PROPERTIES OF COMBINATION USING TRIPLE FACTORIAL

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Abstract:

In this paper an attempt was made to derive few formulae on combinations using the fact called triple factorial.

The results of the combination which had been already proved was solved and proved by me using this concept

Keywords: triple factorial, semi factorial, double factorial, combination, permutation

Introduction:

Initially, this idea was highlighted to me in solving a mathematical puzzle. Then I started to solve properties of permutations and combinations using the fact called sub factorial. Then I got a thought to solve the properties of combinations and permutations using the fact called semi factorial or double factorial. Then I decided to solve the properties of combinations using the fact called triple factorial.

In this paper I have solved few properties of combination using the notation triple factorial which was already proved.

Formula for triple factorial

$n!!! = n \times n-3 \times n-6 \times \ldots..\times 3 \quad for\ n\ which\ is\ \ a\ multiple\ of\ 3$

$n!!! = n \times n-3 \times n-6 \times \ldots..\times 1 \quad for\ even\ n\ other\ than\ a\ multiple\ of\ 3$

$n!!! = n \times n-3 \times n-6 \times \ldots..\times 2 \quad for\ an\ odd\ n\ other\ than\ a\ multiple\ of\ 3$

In particular we assume that

0!!!=1

1!!!=1

2!!!=1

3!!!=3

Result 1

Relation between factorial and double factorial

n!=n!!!(n-1)!!!(n-2)!!!

Example

5!=5!!!4!!!3!!!

=5*2*4*1*3

= 120

Result 2

Expression of combination in terms of triple factorial

$$nC_r = \frac{n!!!(n-1)!!!(n-2)!!!}{(n-r)!!!(n-r-1)!!!(n-r-2)!!!r!!!(r-1)!!!(r-2)!!!}$$

Result 3

$$nC_0 = 1$$

Proof:

$$nC_0 = \frac{n!!!(n-1)!!!(n-2)!!!}{(n-0)!!!(n-1)!!!(n-2)!!!0!!!}$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{n!!!\,(n-1)!!!\,(n-2)!!! * 1}$$

= 1

Result 4

$$nC_n = 1$$

Proof:

$$nC_n = \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-n)!!!\,n!!!\,(n-1)!!!\,(n-2)!!!}$$

$$= \frac{n!!!(n-1)!!!(n-2)!!!}{(0)!!!n!!!(n-1)!!!(n-2)!!!}$$

$$= \frac{n!!!(n-1)!!!(n-2)!!!}{1*n!!!(n-1)!!!(n-2)!!!}$$

$= 1$

Result 5

$nC_1 = n$

Proof:

$$nC_1 = \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-1)!!!\,(n-2)!!!\,(n-3)!!!\,1!!}$$

$$= \frac{n(n-3)!!!}{(n-3)!!!}$$

$= \text{n}$

Result 6

$\text{n}C_{n-1} = n$

Proof:

$$\text{n}C_{n-1} = \frac{n!!!(n-1)!!!(n-2)!!!}{(n-n+1)!!!(n-1)!!!(n-2)!!!(n-3)!!!}$$

$$= \frac{n!!!}{(1)!!!\,(n-3)!!!}$$

$$= \frac{n(n-3)!!!}{1(n-3)!!}$$

$= n$

Result 7

Relation between permutation and combination

$$\text{n}C_r = \frac{nP_r}{r!}$$

Proof:

$$\frac{nP_r}{r!} = \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r)!!!\,(n-r-1)!!!\,(n-r-2)!!!\,r!!!\,(r-1)!!!\,(r-2)!!!}$$

$= nC_r$

Result 8:

$\text{n}C_r + \text{n}C_{r-1} = \text{n+1}C_r$

Proof:

$$\mathrm{n}C_r + \mathrm{n}C_{r-1} = \frac{n!!!(n-1)!!!(n-2)!!!}{(n-r)!!!(n-r-1)!!!(n-r-2)!!!r!!!(r-1)!!!(r-2)!!!} + \frac{n!!!(n-1)!!!(n-2)!!!}{(n-r+1)!!!(n-r)!!!(n-r-1)!!!(r-1)!!!(r-2)!!!(r-3)!!!}$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r)!!!\,(n-r-1)!!!\,(r-1)!!!\,(r-2)!!!}\left[\frac{1}{(n-r-2)!!!\,r!!!} + \frac{1}{(n-r+1)!!!\,(r-3)!!!}\right]$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r)!!!\,(n-r-1)!!!\,(r-1)!!!\,(r-2)!!!}\left[\frac{1}{(n-r-2)!!!\,r(r-3)!!!} + \frac{1}{(n-r+1)(n-r-2)!!!\,(r-3)!!!}\right]$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r)!!!\,(n-r-1)!!!\,(r-1)!!!\,(r-2)!!!}\left[\frac{n-r+1+r}{(n-r+1)(n-r-2)!!!\,r(r-3)!!!}\right]$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r)!!!\,(n-r-1)!!!\,(r-1)!!!\,(r-2)!!!}\left[\frac{n+1}{(n-r+1)(n-r-2)!!!\,r(r-3)!!!}\right]$$

$$= \frac{n!(n+1)}{(n-r)!!!(n-r-1)!!!(r-1)!!!(r-2)!!!(n-r+1)(n-r-2)!!!r(r-3)!!!}$$

$$= \frac{(n+1)!}{(n-r+1)(n-r)!\,r(r-1)!}$$

$$= \frac{(n+1)!}{(n-r+1)!r!}$$

$$= \mathrm{n{+}1}C_r$$

Result 9:

$$\mathrm{n}C_r = \frac{n}{r}\ \mathrm{n{-}1}C_{r-1}$$

Proof:

$$\frac{n}{r}\ \mathrm{n{-}1}C_{r-1} = \frac{n}{r}\ \frac{(n-1)!!!\,(n-2)!!!\,(n-3)!!!}{(n-r)!!!\,(n-r-1)!!!\,(n-r-2)!!!\,(r-1)!!!\,(r-2)!!!\,(r-3)!!!}$$

$$= \frac{n}{r}\ \frac{(n-1)!}{(n-r)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,r!}$$

$$= \text{n}C_r$$

Result 10

$\text{n}C_r + \text{n}C_{r+1} = \text{n+1}C_{r+1}$

Proof:

$\text{n}C_r + \text{n}C_{r+1}$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r)!!!\,(n-r-1)!!!\,(n-r-2)!!!\,r!!!\,(r-1)!!!\,(r-2)!!!} + \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r-1)!!!\,(n-r-2)!!!\,(n-r-3)!!!\,(r+1)!!!\,(r)!!!\,(r-1)!!!}$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r-1)!!!\,(n-r-2)!!!\,(r)!!!\,(r-1)!!!}\left[\frac{1}{(n-r)!!!\,(r-2)!!!} + \frac{1}{(n-r-3)!!!\,(r+1)!!!}\right]$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r-1)!!!\,(n-r-2)!!!\,(r)!!!\,(r-1)!!!}\left[\frac{1}{(n-r)(n-r-3)!!!\,(r-2)!!!} + \frac{1}{(n-r-3)!!!\,(r+1)(r-2)!!!}\right]$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r-1)!!!\,(n-r-2)!!!\,(r)!!!\,(r-1)!!!\,(n-r-3)!!!\,(r-2)!!!}\left[\frac{r+1+n-r}{(n-r)(r+1)}\right]$$

$$= \frac{n!!!\,(n-1)!!!\,(n-2)!!!}{(n-r-1)!!!\,(n-r-2)!!!\,(r)!!!\,(r-1)!!!\,(n-r-3)!!!\,(r-2)!!!}\left[\frac{1+n}{(n-r)(r+1)}\right]$$

$$= \frac{n!\,(n+1)}{(n-r-1)!\,r!\,(n-r)(r+1)}$$

$$= \frac{(n+1)!}{(n-r)!\,(r+1)!}$$

$$= \text{n+1}C_{r+1}$$

Conclusion:

In this paper I have solved few properties of combination using the fact called triple factorial.

References:

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