SQUARE MULTIPLICATIVE LABELING ONCERTAIN GRAPHS

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ABSTRACT: *G* is said to be a *Square multiplicative labeling* if there exists a bijection $f:V(G)\rightarrow\{1,2,3,......p\}$ such that the induced function $f^*:E(G)\rightarrow N$ given by $f^*(uv)=[f(u)]^2.[f(v)]^2$ for every $uv\in E(G)$ are all distinct. A graph which admits Square multiplicative labeling is called *Square multiplicative graph*.In this paper, we show that the barbell graph, square graph of the path graph,middle graph of the path graph,the corona graph $P_n \odot k_1$ admit square multiplicative labeling.

KEYWORDS: Square multiplicative labeling, Barbellgraph, Squaregraph, Middlegraph,corona graph

AMS Subject Classification (2010): 05C78.

1Introduction

Labeling of graph *G* is the assignment of labels, typically represented by integers to edges or vertices or both. A useful survey on graph labeling by J.A.Gallian (2015) can be found in [3].For the past few decades several modifications in the methods of labeling have evolved. One such labeling method is the square multiplicative labeling[4],[5]. In this paper we consider a simple, finite, connected and undirected graph.

Definition 1.1 :*G* is said to be a *Square multiplicative labeling* if there exists a bijection $f:V(G)\rightarrow\{1,2,3,......p\}$ such that the induced function $f^*:E(G)\rightarrow N$ given by $f^*(uv)=[f(u)]^2.[f(v)]^2$ for every $uv\in E(G)$ are all distinct. A graph which admits Square multiplicative labeling is called *Square multiplicative graph*.

Definition 1.2 :A *Barbell graph*[1]*B*(*p*, *n*) is the graph obtained by connecting n-copies of a complete graph K_p by *n-1* bridges.

Definition 1.3 : The square of *G* is a graph constructed from *G* by adding edges between vertices that are at a distance two in*G*.

Definition 1.4 : The Middle graph[7] of *G* denoted by M(*G*) has the vertex set V(*G*)UE(*G*).Two vertices *x*, *y* in the vertex set of M(*G*) are adjacent in M(*G*) in case one of the following holds (i) *x*, *y* are in E(*G*) and *x* , *y* are adjacent in *G*. (ii) *x* is in V(*G*) , *y* is in E(*G*) , *x* and *y* are incident in *G*.

Definition 1.5 : The join of two graphs G and H , denoted by G+H, is the graph with the vertex set $V(G)\cap V(H)$ where $V(G)\cap V(H)=\phi$ and each vertex of G is adjacent to every vertex of H .When H= k_1 , this is the corona graph[6] $k_1 \odot$ G.

2 Related work

Many results have been proved in square multiplicative labeling some of them are, Every Cycle with one chord is a Square multiplicative graph [5], Every Cycle with twin chords are square multiplicative graphs [5], Quadrilateral snakes are square multiplicative graphs [5], Triangular snakes are square multiplicative graph [5], Bistar $B_{n,n}$ is a square multiplicative graph [5], The double fan graph DF_m is square multiplicative[4],The gear graph $G_m$is square multiplicative[4]. The helm H_m is square multiplicative[4],The flower graph Fl_m is square multiplicative[4],The Friendship graph F_m is square multiplicative[4], The fan graph f_m is square multiplicative [4].

3 Main Results

Theorem 3.1 : The Barbell graph *B*(*p*, *n*) is square multiplicative for $3\le p\le 5, n\ge 2$.

Proof: Let $u_1,u_2,........u_{np}$ be the vertices of *B*(*p*, *n*) and let E be the edge set of *B*(*p*, *n*) .

We note that

$|V(B(p,n)|=np$,

$|E(B(p,n)|=np+(n-1), p=3, n\ge 2.$

$|E(B(p,n)|=np+(3n-1), p=4, n\ge 2.$

$|E(B(p,n)|=np+(6n-1), p=5, n\ge 2.$

Define $f : V(B(p, n)) \to \{1, 2, \ldots.np\}$ as given below:

$f(u_i) = i, 1 \le i \le np$.

The function f induces a square multiplicative labeling of $B(p, n)$.

For if, f^* be the induced function defined by $f^* : E \to N$ such that $f^*(u_i u_j) = i^2 j^2$.

For the Barbell graph $B(p, n)$, when $p = 3$

Let $E = E_1 \cup E_2 \cup E_3 \cup E_4$ where

$E_1 = \{ e_i / e_i = u_{3i} u_{3(i+1)}, 1 \le i \le n-1 \}$,

$E_2 = \{ e_i / e_i = u_{3i} u_{3i-1}, 1 \le i \le n \}$,

$E_3 = \{ e_i / e_i = u_{3i} u_{3i-2}, 1 \le i \le n \}$,

$E_4 = \{ e_i / e_i = u_{3i-2} u_{3i-1}, 1 \le i \le n \}$.

To prove that f^* is injective in E.

(i)**Claim 1 :** f^* is injective in E_1.

Let $e_i, e_j \in E_1$

$f^*(e_i) = f^*(u_{3i} u_{3(i+1)})$

$= (f(u_{3i}))^2 (f(u_{3(i+1)}))^2$

$= (3i)^2 (3(i+1))^2$

$f^*(e_i) = 3^4 i^2 (i+1)^2$.

$f^*(e_j) = f^*(u_{3j} u_{3(j+1)})$

$f^*(e_j) = (f(u_{3j}))^2 (f(u_{3(j+1)}))^2$

$= (3j)^2 (3(j+1))^2$

$f^*(e_j) = 3^4 j^2 (j+1)^2$

Clearly for $i \ne j$, $f^*(e_i) \ne f^*(e_j)$.

Hence f^* is injective in E_1.

We note that all the labeling of edges in E_1 are multiples of 3^4.

(ii)**Claim 2 :** f^* is injective in E_2.

Let $e_i, e_j \in E_2$

$f^*(e_i) = f^*(u_{3i} u_{3i-1})$

$= (f(u_{3i}))^2 (f(u_{3i-1}))^2$

$= (3i)^2 (3i-1)^2$

$f^*(e_i) = 3^2 i^2 (3i-1)^2$

$f^*(e_j) = f^*(u_{3j} u_{3j-1})$

$= (f(u_{3j}))^2 (f(u_{3j-1}))^2$

$= (3j)^2 (3j-1)^2$

$f^*(e_j) = 3^2 j^2 (3j-1)^2$

Clearly for $i \ne j$, $f^*(e_i) \ne f^*(e_j)$.

Hence f^* is injective in E_2.

We note that all the labelings of edges in E_2 are multiples of 3^2.

(iii)**Claim 3 :** f^* is injective in E_3.

Let $e_i, e_j \in E_3$

$f^*(e_i) = f^*(u_{3i}u_{3i-2})$

$= (f(u_{3i}))^2 (f(u_{3i-2}))^2$

$= (3i)^2 (3i-2)^2$

$f^*(e_i) = 3^2 i^2 (3i-2)^2$

$f^*(e_j) = f^*(u_{3j}u_{3j-2})$

$= (f(u_{3j}))^2 (f(u_{3j-2}))^2$

$= (3j)^2 (3j-2)^2$

$f^*(e_j) = 3^2 j^2 (3j-2)^2$

Clearly for $i \neq j$, $f^*(e_i) \neq f^*(e_j)$.

Hence f^* is injective in E_3.

We note that all the labelings of edges in E_3 are multiples of 3^2.

(iv)**Claim 4 :** f^* is injective in E_4.

Let $e_i, e_j \in E_4$

$f^*(e_i) = f^*(u_{3i-2}u_{3i-1})$

$= (f(u_{3i-2}))^2 (f(u_{3i-1}))^2$

$f^*(e_i) = (3i-2)^2 (3i-1)^2$

$f^*(e_j) = f^*(u_{3j-2}u_{3j-1})$

$= (f(u_{3j-2}))^2 (f(u_{3j-1}))^2$

$f^*(e_j) = (3j-2)^2 (3j-1)^2$

Clearly for $i \neq j$, $f^*(e_i) \neq f^*(e_j)$.

Hence f^* is injective in E_4.

(v)**Claim 5 :** f^* is injective among E_1, E_2, E_3 and E_4.

We note that all the labeling of edges in E_1 are multiple of 3^4.Hence it is very clear that all the labelings of edges of E_1 are distinct from the labelings of edges of E_2, E_3 and E_4. We also find that all the edge labelings of E_2, E_3 are multiples of 3^2 ,hence all thelabelings of edges of E_2 is distinct from the edge labelings of E_4 and all the labelings of edges of E_3 is distinct from the edge labelings of E_4. Now we have to show that labelings of edges in E_2 and E_3 are distinct.

(vi)**claim 5.1:** f^* is injective among E_2 and E_3.

Let $e_i \in E_2, e_j \in E_3$

$f^*(e_i) = f^*(u_{3i}u_{3i-1})$

$= (f(u_{3i}))^2 (f(u_{3i-1}))^2$

$= (3i)^2 (3i-1)^2$

$f^*(e_i) = 3^2 i^2 (3i-1)^2$

$f^*(e_j) = f^*(u_{3j}u_{3j-2})$

$= (f(u_{3j}))^2 (f(u_{3j-2}))^2$

$= (3j)^2 (3j-2)^2$

$f^*(e_j) = 3^2 j^2 (3j-2)^2$

Clearly for $i \neq j$, $f^*(e_i) \neq f^*(e_j)$.

Hence f^* is injective among E_2 and E_3 .

$\Rightarrow$ All the edge labels in E are distinct.Hence $B(3, n)$ admits square multiplicative labeling.

$\Rightarrow$ $B(3, n)$ is square multiplicative.

Similarly we can prove that all edge labels are distinct in $B(p, n)$, for $p=4$ and $p=5$.

Hence $B(p, n)$is a Square multiplicative Graph .

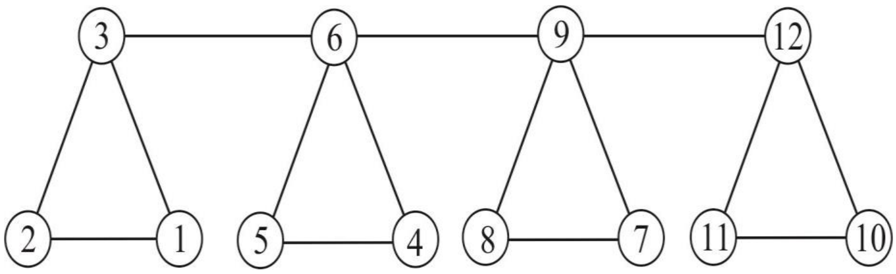

figure 1: square multiplicative labeling of $B(3, 4)$

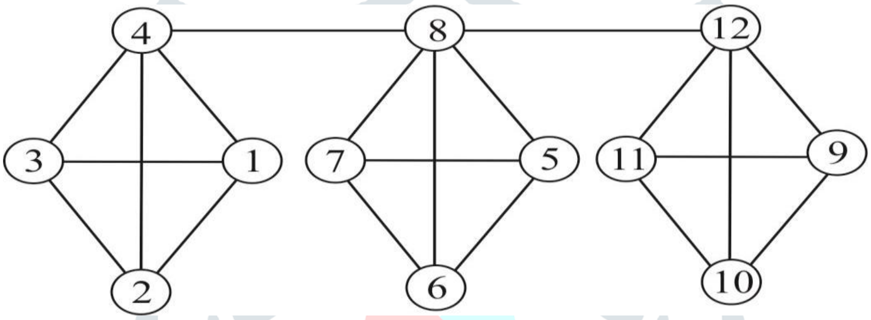

figure 2: square multiplicative labeling of $B(4, 3)$

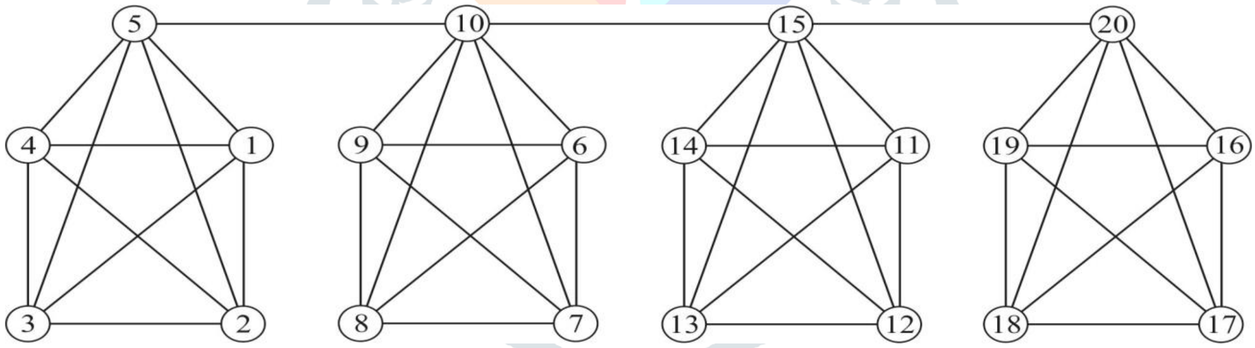

figure 3: square multiplicative labeling of $B(5, 4)$

Theorem 3.2 : For every positive integer n, $P_n^{\,2}$ is square multiplicative.

Proof: Let $u_1, u_2, ...u_n$ be the path vertices of $P_n^{\,2}$.

We note that $|V(P_n^{\,2})| = n$, $|E(P_n^{\,2})| = 2n-3$

Define $f : V(P_n^{\,2}) \to \{1, 2,.....n\}$ as given below:

$f(u_i) = i, 1 \le i \le n$.

The function f induces a square multiplicative labeling of $P_n^{\,2}$.

For if, f^* be the induced function defined by $f^* : E \to N$ such that $f^*(u_i u_j) = i^2 j^2$.

To prove that f^* is injective we have to prove that $f^*(u_i u_j) \ne f^*(u_{i+1} u_{j+1}), i \ne j$

Now $f^*(u_{i+1} u_{j+1}) = (i+1)^2 (j+1)^2$

$f^*(u_i u_j) = i^2 j^2 \ne (i+1)^2 (j+1)^2 = f^*(u_{i+1} u_{j+1})$

$\Rightarrow$ all the edge labels are distinct.

$\therefore\ P_n^{\,2}$ admits square multiplicative labeling

Hence $P_n^{\,2}$ is a Square multiplicative .

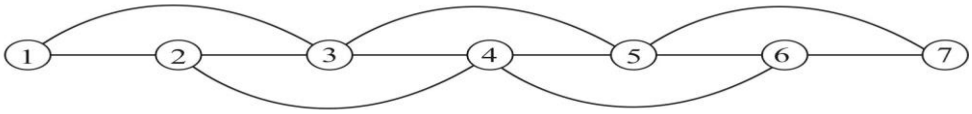

figure 4: square multiplicative labeling of $P_7^{\,2}$

Theorem 3.3 : The Corona graph $P_n \odot k_1$ is square multiplicative.

Proof: Let G denote the graph $P_n \odot k_1$ where P_n is the path graph with n vertices. let the spin vertices be denoted by $v_1, v_2, v_3 \ldots\ldots v_n$ and u_i be the pendent vertices adjacent to $v_1, v_2, v_3 \ldots\ldots v_n$

We note that $|V(G)| = 2n$ and $|E(G)| = 2n-1$.

Let E denote the edge set with $E = E_1 \cup E_2$

Where $E_1 = \{e_i \,/\, e_i = v_i v_{i+1}, 1 \le i \le n-1\}$,

$E_2 = (e_i \,/\, e_i = v_i u_i, 1 \le i \le n\}$.

Define $f : V(G) \to \{1, 2, 3, \ldots\ldots 2n\}$ as given below

$f(v_i) = 2i-1, 1 \le i \le n,$

$f(u_i) = 2i, 1 \le i \le n.$

The function f induces a square multiplicative labeling of G .

For if, f^* be the induced function defined by $f^* : E \to N$ such that $f^*(v_i u_j) = (2i-1)^2 (2j)^2$.

To prove that f^* is injective in E

(i)**Claim 1 :** f^* is injective in E_1 .

Let $e_i, e_j \in E_1$

$f^*(e_i) = f^*(v_i v_{i+1})$

$= (f(v_i))^2 (f(v_{i+1}))^2$

$f^*(e_i) = (2i-1)^2 (2i+1)^2$

$f^*(e_j) = f^*(v_j v_{j+1})$

$= (f(v_j))^2 (f(v_{j+1}))^2$

$f^*(e_j) = (2j-1)^2 (2j+1)^2$

Clearly for $i \ne j$, $f^*(e_i) \ne f^*(e_j)$ .Hence f^* is injective in E_1.

(ii)**Claim 2 :** f^* is injective in E_2

Let $e_i, e_j \in E_2$

$f^*(e_i) = f^*(v_i u_i)$

$= (f(v_i))^2 (f(u_i))^2$

$= (2i-1)^2 (2i)^2$

$f^*(e_i) = 2^2 (i^2)(2i-1)^2$

$f^*(e_j) = f^*(v_j u_j)$

$= (f(v_j))^2 (f(u_j))^2$

$= (2j-1)^2 (2j)^2$

$f^*(e_j) = 2^2 (j^2)(2j-1)^2$

Clearly for $i \ne j$ $f^*(e_i) \ne f^*(e_j)$. Hence f^* is injective in E_2 .

We note that all the labelings of edges in E_2 are multiples of 2^2

(iii)**Claim 3 :** f^* is injective in E_1, E_2

We note that all the labelings of edges in E_2 are multiples of 2^2 .Hence it is very clear that all the edge labels of E_2 are distinct from the edge labels of E_1 .

$\Rightarrow$ all the edge labels in E_1 and E_2 are distinct .

$\therefore$ G admits square multiplicative labeling.

Hence G is a Square multiplicative .

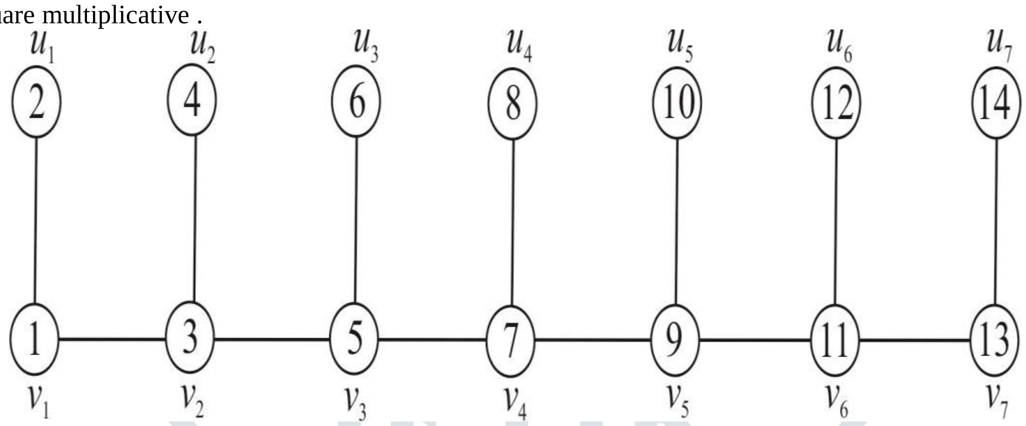

figure 5: square multiplicative labeling of P_7□ k_1

Theorem 3.4 : The Middle graph of the path graph $[M(P_m)]$ is square multiplicative.

Proof: Let $u_1, u_2, ...u_m$ be the path vertices of P_m and $e_1, e_2, ..., e_{m-1}$ be the edges of P_m . Then the vertex set of $M(P_m)$ is $V(M(P_m)) = \{u_1, u_2, u_3, ...u_m, e_1, e_2, e_3, ...e_{m-1}\}$.

Let E denote the edge set with $E = E_1 \cup E_2 \cup E_3$

Where $E_1 = \{ e_i / e_i = e_i e_{i+1}, 1 \le i \le m-2 \}$,

$E_2 = \{ e_i / e_i = u_i e_i, 1 \le i \le m-1 \}$,

$E_3 = \{ e_i / e_i = u_{i+1} e_i, 1 \le i \le m-1 \}$, .

We note that $|V(M(P_m))| = 2m-1$, $|E(M(P_m))| = 3m-4$.

Define $f : V(M(P_m)) \rightarrow \{1, 2, ..., 2m-1\}$ as given below:

$f(u_i) = 2i-1, 1 \le i \le n,$

$f(e_i) = 2i, 1 \le i \le n.$

The function f induces a square multiplicative labeling of $M(P_m)$.

To prove that f^* is injective in E

(i)**Claim 1 :** f^* is injective in E_1 .

Let $e_i, e_j \in E_1$

$f^*(e_i) = f^*(e_i e_{i+1})$

$= (f(e_i))^2 (f(e_{i+1}))^2$

$= (2i)^2 (2(i+1)^2)$

$f^*(e_i) = 2^4 (i)^2 (i+1)^2$

$f^*(e_j) = f^*(e_j e_{j+1})$

$= (f(e_j))^2 (f(e_{j+1}))^2$

$= (2j)^2 (2(j+1)^2)$

$f^*(e_j) = 2^4 (j)^2 (j+1)^2$

Clearly for $i \ne j$, $f^*(e_i) \ne f^*(e_j)$ .Hence f^* is injective in E_1 .

(ii)**Claim 2 :** f^* is injective in E_2

Let $e_i, e_j \in E_2$

$f^*(e_i) = f^*(u_i e_i)$

$= (f(u_i))^2 (f(e_i))^2$

$= (2i-1)^2 (2i)^2$

$f^*(e_i) = 2^2(i^2)(2i-1)^2$

$f^*(e_j) = f^*(u_j e_j)$

$= (f(u_j))^2 (f(e_j))^2$

$= (2j-1)^2 (2j)^2$

$f^*(e_j) = 2^2(j^2)(2j-1)^2$

Clearly for $i \neq j$, $f^*(e_i) \neq f^*(e_j)$ .Hence f^* is injective in E_2 .

(iii)**Claim 3 :** f^* is injective in E_3

Let $e_i, e_j \in E_3$

$f^*(e_i) = f^*(u_{i+1} e_i)$

$= (f(u_{i+1}))^2 (f(e_i))^2$

$= (2i+1)^2 (2i)^2$

$f^*(e_i) = 2^2(i^2)(2i+1)^2$

$f^*(e_j) = f^*(u_{j+1} e_j)$

$= (f(u_{j+1}))^2 (f(e_j))^2$

$= (2j+1)^2 (2j)^2$

$f^*(e_j) = 2^2(j^2)(2j+1)^2$

Clearly for $i \neq j$ $f^*(e_i) \neq f^*(e_j)$.

Hence f^* is injective in E_3 .

(iv) **Claim 4 :** f^* is injective in E_1, E_2 and E_3 .

We note that all the labelings of edges in E_1 are multiple of 2^4 .Hence it is very clear that all the edge labels of E_1 are distinct from the edge labels of E_2 an E_3 .Now we have to show that all the labelings of edges in E_2 and E_3 are distinct.

Let $e_i \in E_2, e_j \in E_3$

$f^*(e_i) = f^*(u_i e_i)$

$= (f(u_i))^2 (f(e_i))^2$

$= (2i-1)^2 (2i)^2$

$f^*(e_i) = 2^2(i^2)(2i-1)^2$

$f^*(e_j) = f^*(u_{j+1} e_j)$

$= (f(u_{j+1}))^2 (f(e_j))^2$

$= (2j+1)^2 (2j)^2$

$f^*(e_j) = 2^2(j^2)(2j+1)^2$

Clearly for $i \neq j$ $f^*(e_i) \neq f^*(e_j)$ .Hence all edge labelings in E_2 and E_3 are distinct.

$\therefore$ all edge labels in E_1 , E_2 and E_3 are distinct.

Hence all the edge labels in *E* are distinct.

$\Rightarrow M(P_m)$ admits square multiplicative labeling .

Hence $M(P_n)$ is a Square multiplicative .

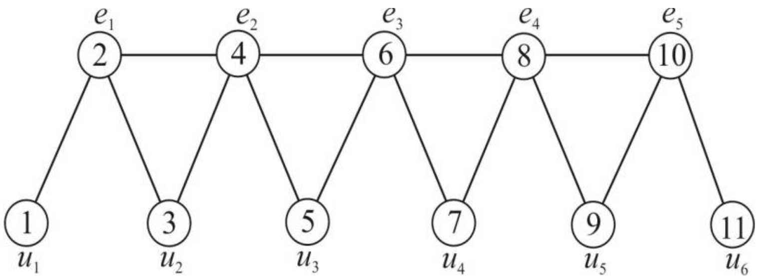

figure 6: square multiplicative labeling of $M(P_6)$

4Conclusion :

Since every graph is not a square multiplicative, it is very absorbing as well as challenging to investigate graphs which admits square multiplicative labeling. Here we have examined four results on square multiplicative labeling.This type of labeling can be extended to some other graphs.

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