THE HARMONIC GENERATION EFFECT IN HIGH TEMPERATURE SUPERCONDUCTOR WITH FEW MODELS

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Abstract:  
In this paper, we discuss about the harmonic generation effect in high temperature superconductor with few models which are systematic discussed.

Keywords: Superconductivity, Low temperature, Harmonic generation, A.M.

Introduction:  
The harmonic generation is due to non-linear response of the magnetization of a high -$T_c$ sample to a periodic magnetic field. When a piece of high-$T_c$ superconductor is subjected to an ac field. When a piece of high-$T_c$ superconductor is subjected to an ac field of amplitude $H_{ac}$ and frequency $f$ such that $H_{ac}> H_{c1}$, where $H_{c1}$ is the lower critical field of the grain boundary region the the vortices sweep in and out of the superconductor during each ac cycle. The field inside the superconductor lags behind the applied ac field in a non-linear fashion. This non-linearity leads to the generation of add harmonics of frequency $(2n+1) f$. The non-linear magnetic response implies that the magnetization in an ac field of frequency $\omega$ will contain a response at higher harmonics $n\omega$, and this magnetization will not be in phase with the applied field. Let the magnetization lag behind the applied field by an angle $\theta$, then

$$M= \chi H= \chi H_{ac} \cos(\omega t- \theta) $$

$$= H_{ac}(\chi \cos\theta \cos\omega t+ \chi \sin\theta \sin\omega t) $$

Defining

$$\chi'= \chi \cos\theta$$

$$\chi''= \chi \sin\theta$$

Then from equation (2),

$$M= H_{ac}(\chi' \cos n \omega t+ \chi'' \sin n \omega t)$$

$\chi'$ is associated with the in phase and $\chi''$ with the out of phase signal with respect to the applied field. The complex susceptibility can be as $\tilde{\chi}= \chi' + \chi''$. The addition of a dc field induces the generation of even harmonics also of frequency $nf$. The amplitude of the both families of harmonics is found to modulate with the variation of dc field. The harmonics signal vanishes whenever the superconductor goes into normal state due to increase of either the temperature or the applied field. The harmonic generation depends on temperature, applied field and the type of superconductor. The several basic models such as Bean’s macroscopic model, Anderson and Kim Model and other model such as Jeffries model have been proposed to explain these phenomena.

A. Critical state model

1. Bean’s macroscopic model:  
C.P.Bean first considered the concept of critical state and hence the state is known also known as Bean critical state. Bean assumed the following simple model.  
1. For $H<H_{c1}$, the magnetic field is screened within a very short distance of the surface and we may take $B=0$ within the superconductor.  
2. For $H>H_{c1}$ and constant $J_c$, the flux penetrates the superconductor with density that decreases linearly with distance down to $H_{c1}$, at which point B drops to zero.
When a current flows through a type II superconductor, it exerts a Lorentz force on the vortices. As the result the vortices tend to move from their equilibrium position. The movement is, however, opposed by the presence of so called pinning forces. A critical state is reached when the local Lorentz force become equal to the local pinning force. The condition in the critical state is given by
\[ f_p(r) = f_(r) \] (4)
where \( f_p(r) \) is the pinning force density determined by the sample preparation methods, \( f_(r) \) is the Lorentz force density written as
\[ f_(r) = \frac{J_c(r) X B}{c} \] (5)
According to Ampere law
\[ \nabla \times B(r) = \left(\frac{4\pi}{c}\right)J_c(r) \] (6)
This is basic equation describing the critical state of hard superconductors. Bean pointed out that, the critical current density should be directly governed by the microstructure of superconductors. However to simplify the problem, a field independent \( J_c(T) \) was assumed by Bean, i.e.
\[ J_c[B(r),T] = J_c(T) \] (7)
Since the critical current density is independent of the position, the local field has a linear relation to the position.

2. Y. B. Kim and Anderson Model:

The field dependence of the critical state model was first proposed by Y.B. Kim et al. They assumed that the critical current density is inversely proportional to the local field (\( H_T \)). In this case \( J_c \) is given by
\[ J_c(T,B(x)) = \frac{J_c(T)}{1 + \frac{B(x)}{H_0(T)}} \] (8)
Where, \( H_0 \) is a material parameter with magnetic field dimension which can be determined experimentally
Eq. (8) shows that the field dependence of \( J_c \) is associated with term \( B(x)/H_0 \) which may vary considerably among different system. Anderson and Kim’s model has been used to calculate the virgin magnetization curve for a superconducting slab. Ullmaiser and Kernohan developed a power-law relation of the critical current density. For flux density distribution in hard superconductors. After considering specific pinning mechanism Irie and Yamafuji independently proposed the power-law relation which can be written as
\[ J_c(T,B(x)) = J_c(T) \left(\frac{H_0}{B(x)}\right)^n \] (9)
Where \( n \) is a material parameter and directly reflects pinning strength.
This model has been solved for slab and cylindrical geometries. Xu et al. proposed the general form of various critical state relations by an expression
\[ J_c \{T,B(x)\} = J_c(T) \left(\frac{H_0}{B(x)}\right)^n \] (10)
Where \( \beta \) is a dimensionless constant. It is clear that \( \beta=0 \) and 1 gives the Bean’s and Kim’s model, respectively.
An exponential model was introduced by Ravi Kumar and Chaddah for HTSC according to which
\[ J_c(B) = J_c(0)\exp\left(-\frac{|B|}{B_0}\right) \] (11)
And this has been solved for slab, circular and rectangular geometry’s. Bhagwat and Chaddah have developed a theoretical critical state model for the general ellipsoid, which can be greatly simplified for the case of extreme oblateness with respect to an axis of circular symmetry. This is most complete model to date, which can be applied to discs. It contains the important result that the circulating currents are not constant throughout the cross section, even in absence of any possible field dependence of \( J_c \) but reach their critical limit only in the equatorial plane of the ellipsoid. Muller et al. measured harmonics susceptibilities \( X_a \) and \( X_a^* \) instead of harmonics voltage, \( V_a \) and compared the experimental observation with the magnetization equation of Ji et al based on Kim’s model. The comparison shows good agreement. The simplified Kim model with \( H_0=0 \) tends to exaggerate the magnetization near zero field. Using complete Kim model, Muller et al. obtained better agreement in theory and experiments. This critical state model explains the appearance of even harmonics in the presence of dc field, the modulation of higher harmonics with \( H_0 \) and the variation of the amplitude of third harmonics (\( V_3 \)) with
H$_{ac}$. For a good quality sample it has been observed that initially $V_3\alpha$ $H_{ac}^2$ but later $V_3\alpha$ $H_{ac}^3$. This model explains this observation in terms of cross over from Bean’s regime to Kim’s regime. Bean’s model predicts $V_3\alpha$ $(H_{ac})^3$ whereas Anderson & Kim’s model predicts $V_3\alpha$ $(H_{ac})^3$. Where $H_{ac}$ crosses over from Bean regime to Anderson- Kim regime, The generation of even harmonics becomes observable.

B. Other model such as C. D. Jeffries model:

C.D. Jeffries and Coworker’s$^{13,14,15}$ proposed a model in which the granular superconducting sample is assumed to be essentially composed of a collection of current loops formed by superconducting “grains” in contact through Josephson junction or other weak links. These loops are assumed not to be coupled and to behave independently. According to this model, the magnetic non-linearity results from the non-liner relationship between Josephson current in these loops and the magnetic flux enclosed by the loops. The Josephson current phase relation is

$$I(t) = I_c \sin (\alpha + \beta \sin \omega t)$$  \hspace{1cm} (12)

Where $\alpha = 2mS_0 H_{dc} / \Phi_0$, $\beta = 2mS_0 H_{ac} / \Phi_0$, where $S_0$ is characteristic loop area, $H_{dc}$ and $H_{ac}$ are amplitudes of dc and ac fields respectively. Due to non linearity of the current phase relation, harmonics are generated in the loop current and would be detected by pick up coil. Assuming that a particular ensemble, the loop area is characterized by a distribution function $F(A)$, with $A=S/S_0$, then integrating over all the loops, the signal amplitude of the $n^\text{th}$ harmonics as detected by the pickup coil can be expressed as

$$\langle V_n \rangle = n\omega \int_{A_\delta}^{\infty} A \, J_n(A\beta) \cos(n\alpha) \, F(A) \, dA / G ; \text{ odd } n$$  \hspace{1cm} (13)

$$\langle V_n \rangle = n\omega \int_{A_\delta}^{\infty} A \, J_n(A\beta) \sin(n\alpha) \, F(A) \, dA / G ; \text{ even } n$$  \hspace{1cm} (14)

Where $G = \int_{A_\delta}^{\infty} F(A) \, dA$ (15)

And $\delta > 0$ is lower cut off. If $H_{dc}$ then $\alpha = 0$ and only odd harmonics are generated where as for$H_{dc} \neq 0$, even harmonics are also generated. Jeffries et al observed than both even and odd higher harmonics showed series of sharp dips with average spacing $\delta H_{dc}$ which is inversely proportional to $n$. These dip have been described as distinct evidence foe flux quantisation in superconducting loops in the granular samples. As $H_{dc} \to 0$, the even harmonics power shows a sharp dip. The model has also explained this feature.

M. Golosovsky et al$^{16,17}$ presented a simple model according to which harmonic generation is due to non-linear response of the complex resistivity of the superconducting film to the external periodic magnetic field. The total complex resistivity is written as

$$\rho = \rho_0 + \rho_f + \rho_j$$  \hspace{1cm} (16)

Where $\rho_0 = \left\{ \frac{\sigma_1 - iC_2}{4\pi \omega \mu_{\text{eff}}} \right\}$, $\sigma_1$ is the real part of the complex resistivity, $\mu$ is effective permeability of medium and $\lambda_{\text{eff}}$ is the effective penetration depth, $\rho_j$ is a resistivity term associated with dissipation due to damped fluxion motion in the microwave field and $\rho_f$ is the effective resistivity associated with the internal Josephson junction. It has been demonstrated that for granular high-$T_c$ superconducting films the harmonics generation by magnetic field modulation of the microwave transmission in mainly due to the viscous flux motion. The contribution of the internal josephson junction is found to be of less important whereas in case of current modulation the contribution of $\rho_j$ is more important.

The dependence of resistivity ($\rho_f$) on magnetic field is expressed as

$$\rho_f(t) = \rho_0 + \alpha [H_{dc} + H_1 \cos(2\pi f_0 t)]$$  \hspace{1cm} (17)

Where $\alpha = \beta \Phi_0 / \pi C^2$, $\beta$ is fraction of weakly pinned fluxion, $\Phi_0$ is the flux quantum, $n$ is the viscosity coefficient of damped fluxon, $H_{dc}$ is dc field and $H_1$ Cos $2\pi f_0 t$ is ac field modulation. Golosovski et al$^{18}$ suggested that the dependence of complex resistivity on bias current is due to $\rho_j$ whose Fourier transform can be given as $\rho_j = \Sigma V_n(I_{ac}) \cos 4\pi n f_0 t$ (18)

Where $V_n$ is amplitude of the $n^\text{th}$ harmonics. Due to modulation of $\rho_j$ by the ac bias current, generation of higher harmonics occurs.

DISCUSSION:

C.P. Beans model is relevant to hard superconductors that show a large hysteresis. Kim’s model was found to agree fairly well the experimental results on some A-15 conventional superconductors. The power-law relation of the critical current density for flux density distribution in hard superconductors has been solved for slab and cylindrical geometries. An exponential model for high temperature superconductor has been solved for slab, circular and rectangular geometry’s. Jeffries model predicts the generation of odd harmonics on the application of ac field and that of even harmonics on application of dc field.
REFERENCES