, S.Tsujikawa, "Properties of singularities in (phantom) dark energy universe" Phys.Rev. D, vol-71, 063004, March 2005