ON r-KINGS IN FUZZY DIGRAPHS

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Abstract:

In this paper we introduce Fuzzy out neighbourhood of a set, Fuzzy In neighborhood of a set r-Kings in Fuzzy Digraphs. Also,Diameter and radius of Fuzzy Digraph are discussed.

Keywords:

Fuzzy digraph, Round Digraph,Fuzzy out neighbourhood of a set, Fuzzy In neighborhood of a set,Diameter of a fuzzy Digraph, Radius of a fuzzy Digraph.,r-Kings Fuzzy Digraph.

I INTRODUCTION

The concept of fuzzy graph was introduced by Rosenfeld [1] in 1975. Fuzzy graph theory has a vast area of applications. It is used in evaluation of human cardiac function, fuzzy neural networks, etc. Fuzzy graphs can be used to solve traffic light problem, time table scheduling, etc. In fuzzy set theory, there are different types of fuzzy graphs which may be a graph with crisp vertex set and fuzzy edge set or fuzzy vertex set and crisp edge set or fuzzy vertex set and fuzzy edge set or crisp vertices and edges with fuzzy connectivity, etc. A lot of works have been done on fuzzy graphs [3], [4], [5]..

The Fuzzy Directed Graph, Fuzzy competition digraphs are well known topic.The fuzzy round digraph,Quasi transitive digraph was defined and discssed.[6],[7] .In this article Fuzzy in neighbourhood of a set, fuzzy out neighbour hood of a set ,diameter and radius of a fuzzy digraph,,diameter and radius of a Fuzzy Round digraphs are defined and their properties are investigated.

II PRELIMINARIES

Definition 2.1:

Fuzzy digraph $\vec{\xi}$ = (V,σ,$\vec{\mu}$) is a non-empty set V together with a pair of functions σ : V → [0,1] and $\vec{\mu}$: V ×V → [0,1] such that for all x, y ∈ V , $\vec{\mu}$ (x,y) ≤ σ(x)∧σ(y).Since $\vec{\mu}$ is well defined, a fuzzy digraph has at most two directed edges (which must have opposite directions) between any two vertices. Here $\vec{\mu}$ (u,v) is denoted by the membership value of the edge $\overrightarrow{(u,v)}$.The loop at a vertex x is represented by $\vec{\mu}$ (x,x) ≠ 0. Here $\vec{\mu}$ need not be symmetric as $\vec{\mu}$ (x,y) and $\vec{\mu}$ (y,x) may have different values. The underlying crisp graph of directed fuzzy graph is the graph similarly obtained except the directed arcs are replaced by undirected edges

Definition 2.2:

Fuzzy out-neighbourhood of a vertex v of a directed fuzzy graph $\vec{\xi}$= (V,σ, $\vec{\mu}$) is the fuzzy set $N^+(v) = ({X_v}^+, {m_v}^+)$ where ${X_v}^+$ = {u | $\vec{\mu}$ (v,u) > 0} and ${m_v}^+$: ${X_v}^+$ → [0,1] defined by ${m_v}^+$ (u) = $\vec{\mu}$ (v,u).

Similarly, ***fuzzy in-neighbourhood*** of a vertex v of a directed fuzzy graph $\vec{\xi} = (V,\sigma,\vec{\mu})$ is the fuzzy set $N^-(v) = (X_v^-, m_v^-)$ where $X_v^- = \{u \mid \vec{\mu}(u,v) > 0\}$ and $m_v^- : X_v^- \to [0,1]$ defined by $m_v^-(u) = \vec{\mu}(u,v)$.

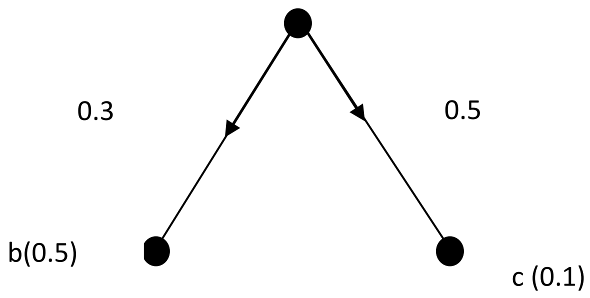

Figure:1

Example 2.2.1:

Let $\vec{\xi}$ be a directed fuzzy graph. Let the vertex set be {a,b,c} with membership values σ(a) = 0.4, σ(b) = 0.5, σ(c) = 0.1. The membership values of arcs are $\vec{\mu}$ (a,b) = 0.3, $\vec{\mu}$ (b,c) = 0.5. So $N^+(a)$ = {(b,0.3),(c,0.1)}. $N^-(c)$ = {(a,0.4)}. (Note that (a,σ(a)) represents the vertex a with membership value σ(a)). It is shown in Figure 1.

Definition 2.3:

Let D be a fuzzy Digraph $\vec{\xi} = (V,\sigma,\vec{\mu})$,for a set W ⊆ V , $N_D^+(W) = \bigcup_{(\omega,\sigma(\omega))\epsilon W} N^+\left(\omega,\sigma(\omega)\right) - W$, then the $N_D^+(W)$ is known as the ***fuzzy out neighbourhood*** of a set W.

Definition2.4

Let D be a fuzzy Digraph $\vec{\xi} = (V,\sigma,\vec{\mu})$,for a set W ⊆ V ,

$N_D^-(W) = \bigcup_{(\omega,\sigma(\omega))\epsilon W} N^-\left(\omega,\sigma(\omega)\right) - W$, then the $N_D^-(W)$ is known as the ***fuzzy in neighbourhood*** of a set W.

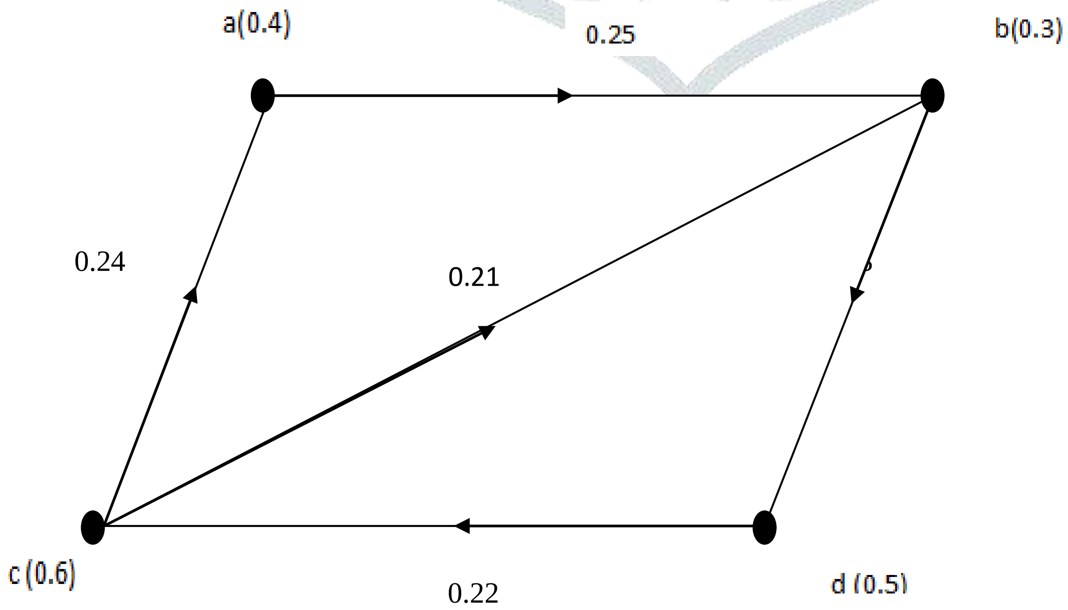

figure:2

Example 2.2.2:

The fuzzy out neighbourhood of the set W={a,b} are

N^{+} ({a,b})={d(0.23)}

N^{+2}({a,b})={c(0.22)}

The fuzzy in neighbourhood of the set W={a,b} are

N^{-} ({a,b})={c(0.240,c(0.25)}

N^{-2}({a,b})={d(0.22)}

III r-Kings in fuzzy Digraphs

Definition 3.1:

The ***distance from a set X to Y*** of vertices in fuzzy digraph D is dist(X,Y)=Max {dis((x,σ(x)),(y,σ(y)) : ((x,σ(x)∈ X (y,σ(y) ∈Y }

Definition 3.2:

The ***diameter*** of a fuzzy digraph D is diam(D) =dist(V,V)

Definition 3.2:

The ***out radius r***$^{+}$(D) is defined as

r^{+}(D)= min{ dist((x,σ(x)),V) : (x,σ(x)) ϵV}

The ***in radius*** r-(D) is defined as

r^{-}(D)= min{ dist(V,(x,σ(x))) : (x,σ(x)) ϵ V}

Definition 3.3:

The ***radius*** of the fuzzy digraphs rad(D)is defined as

Rad (D) = min {(dist((x,σ(x)),V)+ dist(V,(x,σ(x)))/2 :(x,σ(x)) ϵ V}

Example 3.1.1:

In the figure 2 the diameter diam(D)=3,r^{+}(D)=1,r^{-}(D)= 1 and Rad(D)=2.

Definition 3.4:

Let D be a fuzzy Digraph $\vec{\xi}$ = (V,σ, $\vec{\mu}$) , for any integer r, a veryex (v, σ(v)) is an r-King if dist((v, σ(v)),V)≤ r.

Example 3.1.2:

In the figure 2 the vertex ‘d’ is 2-King.

Preposition:3.4:

A weighted fuzzy digraph D has a finite out -radius if and only if D has a unique initial strong component.

Proof:

A fuzzy digraph with two or more initial strong component is obviously of infinite out radius. If D has only one initial strong component , then D contains an out-branching. Thus $r^{+}(D) < \infty$

Theorem:3.5:

Let D be a connected bridgeless fuzzy Digraph and D' be the orientation of D. Then the diameter of D' is equal to the length of the longest path of D.

Proof:

For every strongly connected orientation D_o of D diam(D_o)≤LP(D).Hence to prove this theorem it suffices to construct some orientation D_1 of D with the property diam(D_1)=LP(D).

Let P =($x_1\ \sigma(x_1), x_2\sigma(x_2), \ldots, x_k\sigma(x_k)$),be a longest path of D and associated with each vertex $x_i\ \sigma(x_i)$ with a label r($x_i\ \sigma(x_i)$)= i. Since D has no bridge the edge($x_{k-1}\ \sigma(x_{k-1}), x_k\ \sigma(x_k)$) is not a bridge .Consequently there exist an ($\{x_1\ \sigma(x_1), x_2\sigma(x_2), \ldots, x_{k-1}\ \sigma(x_{k-1})\}, x_k\sigma(x_k)$) path R_1 which is different from the path ($x_{k-1}\ \sigma(x_{k-1}), x_k\ \sigma(x_k)$).Let ($x_i\ \sigma(x_i)$) be the initial vertex of R_1.Define r(v,$\sigma(v)$)=i for all vertices v$\sigma(v)$ ϵV(R_1)-{ $x_k\sigma(x_k)$}.Since ($x_{i-1}x_i$) is not a bridge there exist an ($\{x_1\ \sigma(x_1), x_2\sigma(x_2), \ldots, x_{i-1}\sigma(x_{i-1})\}, \{x_i\ \sigma(x_i), x_{i+1}\sigma(x_{i+1}), \ldots, x_k\sigma(x_k)\} \cup V(R_1)$)-path R_2 which is different from the path ($x_{i-1}x_i$). If $x_j\sigma(x_j)$ is the initial vertex of R_2,then define r(v,$\sigma(v)$)=j for all vertices v$\sigma(v)$ ϵV(R_2) besides the terminal one. Similarly build paths R_3, R_4 ....untill obtain a path R_s with initial vertex $x_i\ \sigma(x_i)$ and the set r(v,$\sigma(v)$)=1 for all vertices v$\sigma(v)$ in R_s but the terminal one.

Now orient the path P from $x_1\ \sigma(x_1)$ to $x_k\sigma(x_k)$ and each path R_i from its end vertex having bigger label to its other end vertex. To check that the oriented graph induced by the arcs of the paths $\bigcup_{i=1}^{s} Q_i \cup Q$ is strong .Define $X = V(D) - (\bigcup_{i=1}^{s} V(R_i) \cup V(P))$ and suppose that X≠ ∅ .Since D has no bridges there exist some vertex $v\sigma(v) \in X$ and a pair of paths from $v\sigma(v)$ to vertices in V(D) –X with no common vertices. Then merge these two paths to one. Now orient the last path from its end vertex having the bigger label to the one having the smaller label .If the labels of all other vertices of the path S_1 are as the label of the terminal vertex of this path.

If $X - V(s_1) \neq \emptyset$ continue the construction of the paths $s_2, s_3, \ldots$ passing over the rest of the vertices of X until $\bigcup_{i=1}^{t} V(s_i) = X$ where the orientation and the labels are chosen in the same manner.Finally orient each unoriented edge($u\sigma(u)v\sigma(v)$) from $u\sigma(u)$ to $v\sigma(v)$ if r($u\sigma(u)$)≥r($v\sigma(v)$) and from $v\sigma(v)$ to $u\sigma(u)$ otherwise.

Let D denote the obtained oriented graph. The fuzzy digraph D contains a strongly connected spanning subgraph .Therefore D is strongly connected .Since all arcs ($u\sigma(u)w\sigma(w)$) of D besides those in P, are oriented such that r($u\sigma(u)$) ≥r(w$\sigma(w)$),there is no path from $x_1\ \sigma(x_1)$ to $x_k\sigma(x_k)$ having less than k-1. Hence diam (D)=k-1.

IV CONCLUSION

Finally we defined the diameter, radius,and distance of the fuzzy digraph and analyzed their properties. This study will help full in the system where fuzzy digraphs are applied.

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