ANTI-FUZZY SUB ALGEBRAS AND ANTI FUZZY K-IDEALS IN INK-ALGEBRAS

R.Rajakumari[1] and KR.Balasubramanian[2]
Research scholar[1] and Assistant Proferssor[2,]
[1,2]Department of mathematics, H. H. The Rajah's college,
Pudukkottai-622-001, Tamil Nadu, India

Abstract:

The aim of this paper is introduce the notion of anti-fuzzy sub algebras and anti-fuzzy K-ideals in INK-algebras, several theorems, properties are stated and proved. The fuzzy relations on INK-algebras are also studied.

Keywords:

INK- algebras, Anti homomorphism, Anti fuzzy K- ideal, anti-fuzzy subgroup, anti-fuzzy sub algebra, Cartesian product, level subset, conditions stated.

1. Introduction

K. Iseki and Tanaka introduced two classes of abstract algebras BCI-algebras and BCK-algebras in[2]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI- algebra. Q.P. Hu and X. Li introduced a wide class of abstract algebra namely BCH- algebras. L.A. Zadeh ,(1965), introduced the notion of fuzzy sets in [7]. This idea has been applied to several mathematical branches. Xi applied this concept to BCK- algebra.W.A. Dudek (1992) fuzzified the ideals in BCC- algebras. Y.B.Jun (2009) contributed a lot to develop the theory of fuzzy sets. M.Kaviyarasu et.all(2017), introduced a new notion called INK-algebra in[11], which is a generalization of TM/Q/ BCK/ BCI/ BCH-algebra and investigated some properties. In this study, we introduce the concepts of anti-fuzzy sub algebras and anti-fuzzy K-ideals in INK-algebra and investigate some of their properties.

2 Preliminaries

In this section we first review some elementary aspects which are useful in the sequel.

Definition: 2.1[2]

A BCK-algebra is algebra (X,*, 0) of type (2, 0) satisfying the following conditions:

i) $((x * y) * z = x * (z * (0* y))$,
ii) $(x *(x * y)) * y = 0$,
iii) $x * x = 0$,
iv) $0 * x = 0$,
v) $x * y = 0$and $y * x = 0$ imply $x = y$ for all $x, y, z \in X$ .We can
Define a partial ordering $\leq$on X by $x \leq y$ is defined by $x * y = 0$.

Definition 2.2.[11]

A INK-algebra (X,*,0) is a non-empty set X with a constant '0' and a binary operation * satisfying the following axioms :

i) $x * x = 0$,
ii) $x * 0 = x$, for $x \in X$.
iii) $0 * x = x$,
iv) $(x * y) * z = x * (z * (0 * y))$

In X we can define a binary relation $\leq$ by $x \leq y$ if and only if $x * y = 0$.

Example: 2.3.[11]

Consider the set X = {0, 1, 2, 3} with the following table

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then we easily check that (X,*, 0) is a INK-algebra since Then we easily can check that $(X,*,0)$ is a k-algebra, since we have $x* x = 0$, $x * 0 = 0$ and

$(x * y) * z = x * (z * (0 * y))$, for all $x, y, z \in X$ .But $(X,*,0)$ is not a INK-algebra,

Since $0 *1 \neq 0$

x=0,y=1,z=3

(0*1)*3=0*(3*(0*1))

(1*3) = (3*1)

2=2 hence verified,

3. Anti-Fuzzy sub algebras of k-ideals

Definition:3.1.

A fuzzy set μ of X *is* called a anti-*fuzzy sub algebra* of *K* if $\mu(x * y) \leq max\{\mu(x), \mu(y)\}$, for all $x, y \in X$`

Definition: 3.2.

A fuzzy set μ of X *is* called a anti fuzzy *ideals* of *K* if it satisfies:

(i) $\mu(0) \leq \mu(x)$,

(ii) $\mu(x) \leq max\{\mu(x * y), \mu(y * (y * x))\}$ for all x , y ϵX

Clearly x = 0 gives μ is a anti-fuzzy ideal of X.

Definition: 3.3.

Let G be a group. A fuzzy subset μ of a group G is called a anti fuzzy subgroup of the group if

(i) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$ for every x,y Є G and

(ii) $\mu(x^{-1}) = \mu(x)$ for every x Є G

Example:3.4.

Consider the set X = {0, 1, 2, and 3} with the following *cayley's* table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define a fuzzy set μ in X as by $\mu(0) = 0.5$,

$\mu(1) = 0.6$, $\mu(2) = 0.7$ $\mu(3)=0.9$.

. It is easy to verify that μ is a anti fuzzy K-ideal of X.

Theorem: 3.5.

If μ is a anti fuzzy sub algebra of a INK –algebra X, then $\mu(0) \leq \mu(x)$, for any $x \in X$.

Proof. Subsequently x * x = 0, for any $x \in X$, then:

$\mu(0) = \mu(x * x) \leq \max\{\mu(x), \mu(x)\}$

$\Rightarrow \mu(0) = \mu(x)$.

Theorem: 3.6.

If μ is a anti fuzzy sub algebra of a INK –algebra X, then $\mu(0) \leq \mu(x)$,for any $x \in X$.

Proof:

Subsequently $x * x = 0$, for any $x \in X$, then:

$$\mu(0) = \mu(x * x) \leq \max\{\mu(x), \mu(x)\}$$

$$\Rightarrow \mu(0) = \mu(x).$$

Theorem: 3.7.

A fuzzy set μ of an INK-algebra X is a anti fuzzy sub algebra if and only if for every $t \in [0,1]$ μ_t is either empty of a anti fuzzy sub algebra of X.

Proof.

Assume that μ is a anti fuzzy sub algebra of X. and ≠ ∅

Then for any $x, y \in \mu_t$

we have, $\mu(x * y) \leq \max\{\mu(x), \mu(y)\} \leq t$.

Therefore $x, y \in \mu_t$

μ_t. Hence μ_t is anti-fuzzy sub algebra of X. Conversely, μ_t is a anti fuzzy sub algebra of X. Let $x, y \in X$.

Take $t = \max\{\mu(x), \mu(y)\}$. Then by assumption μ_t is anti-fuzzy sub algebra of X implies:

$x * y \in \mu_t$. Therefore $\mu(x * y) \leq t = \max\{\mu(x), \mu(y)\}$

.Hence μ is a anti fuzzy sub algebra of X.

Definition: 3.8.

Let G be a group .A fuzzy subset μ of a group G is called a anti fuzzy subgroup of the group if

(i)$\mu(xy) \leq \max\{\mu(x), \mu(y)\}$), for every $x, y \in G$

(ii)$\mu(x^{-1}) = \mu(x)$, for every $x, y \in G$

Theorem: 3.9.

Anti-fuzzy sub algebra is a anti-fuzzy subgroup.

Proof:

Consider a anti-fuzzy sub algebra on the algebra A then there exists a Fuzzy set $\mu \in K^A$ then for

(i) nullary operation

$\mu(f(x_1 \ldots\ldots\ldots x_n)) \leq \mu(x_1) * \ldots\ldots\ldots\ldots * \mu(x_n)$ for all $x_1 \ldots\ldots\ldots x_n \in A$------------ (1)

From (1) let $x_2 = y$

$\mu(f(xy)) \leq \mu(x) * \mu(y)$ ---------(2)

since $\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$ ---------(3)

from (2) and (3)

$\mu(x) * \mu(y) = \mu(x * y) \leq \max\{\mu(x), \mu(y)\}$-------(4)

Let $\mu: X \rightarrow [0, 1]$

Therefore $\mu: X^{-1} \rightarrow [0, 1]$.

$\Rightarrow \mu(x^{-1}) = \mu(x)$ --------------(5)

(ii) nullary operation (for any constant)

If there exist any constant $\mu(c)$ then

$\mu(c) * \mu(x) = \mu(c * x) \leq \max\{\mu(c), \mu(x)\} = \mu(x)$------------(6)

$\mu(c) \leq \mu(x)$, for all $x \in A$.

∵ all the element of $\mu(x)$ lies between 0 & 1. Also $\mu(c)$ is any constant and ≤ 1. Using the results (4), (5) and (6), it is clear that anti fuzzy sub algebra is a anti fuzzy subgroup.

Theorem: 3.10.

A anti fuzzy sub algebra of a group G is a anti-fuzzy subgroup of the group G iff $\mu(xy^{-1}) \leq \max\{ \mu(x) , \mu(y) \}$ for every x,y Є G.

Proof:

Real part:

Consider a anti-fuzzy sub algebra of a group G is a anti fuzzy subgroup on the algebra A then we have to prove that

$$\mu(xy^{-1}) \leq \max \{ \mu(x) , \mu(y) \}$$

For anti-fuzzy sub algebra there exist a fuzzy set $\mu \in K_A$, then

(i) For n – ary operation

$$\mu(f(x_1 \ldots\ldots x_n)) \leq \mu(x_1) * \ldots\ldots * \mu(x_n) \text{ for all } x_1 \ldots\ldots x \; n \in A$$

Let $x_1, x_2 \in k$ where $k \in G$ .Consider $x_1 = x$ and $x_2 = y^{-1}$

Since it is a anti fuzzy subgroup using theorem (1)

Therefore $\mu(f(x_1\, x_2)) = \mu(f(x\, y^{-1}))$

$\leq \mu(x) * \mu(y^{-1})$

$\mu(x) * \mu(y^{-1}) = \mu(x * y^{-1})$

since anti fuzzy algebra satisfies fuzzy relation

so $\mu(x * y^{-1}) \leq \mu(x) \vee \mu(y^{-1})$ using equation (5)

$\leq \mu(x) \vee \mu(y)$

$\leq \max \{\mu(x), \mu(y)\}$

i) For nullary operation

$$\mu(c) \leq \mu(x) \text{ for all } x \in A.$$

Using theorem (1), it is a subgroup

∴ Let $\mu(e)$ be an identity element ,

We know that $\mu(c) \leq \mu(x)$

$\mu(c) = \mu(c) * \mu(e^{-1}) \leq \mu(c) * \mu(e)$ using equation (5)

$\leq \mu(c) \vee \mu(e)$

$\leq \max\{\mu(c), \mu(e)\} = \mu(e)$----------(7)

C is any constant it may be ≤ identity element

$\mu(x) = \mu(x) * \mu(e^{-1}) \leq \mu(x) * \mu(e)$ using equation (5)

$\leq \mu(x) \vee \mu(e)$

$\leq \max \{\mu(x), \mu(e)\} = \mu(x)$----------(8)

Converse part:

If $\mu (xy^{-1}) \leq \max \{\mu(x), \mu(y)\}$ for every x, y Є k and k Є G then a fuzzy partially ordered subset μ of a group is a anti fuzzy subgroup of the group G.

Proof:

$\mu (xy^{-1}) \leq \max\{ \mu(x) , \mu(y^{-1}) \}$

$\leq \max \{ \mu(x) , \mu(y) \}$ using (5)

$\leq \mu(x) \vee \mu(y)$

$\mu (xy^{-1}) = \mu(x * y^{-1})$ using fuzzy algebra relation.

$\mu(x * y^{-1}) = \mu(x) * \mu(y^{-1}) = \mu(x) * \mu(y)$

$\leq \mu(f(x\, y))$ (n- ary operation)

$\Rightarrow \mu(f(x\, y)) \leq \mu(x) * \mu(y)$-----------(9)

Let $\mu(c)$ and $\mu(e)$ be a any constant& identity element, then

Assume that $\mu(c) = \mu(x)$

$\mu(c) * \mu(e^{-1}) = \mu(x) * \mu(e^{-1})$

Using (5) $\mu(c) * \mu(e^{-1}) = \mu(x) * \mu(e^{-1})$

$\mu(c) * \mu(e) \leq \max \{ \mu(c) , \mu(e) \} = \mu(e)$

$\mu(x) * \mu(e) \leq \max\{ \mu(x) , \mu(e) \} = \mu(x)$

C is constant it may be ≤ identity element and x

Therefore, $\mu(c) \leq \mu(x)$ --------- (10)

From (9) and (10) we get, If $\mu (xy^{-1}) \leq \max\{ \mu(x) , \mu(y) \}$

Hence anti fuzzy sub algebra is a anti fuzzy subgroup

Result: 3.11.

A fuzzy subset μ of a group G is a anti fuzzy subgroup G⟺ $\mu(xy^{-1}) \leq \max \{ \mu(x) , \mu(y) \}$ for every x , y Є G

4. Anti-Fuzzy k-ideals in INK-algebra:

Definition: 4.1.

A fuzzy subset μ in a INK-algebra X is called a anti fuzzy ideal of X, if:

i) $\mu(0) \leq \mu(x)$

ii) $\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$,for all x, y, z Є X.

Definition:4.2.

A fuzzy subset μ in a INK-algebra X is called a anti fuzzy K- ideal of X, if:

(i)$\mu(0) \leq \mu(x)$

(ii)$\mu(x) \leq \max\{\mu(z * x) * (z * y), \mu(y)\}$, for all x, y, z Є X.

Clearly z = 0 gives μ is a anti fuzzy K- ideal of X.

Example: 4.3.

Consider INK-algebra X = {0, 1,2,3} with the following cayley table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define $\mu: X\to[0,1]$ *by* $\mu(0) = 0.2, \mu(1)= 0.3, \mu(2) = \mu(3)=0.4$

$\mu(0) \leq \max\{\mu(3 * 0), \mu (3*1), \mu(1)\}$

$= \max\{ \mu(3 * 1), \mu(1)\}$

$= \max\{\mu(2), \mu(1)$

$0.2 = \max\{0.4,0.3\}$

$\therefore 0.2 \leq 0.4$

Then it is easy to verify that μ is anti fuzzy K- ideal.

Theorem: 4.4.

Let μ be a anti fuzzy K- ideal of a INK-algebra X. Then the following are equivalent:

(K_1) μ is a anti fuzzy k-ideal of X,

(K_2) $\mu((x * y) * z) \leq \mu(x * (y * z))$ for all x, y, z ∈ X,

(K_3) $\mu(x * y) \leq \mu(x * (0 * y))$.

Proof.

(K_1)→ (K_2) since μ is a anti fuzzy K- ideal of X, we have

$\mu((x * y) * z) \leq \max\{\mu((x*y)*(0*z)), \mu(0)\}$
$= \mu((x*y)*(0*z))$. On the other hand
$(x*y)*(0*z) = (x * y) * ((y * z) * y)$
$\leq (x * (y * z)$,
Consequently $\mu((x * y) * z)) \leq \mu((x * y) * (0 * z))$.
$(K_2) \rightarrow (K_3)$ Letting $y = 0$ and $z = y$,
$(K_3) \rightarrow (K_1)$ since $(x * (0 * y)) * (x * (z * y)) \leq (z * y) * (0 * y) \leq z$,
we have $\mu(x * (0 * y)) \leq \max\{\mu(x * (z * y)), \mu(z)\}$.
Hence by hypothesis

$$\mu(x * y) \leq \max\{\mu(x * (z * y)), \mu(z)\}.$$

Therefore μ is a anti fuzzy K-ideal of X.

Theorem: 4.5.

Every anti fuzzy K-ideal μ of a INK-algebra X is order reversing, that is if $x \geq y$then: $\mu(x) \leq \mu(y)$, for all $x, y \in X$.

Proof.

Let $x, y \in X$ such that $x \geq y$. Therefore $x * y = 0$, Put $z = 0$,Now,

$\mu(x) = \mu(0 * x)$
$\leq \max\{\mu((z * x) * (z * y)), \mu(y)\}$
$= \max\{\mu((0 * x) * (0 * y)), \mu(y)\}$
$= \max\{\mu(x * y), \mu(y)\}$
$= \max\{\mu(0), \mu(y)\}$
$\mu(x) = \mu(y)$. Hence complete the proof.

Theorem: 4.6.

Let μ be an anti-fuzzy ideal of INK-algebra X. If $\mu(x * y) \leq \mu(x)$ for all $x, y \in X$, then μ is a K-ideal of X.

Proof:

Since μ is an anti-fuzzy ideal of X, by hypothesis we have
$\max\{\mu(x*(y*z)), \mu(y)\} \leq \max\{\mu((x * z) * (y * z)), \mu(y * z)\}$
$\leq \mu(x * z), \forall x, y, z \in X$.
Let X is an INK-algebra. A fuzzy set μ in X is called an anti-fuzzy sub algebra of X if $\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$ for all $x, y, z \in X$

Theorem: 4.7.

A fuzzy set μ in a INK-algebra X is a anti fuzzy K-ideal if and only if it is a anti fuzzy ideal of X.

Proof: Let μ be a anti fuzzy K-ideal of X.

Then i) $\mu(0) \leq \mu(x)$ ii) $\mu(x)$
$\leq \max\{\mu(x * y), \mu(y)\}$ for all $x, y, z \in X$.
Putting $z = 0$ in (ii) we have, $\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$.
Hence μ is a anti fuzzy ideal of X. Conversely, μ is a anti fuzzy ideal of X.

$$\mu(x) \leq \max\{\mu(x * y), \mu(y)\}$$

Then: $\mu(x) \leq \max\{\mu((0 * x) * (0 * y)), \mu(y)\}$.
if we replace z for 0,we have $\mu(x) \leq \max\{\mu((z * x) * (z * y)), \mu(y)\}$.
Hence a fuzzy set μ in a INK-algebra.

5 Cartesian products of anti-fuzzy K-ideals of INK-algebras

Definition: 5.1.

Let μ and ϑ be the fuzzy sets in a set X. The

Cartesian product $\mu \times \vartheta$: $X \times X \rightarrow [0, 1]$ is defined by:
$\mu \times \vartheta$ (x, y) = max {μ(x), ϑ (y)} , for all x, y 2 X.

Theorem: 5.2.

If μ and ϑ are anti-fuzzy K-ideals in a INK- algebra X, then $\mu \times \vartheta$ is a anti-fuzzy K-ideal in $X \times X$.

Proof.

If any $(x, y) \in X \times X$, we have:

i)$(\mu \times \vartheta)(0, 0)= \max \{\mu(0), \vartheta(0)\}$
$\leq \max \{\mu(x), \vartheta(y)\}$
$= (\mu \times \vartheta)(x, y)$.

ii) Let (x_1, x_2), (y_1, y_2) and $(z_1, z_2) \in X \times X$.

$(\mu \times \mu)(x_1, x_2) = (\mu \times \vartheta)(x_1 * x_2)$
$= \max \{\mu(x_1), \vartheta(x_2)\}$
$\leq \max \{\max \{((z_1 * x_1) * (z_1 * y_1)), \mu(y_1)\},$
$\max\{ \vartheta((z_2 * x_2) * (z_2 * y_2)), \vartheta(y_2)\}$
$= \max\{\max\{\mu((z_1 * x_1) * (z_1 * y_1)), \vartheta((z_2 * x_2) * (z_2 * x_2))\},$
$\max \{\mu(y_1), \vartheta(y_2)\}$
$= \max\{(\mu \times \vartheta)((z_1 * x_1) * (z_1 * y_1)), \vartheta((z_2 * x_2) * (z_2 * y_2)),$
$(\mu \times \vartheta)(y_1, y_2)\},$
$= \max\{(\mu \times \vartheta)((z_1, z_2) * (x_1, x_2)), \vartheta((z_1, z_2) * (y_1, y_2)), (\mu \times \vartheta)(y_1, y_2)\}.$

Hence $(\mu \times \vartheta)$ is a anti fuzzy K-ideal of a INK-algebra in $X \times X$.

Theorem: 5.3.

Let μ and ϑ be fuzzy sets in a INK-algebra X such that $\mu \times \vartheta$ is a anti fuzzy K-ideal of $X \times X$.

i) Either $\mu(0) \leq \mu(x)$ or $\vartheta(0) \geq \vartheta(x)$,for all x 2 X
ii) If $\mu(0) \leq \mu(x)$,for all $x \in X$, either $\vartheta(0) \leq \mu(x)$ or $\vartheta(0) \leq (x)$
iii) If $\vartheta(0) \leq \vartheta(x)$,for all $x \in X$, either $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \vartheta(x)$
iv) Either μ or ϑ is a anti fuzzy K-ideal of X.

Proof.

$\mu \times \vartheta$ is a anti fuzzy K-ideal of $X \times X$. Therefore $(\mu \times \vartheta)(0, 0) \leq (\mu \times \vartheta)(x, y)$,for all $(x, y) \in X \in X$ .And $(\mu \times \vartheta)(x_1, x_2)$
$\leq \max \{(\mu \times \vartheta)((z_1, z_2) * (x_1, x_2)) * ((z_1, z_2) * (y_1, y_2)), (\mu \times \vartheta)(y_1, y_2)\}$,for all (x_1, x_2), (y_1, y_2) and $(z_1, z_2) \in X \times X$.
Suppose that $\mu(0) > \mu(x)$ and $\vartheta(0) > \vartheta(y)$, for some $x, y \leq X$.
Then: $(\mu \times \vartheta)(x, y) = \max \{\mu(x), \vartheta(y)\} < \max \{\mu(0), \vartheta(0)\} = (\mu \times \vartheta)(0, 0)$. a contradiction.
Therefore either $\mu(0) \leq \mu(x)$ or $\vartheta(0) \leq \vartheta(x)$,for all $x \in X$.
Assume that $\exists$ x, y $\in$ X such that:
$\vartheta(0) > \mu(x)$ and $\vartheta(0) < \vartheta(y)$. Then: $(\mu \times \vartheta)(0, 0) = \max \{\mu(0), \vartheta(0)\} = \vartheta(0)$
and hence $(\mu \times \vartheta)(x, y) = \max \{\mu(x), \vartheta(y)\} < \vartheta(0)$
$= (\mu \times \vartheta)(0, 0)$, a contradiction. Hence if $\mu(0) \leq \mu(x)$,for all $x \in X$,
then either: $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \vartheta(x)$
Similarly we can prove that if$\vartheta(0) < \vartheta(x)$, for all $x \in X$,
then either $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \vartheta(x)$.
First we prove that ϑ is a anti fuzzy k-ideal of X.
Since ,by (i),either $\mu(0) \leq \mu(x)$ or $\vartheta(0) \leq \vartheta(x)$, for all $x \in X$.
Assume that $\vartheta(0) \leq \vartheta(x)$ for all $x \in X$. It follows from (iii)
That either $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \vartheta(x)$. If $\mu(0) \leq \vartheta(x)$, for any $x \in X$,
then: $\vartheta(x) = \max\{\mu(0), \vartheta(x)\}$

$= (\mu \times \vartheta)(0, x).\ \vartheta(x)$
$= \max\{\mu(0), \vartheta(x)\} = (\mu \times \vartheta)(0, x)$
$\leq \max\{(\mu \times \vartheta)((0, z) * (0, x)) * ((0 * z) * (0, y)), (\mu \times \vartheta)(0, y)\}$
$= \max\{(\mu \times \vartheta)(((0 * 0), (z * x)) * ((0 * 0), (z * y))), (\mu \times \vartheta)(0, y)\}$
$= \min\{(\mu \times \vartheta)(((0 * 0) * (0 * 0)), ((z * x) * (z * y))), (\mu \times \vartheta)(0, y)\}$
$= \min\{(\mu \times \vartheta)(0, ((z * x) * (z * y))), (\mu \times \vartheta)(0, y)\}$
$\vartheta(x) = \max\{\vartheta(z * x) * (z * y), \vartheta(y)\}$.
Hence ϑ is a anti fuzzy K-ideal of X.
Now we will prove that μ is a anti fuzzy K-ideal of X. Let $\mu(0) \leq \mu(x)$.
By(ii) either $\vartheta(0) \leq (x)$ or $\vartheta(0) \leq \vartheta(x)$.Assume that $\vartheta(0) \leq \vartheta(x)$,
Then: $\mu(x) = \max\{\mu(x), \vartheta(0)\}$
$= (\mu \times \vartheta)(x, 0)$.
$\mu(x) = \max\{\mu(x), \vartheta(0)\}$
$= (\mu \times \vartheta)(x, 0)$
$\leq \max\{(\mu \times \vartheta)(((z, 0) * (x, 0)) * ((z * 0) * (y, 0))), (\mu \times \vartheta)(y, 0)\}$
$= \max\{(\mu \times \vartheta)(((z * x), (0 * 0)) * ((z * y), (0 * 0))), (\mu \times \vartheta)(y, 0)\}$
$= \max\{(\mu \times \vartheta)(((z * x) * (z * y)), ((0 * 0) * (0 * 0))), (\mu \times \vartheta)(y, 0)\}$
$\mu(x) = \max\{\mu((z * y) * (y * z)), \mu(y)\}$.
Hence μ is a anti fuzzy K-ideal of X.

6. Anti-homomorphism of INK-algebras

Definition: 6.1.
Let X_1 and X_2 be K-algebras. A mapping
$g : X_1 \rightarrow X_2$ is said to be a anti homomorphism if it satisfies:
$f(x_1 * x_2) = f(x_2) * f(x_1)$, for all $x, y \in X$.

Definition:6.2.
Let $g : X^1 \rightarrow X^1$ be an endomorphism and μ
a fuzzy set in X^1. We define a new fuzzy set in X by μ_g in X by
$\mu_g(x) = \mu(g(x))$, for all x in X.

Theorem: 6.3.
Let f be an endomorphism of a INK- algebra
X^1. If μ is a anti fuzzy K-ideal of X^1, then so is μ_g.

Proof :
$\mu g(x) = \mu(g(x)) * \mu(0)$
$= \mu(g(0))$
$= \mu g(x)$. for all $x \in X^1$. Let $x, y, z \in X^1$.
Then: $\mu_g(x) = \mu(g(x))$
$\leq \max\{\mu((g(z) * g(x)) * (g(z) * g(y))), \mu(g(y))\}$
$= \max\{\mu((g(z * x)) * g(z * y)), \mu(g(y))\}$
$= \max\{\mu(g((z * x) * (z * y)), \mu(g(y))\}$
$= \max\{\mu(g((z * x) * (z * y)), \mu(g(y))$
$= \max\{\mu g((z * x) * (z * y)), \mu g(y))\}$.
Hence μ_g is a fuzzy k-ideal of X^1.

References

[1] K. Iseki, On BCI-algebras, Math. Seminar Notes, 8 (1980), 125130.
[2] K. Iseki and S. Tanaka, An introduction to the theory of BCK -algebras, Math. Japonica, 23 (1978), 126,
[3] J.Neggers and H.S.Kim , "On B-algebras",math,vensik,54 (2002), 21-29.
[4] Y.B.Jun., 2009."Generalization of (∈, ∈ ∧q) -fuzzy subalgebras in BCk /BCI-algebras". Comput.Math. Appl., 58: 1383-1390. DOI: 10.1016/j.camwa.2009.07.043
[5] A.Tamilarasi , K. Megalai, TM-algebra an introduction,CASCT,2010.
[6] K. Megalai . and A. Tamilarasi, "Fuzzy Subalgebras and Fuzzy T- ideals in TM- algebra",, Journal of Mathematics and Statistics 7(2) (2011) 107-111.
[7] L. A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965) 338-353.
[8] Laszlo filep.Study of fuzzy algebras and relations from a general viewpoint, acta Mathematica Academiae Paedagogicae Nyrlregyhaziensis,Tonus 14(1998), 49-55.
[9] W.B.Vasantha kandasamy. Smarandache fuzzy algebra.american research press, Rehoboth 2003.11-12.
[10] I. M. Anthony and H. Sherwood. Fuzzy groups redefined, Journal Math. Anal. Appl.,(1979) 69:124-130,
[11] M.Kaviyarasu, K.Indhira and V.M.Chandrasekaran (2017) " Fuzzy Sub algebras and Fuzzy K-ideals in INK-Algebras" Inter national Journal of Pure and Applied Mathematics Volume 113 No. 6 2017, 47 - 55
[12]W.A.Dudek, On proper BCC-algebra,Bull.Inst.Math Academia sinica,**20** (1992),137-150