THE STRONG NON SPLIT LINE DOMINATION OF A JUMP GRAPH

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ABSTRACT

A dominating set of a jump graph J(G)= (V,E) is a strong line dominating set if the induced sub graph < E-D > is complete. The strong non split line domination number γ'_{ns}(J(G)) of J(G) is minimum cardinality of a strong non split line dominating set. In this paper we relate this parameter to the parameters of jump graph J(G) and obtain its exact values for some standard graphs.

Key words: Dominating set of a jump graph, split line domination, non split line domination

INTRODUCTION:

All the graphs considered here are assumed to be finite, undirected, nontrivial and connected without loops or multiple edges. Any undefined term in this paper may be found in Haynes et.al.,[2]

Let J(G) be a jump graph. A set D <⊆ V is a dominating set of J(G) if every vertex in V-D is adjacent to some vertex in D. The domination number γ(J(G) of J(G) is the minimum cardinality of a dominating set.

N.Pratap Babu Rao and Sweta. N introduced the concept of Non split domination in jump graphs.

A dominating set D of a graph S_1={1} is a non split dominating set if the induced sub graph <V-D> is connected. The non split dominating number γ_{ns}(J(G)) of J(G) is the minimum cardinality of a non split dominating set.

Dominating sets whose complements induces a complete sub graphs have a great diversity of applications one such application is the following.

In setting up the communication links in a network One might want a strong core group that can communicate with each other . Member of the core group and so that every one in the group receives the message from some one outside the group and communicate it to every other in the group. This suggest the following definition.

Definition: A dominating set of vertices a γ-set if it is a dominating set with cardinality γ(J(G)). Similarly a γ_{ns}-set and a γ_{sns}-set are defined unless and otherwise stated, the graphs has p vertices and q edges.

2.RESULTS:

Theoem 1: For any graph J(G)

$$\gamma'(J(G) \leq \gamma'_{ns}(J(G)) \leq \gamma'_{sns}(J(G))$$

Proof; This follows from the fact that every strong non split dominating set of J(G) is a non split dominating set and every non split dominating set is a dominating set.

The following characterization is easy to see hence we omit its proof,

Theorem 2: A strong non split dominating set D of J(G) is minimal if and only if for each vertex v ∈ D, one of the following conditions is satisfied.

i) There exists a vertex u ∈ V-D such that N(v) ∩ D = {v}

ii) V is an isolated vertex in <D>

iii) There exists a vertex w ∈ V-D such that w is not adjacent to v

. Now, we obtain a relationship between γ'_{sns}(J(G)) and γ'_{sns}(J(H)) where J(H) is any spanning sub graph of J(G)

Theorem 3: For any spanning sub graph J(H) of J(G)

$\gamma'_{sns}(J(G)) \leq \gamma'_{sns}J(H))$

Proof: Since every strong non split dominating set of J(H) is a strong non split dominating set of J(G), Above inequality holds.

Next we obtain a lower bound in γ'_{sns}(J(G))

Theorem 4 : For any jump graph J(G)

$\beta_0(J(G)) \leq \gamma'_{sns}(J(G))$ where β_0(J(G)) is the independent number of J(G)

Proof : Let D be a γ'_{sns}(J(G))-set of J(G) and S be an independent set of vertices in J(G), Then either S⊆ D or S contains at most one vertex for V-D and at most |D|-1 vertices from D.

This implies $\beta_0(J(G)) \leq \gamma'_{sns}(J(G))$

The bound given above inequality is shows,

For example for a complete jump graph J(K_p) $\gamma'_{sns}(J(G)) = 1 = \beta_0(K_p)$

Now we prove the major result of this paper from which we can deduce the exact values of γ'_{sns}(J(G)) for some standard graphs.

Theorem 5: For any graph J(G), $p - w(J(G)) \leq \gamma'_{sns}(J(G))$

$\gamma'_{sns}(J(G)) \leq p - w(J(G)) + 1$

Where w(J(G)) is a clique number of J(G)

Proof: Let D be a γ_{sns}-set of J(G) since V-D is complement

$w(J(G)) \geq |V-D|$ …………………..(A)

Let <S> be a complete graph with |S\= W(J(G)) Then for any vertex u ∈ S, <V-S> ∪ |u| is a strong non split dominating set of J(G) and

$\gamma'_{sns}(J(G)) \leq q - w(J(G)) + 1$……………….(B)

From (A) and (B)

$q\text{-}w(J(G) \leq \gamma'_{sns}(J(G)) \leq q - w(J(G)) + 1$

Hence the result.

Corollary 5.1: let J(G) be a graph with $w(J(G)) \geq \delta'(J(G))$ then $\gamma'_{sns}(J(G)) \leq q - \delta'(J(G))$ where δ'(J(G))) is the minimum degree of L(J(G)). Further, the bound is attained if and only if one of the following is satisfied.

i) $W(J(G)) = \delta'(J(G))$

ii) $W(J(G)) = \delta'(J(G)) + 1$ and every w-set S contains a vertex not adjacent to a edge of E – S.

Proof: Suppose $W(J(G)) = \delta'(J(G)) + 1$. Then from Theorem (4) and (5)

$\gamma'_{sns}(J(G)) \leq q - w(J(G))$ suppose $w(J(G)) = \delta'(J(G))$ and let S be a w-set of J(G).Then E – S is a strong non split line dominating set of J(G) and hence

$\gamma'_{sns}(J(G)) \leq q - \delta(J(G))$.

Now we prove that the second part

Suppose one of the given conditions is satisfied. Then from Theorems (4) AND (5) it is easy to see that

$\gamma'_{sns}(J(G)) \leq q - \delta'(J(G))$ or $\gamma'_{sns}(J(G)) \leq q - \delta'(J(G)) + 1$ suppose there exists a w-set S with

|S| = δ'[(J(G)) + 1 such that every vertex in S is adjacent to some edge in E – S. Then E-S is a strong non split line dominating set of J(G), and hence $\gamma'_{sns}(J(G)) \leq q - \delta'(J(G)) - 1$, which is a contradiction.

Hence one of the given condition is satisfied.

In the next result we list the exact values of $\gamma'_{sns}(J(G))$ for some standard graphs

Proposition 6:

i) For any complete graph $J(K_p)$ with $p \geq 2$ vertices $\gamma'_{sns}(J(K_p)) = 1$

ii) For any complete bipartite jump graph $K_{m,n}$ with $2 \leq m \leq n$

iii) $\gamma'_{sns}(J(K_{m,n})) = m + n - 2$

iv) For any cycle $J(C_p)$ with $p \geq 3$ vertices $\gamma'_{sns}(J(C_p)) = p - 2$

v) For any path $J(P_p)$ with $p \geq 4$ vertices $\gamma'_{sns}(J(P_p)) = p - 2$

vi) For any wheel $J(W_p)$) with $p \geq 4$ vertices $\gamma'_{sns}(J(W_p)) = p - 3$.

A set D of vertices in a jump graph J(G) is a edge set dominating set if for any set $S \subseteq E-D$, there exists a edge x in D such that the induced sub graph $< S \cup \{x\} >$ is connected. The edge set domination number $\gamma'_{vs}(J(G)))$ of J(G) is the minimum cardinality of a edge set dominating set [8]

Theorem 7: If a graph J(G) has independent strong non split line dominating set then diam $(J(G)) \leq 3$ where diam (J(G)) is the diameter of J(G).

Proof: let D be an independent strong non split line dominating set of G.

We consider the following cases;

Case i) let $x, y \in E - D$ then $d(x, y)=1$

Case ii) let $x \in D$ and $y \in E - D$ Since D is independent there exists a edge $z \in E - D$ such that x is adjacent to z Thus $d(x, y) \leq d(x, z) + d(z, y) \leq 2$

Case iii) Let $x, y \in D$ As above there exists two edges $z_1, z_2 \in E - D$ such that x is adjacent z_1 and y is adjacent to z_2 Thus $d(x, y) \leq d(x, z_1) + d(z_1, z_2) + dz_2, y) \leq 3$

Thus for all edges $x, y \in E$, $d(x, y) \leq 3$

Hence $diam(J(G)) \leq 3$.

Corollary 7.1 If $\gamma'(J(G)) = \gamma'_{sns}(G)$, then diam $(J(G)) \leq 3$

Proof: let D be a γ'_{sns}-set of J(G). Since d is also a γ'-set every edge $y \in D$ is adjacent to at least one edge $x \in E - D$. As in the proof of theorem 7 one can show the $d(x, y) \leq 3$ for all edges $x,y \in E$.

Thus diam $(J(G)) \leq 3$

Theorem 8 : Let D be an independent set of vertices in J(G) if $|D| < 1 - \Delta'(J(\bar{G}))$ then E – D is a strong non split line dominating set of $J(\bar{G})$, Where $J(\bar{G})$ is the complement of J(G).

Proof: Since each edge $y \in D$ is not adjacent to at least one edge in E – D, it implies that E – D is a line dominating set of $J(\bar{G})$ and further it is a strong non split line dominating set as < D > is complete in $J(\bar{G})$.

A line dominating set D of a connected graph J(G) is a split line dominating set if the induced sub graph <E – D> is connected [4] . In [5] Kulli V. Rand B.Janakiram extended the concept of split domination to strong split domination as follows.

A line dominating set D of a connected graph G is a strong split line dominating set if induced sub graph <E-D> is totally disconnected with at least two vertices. The strong split line domination number $\gamma'_{sns}(\bar{G})$ of G is minimum cardinality of a strong split line dominating set.

Theorem 9: Let D be a γ'_{sns}-set of J(G), Then by (1) and (5) E-D has at least two edges Also by (ii) every edge in E-D is not adjacent to at least one edge in D. This implies that D is a line dominating set of $J(\bar{G})$

and further it is a strong split line dominating set , a <E-D> is totally disconnected with at least two edges in J($\bar{G}$).

Thus $\gamma'_{ss}(J(\bar{G})) \leq \gamma'_{sns}(J(G))$

Kulli V.R. and B.Janakiram [6] introduced then following concept.

A dominating set D of a graph G=(V,E) is a regular set dominating set if for any set $I \subseteq V - D$, there exists a set S⊆ D such that the induced sub graph < I ∪ D > is regular. The regular set domination number $\gamma_{rs+}(G)$ of G is the minimum cardinality of a regular et dominating set.

Theorem 10: For any graph J(G)

$\gamma'_{ns}(J(G)) \leq \gamma'_{sns}(J(G)) + 1.$

Proof: let D be a γ'_{sns}-set of J(G). Since <E-D> is complete for any vertex x ∈ E-D, D ∪{x} is a regular set line dominating set of J(G) This proves the result.

Theorem 11: If diam(J(G)) ≤ 3 then

$\gamma'_{ns}(J(G)) \leq q - m.$ where m is the number of cut edges of J(G)

Proof: If J(G) has no cut edges, then the result is trivial. Let S be the set of all cut edges with |S|=m, let x,y ∈ S suppose x and y are not adjacent. Sionce there exists two edges x_1 and y_1 such that x_1 is adjacent to x and y_1 is adjacent to y it implies that $d(x_1, y_1) \geq 4$. a contradiction. Hence every two edges in S are adjacent and every edge in S is adjacent to at least one edge in E-S. This proves that E-S is a strong non split line dominating set of J(G).

Hence the result.

Theorem 12: Let J(G) be a jump graph such that every edge of J(G) is either a cut edge or an end edge if V(J(G))= m then $\gamma'_{ns}(J(G)) = \gamma'_{sns}(J(G))$ = q-m where m is the number of cut edges of J(G)**.**

Proof: Let S be the set of all cut edges with |S|=m since w(G) = m it implies that every two edges in S are adjacent and hence every edge in S is adjacent to an end edge This proves that E-S is a γ'_{ns}-set of J(G) and further it is a $\gamma'_{sns}(J(G))$-set as <S> is complete

Hence the result.

The following definition is used to prove our next result.

A dominating set D of a graph J(G) is an efficient dominating set if every vertex in V-D is adjacent to exactly one vertex in D. This concept was introduced by Cockayne et.al.,.

Theorem13 : Let J(G) be an n-regular graph with 2n edges If D is an efficient dominating set of J(G)with n-edges then both D and E-D are strong non split line dominating sets of J(G).

Proof: Since every edge in E-D is adjacent to exactly one edge in D. it implies that every two edges in E-D aree adjacent. As J(G) is n-regular every edge in D is adjacent to some edge in E-D. Suppose there exists a edge x ∈ D such that x is adjacent to two or more edges in E-D. Then there exixts a edge y∈ D such that deg y≤ n-1, a contradiction. Hence every edge in D is adjacent to exactly one edge in E-D. Thus as above every two edges in D are adjacent. Hence D and E-D are strong non split line dominating sets of J(G).

Theorem 14: Let J(G) be a gaph with $\Delta'(J(G)) \leq$ q-2 when D be a strong non split line dominating set of G such that <D> is complete and $|D| \leq \delta'(J(G))$. Then (1) D is minimal (2) E-D is also a minimal strong non split line dominating set of J(G).

Proof: Since <D> is complete, it implies that for each edge y ∈ D there exists a edge x∈ V-D such that y is not adjacent to x. Then by theorem2 D is minimal.

$|D| \leq \delta'(J(G))$, it implies that every edge in D is adjacent to some edge in E-D. Then E-D is strong non split line dominating set of J(G) and further as above it is minimal.

Theorem 15: If $\Delta'(J(G)) < \alpha_1(J(G))$ then $\surd'_{sns}(J(G))$ = q- w(J($\bar{G}$)) where $\alpha_1(J(G))$ is the edge covering number of J(G).

Proof: Let S be a vertex cover of J(G) with $|S| = \alpha_1(J(G))$. Since $\Delta'(J(G)) < \alpha_1(J(G))$,

$J(G) \neq J(K_p)$ and E-S is an independent set with at least two edges such that every edge in E-S is not adjacent to at least one edge in S This proves that S is a strong non split line dominating set of $J(\bar{G})$

Thus $\surd'_{sns}(J(\bar{G})) \leq |S|$

$\leq \alpha_1(J(G))$.

$\leq q - \beta_1(J(G))$

$\leq q - w(J(\bar{G}))$

Result follows from theorem 5.

Next we obtain Nordhus Gaddum type result [7]

Theorem 16: Let J(G) e a graph such that both J(G) and $J(\bar{G})$ are connected . $w(J(G)) \geq \delta'(J(G))$ and $w(J(\bar{G})) \geq \delta'(J(\bar{G}))$

$\surd_{sns}(J(G) + \surd_{sns}(J(\bar{G})) \leq q + 1 + \Delta'(J(G)) - \delta'(J(G))$

Proof: By corollary 5.1 $\surd'_{sns}(J(G)) \leq q - \delta'(J(G))$

$\surd'_{sns}(J(\bar{G})) \leq q - \delta'(J(\bar{G})) \leq 1 + \Delta'(J(G))$

Hence the result.

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