# MAGNETIC EFFECT ON JEFFREY FLUID THROUGH AN ARTERY WITH MULTIPLE STENOSIS

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**ABSTRACT:** *The present paper deals with the theoretical analysis of magnetic effect on Jeffrey fluid flow through a multiple stenosed artery. The non-linear governing equations have been simplified under the assumption of mild stenosis. The analytical solutions are obtained for velocity, pressure drop, volumetric flow rate, resistance to the flow and wall shear stress. The influence of various parameters on flow characteristics such as resistance to the flow and wall shear stress have been studied and analyzed through graphs.*

**KEY WORDS**: *Jeffrey fluid, Jeffrey fluid parameter, wall shear stress, magnetic parameter.*

## I. INTRODUCTION

An abnormal narrowing in a blood vessel is known as stenosis or atherosclerosis and is one of the frequently occurring cardiovascular diseases. The heart related diseases mainly occur due to temporary deficiency of oxygen or blood supply to the heart. The stenosed arteries pertaining to the brain can cause cerebral strokes and the coronary stenosed arteries can cause myocardial infarction which leads to heart attack. Due to stenosis, the flow of blood is disturbed and the resistance to flow is more comparing to the normal artery. Hence the studies on blood flow phenomena are helpful in diagnosis of the diseases.

Many theoretical studies related to blood flow through arteries with single senosis are done by [Young and Tsai[1] , Misra and Chakravarthy [2]]. In these studies the blood is assumed as a Newtonian fluid. However, from the experimental results that blood, being suspension of cells, it is observed that blood behaves like a non-Newtonian fluid in small arteries at low shear rates [Chaturani and Ponalagusamy [4]]. In realistic situations, the constrictions may develop in series or irregular shapes or composite or multiple. Effect of magnetic field on Herschel –bulkley fluid through multiple stenoses has been investigated by Maruthi prasad el al.[3].

Jeffrey fluid is a generalization of Newtonian fluid and it serves as a better model for physiological fluids (Hayat et al., [5]). Hence Newtonian fluid can be deduced from the Jeffrey fluid model. Several researchers have investigated Jeffrey fluid flows under different conditions. Akbar and Nadeem [6] have investigated the effect of variable viscosity on blood flow through a tapered artery with stenosis by considering blood as Jeffrey fluid. Furthermore, the study of magneto hydrodynamics flow problems has gained considerable interest because of its extensive engineering and medical applications.

The MHD deals with the motion of electrically conducting fluids. The action of the magnetic field on the conducting fluid generates electric currents and changes the mechanical forces which modify the flow of the fluid [Ferraro and Plumpton[7]]. Some researchers have studied the effects of MHD on Newtonian and non-Newtonian fluids in different conditions. The peristaltic flow of Jeffrey fluid under the effect of magnetic field was investigated by Hayat and Ali [8]. Eldable et al., [9] developed a mathematical model for the peristaltic flow of non-Newtonian fluid with heat and mass transfer through porous medium under the effect of magnetic field. Bhvana Vijaya et al., [10] studied the magnetic effect on Jeffrey fluid through a stenosed tube .

The aim of the present paper is to study the effect of magnetic field on Jeffrey fluid through an artery with multiple stenoses. The governing equations of Jeffrey fluid are presented. The non-linear partial differential equations are simplified by considering mild stenosis. The analytical solutions are obtained subject to the boundary conditions. The effects of various parameters on the flow characteristics have been studied.

## II. MATHEMATICAL FORMULATION

Consider the steady flow of an incompressible Jeffrey fluid through a circular tube of length $L$ with two stenoses. Cylindrical polar coordinates $(r, \theta, z)$ are taken into consideration. The axis of the tube is taken about the $z$-axis. It is further assumed that the stenoses to be mild and axially symmetric. The shapes of the stenoses are represented by cosine curves. It is assumed that the flow is axi-symmetric, the stenosis is developed in axi-symmetric manner and a uniform magnetic field $B_0$ is applied transversely to the flow.

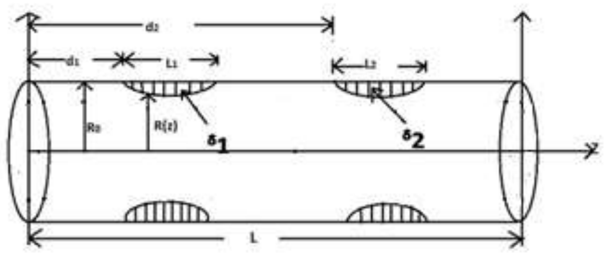


**Fig- 1: Geometry of the tube with two tenoses.**

The radius of the cylindrical tube is given as

$$h = \frac{R(z)}{R_0} = \begin{cases} 1, & 0 \le z \le d_1 \\ 1 - \frac{\delta_1}{2}\left(1 + Cos\frac{2\pi}{L_1}(z - d_1 - \frac{L_1}{2}\right), & d_1 \le z \le d_1 + L_1, \\ 1, & d_1 + L_1 \le z \le d_2 \\ 1 - \frac{\delta_2}{2}\left(1 + Cos\frac{2\pi}{L_2}(z - d_2 - \frac{L_2}{2}\right), & d_2 \le z \le d_2 + L_2 \\ 1, & d_2 + L_2 \le z \le L \end{cases} \quad (1)$$

Where $R(z)$ is the radius of the tube with stenosis, $R_0(z)$ is the radius of the tube without stenosis. Here $L_i$ and $\delta_i (i = 1,2)$ are the lengths and heights of the two stenosis (the suffixes 1 and 2 indicate the first and second stenoses respectively).

The constitutive equations for an incompressible Jeffrey fluid are

$$T = -pI + S \quad (2)$$

$$S = \frac{\mu}{1+\lambda_1}\left(\frac{\partial \gamma}{\partial t} + \lambda_2 \frac{\partial^2 \gamma}{\partial t^2}\right) \quad (3)$$

Where $T$ and $S$ are Cauchy stress tensor and extra stress tensor respectively, $p$ is the pressure, $I$ is the identity tensor, $\lambda_1$ is the ratio of the relaxation to retardation times, $\lambda_2$ is the retardation time, $\mu$ is the dynamic viscosity and $\bar{\gamma}$ is the shear rate.

The governing equations for the present problem are as follows

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

$$\rho\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)u = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rr}) + \frac{\partial}{\partial z}(S_{rz}) - \frac{S_{\theta\theta}}{r} \quad (5)$$

$$\rho\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)w = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rS_{rz}) + \frac{\partial}{\partial z}(S_{zz}) - \sigma B_0^2 w \quad (6)$$

Where the extra tensor components are

$$S_{rr} = \frac{2\mu}{1+\lambda_1}\left(1 + \lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\frac{\partial u}{\partial r}\right.$$

$$S_{rz} = \frac{\mu}{1+\lambda_1}\left(1 + \lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\right)\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)$$

$$S_{zz} = \frac{2\mu}{1+\lambda_1}\left(1 + \lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\frac{\partial w}{\partial z}\right.$$

$$S_{\theta\theta} = \frac{2\mu}{1+\lambda_1}\left(1 + \lambda_2\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\frac{u}{r}\right.$$

Further, $u$ and $w$ are the velocities in $r$ and $z$-directions respectively, $\sigma$ is the electrical conductivity of the fluid.

Using the following non dimensional variables

$$\bar{z} = \frac{z}{L}, \bar{\delta} = \frac{\delta}{R_0}, \bar{R} = \frac{R}{R_0}, \bar{P} = \frac{P}{\frac{\mu U L}{R_0^2}}, \bar{u} = \frac{u}{U},$$

$$\bar{w} = \frac{L}{U\delta}w, Re = \frac{\rho R_0 U}{\mu}, \bar{\mu} = \frac{\mu}{\mu_0} \quad (7)$$

In Eqs. (4) - (6), under the assumption mild stenosis $\frac{\delta}{R_0} \ll 1, Re\left(\frac{2\delta}{L_0}\right) \ll 1$ and $\frac{2R_0}{L_0} \sim O(1)$ takes the form (after dropping the bars)

$$\frac{\partial p}{\partial r} = 0 \quad (8)$$

$$\frac{\partial p}{\partial z} = \frac{1}{1+\lambda_1}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - M^2 w \quad (9)$$

Where $M^2 (= \sigma B_0^2)$ is the magnetic parameter.

The corresponding non dimensional boundary conditions are given by

$$\frac{\partial w}{\partial r} = 0 \text{ at } r = 0 \quad (10)$$

$$w = 0 \text{ at } r = h(z) \quad (11)$$

## III. SOLUTION:

Solving equation (9) subject to the boundary conditions (10) and (11), we get the velocity as,

$$w = \frac{1}{M^2}\frac{dp}{dz}\left[\frac{I_0(M\sqrt{1+\lambda_1}r)}{I_0(M\sqrt{1+\lambda_1}h)} - 1\right] \quad (12)$$

Here $I_0$ is the modified Bessel function of first kind of order zero.

The volumetric flow rate is defined by

$$Q = 2\pi \int_0^h wr\, dr \qquad (13)$$

Integrating Eq. (13) we get

$$Q = \frac{2\pi}{M^2}\frac{dp}{dz}\left[\frac{hI_1(M\sqrt{1+\lambda_1}\,h)}{\sqrt{1+\lambda_1}MI_0(M\sqrt{1+\lambda_1}\,h)} - \frac{h^2}{2}\right] \qquad (14)$$

Where $I_1$ is the modified Bessel function of first kind of order one.

$$\frac{dp}{dz} = \frac{M^2Q}{2\pi\left[\frac{hI_1(M\sqrt{1+\lambda_1}h)}{M\sqrt{1+\lambda_1}I_0(M\sqrt{1+\lambda_1}h)} - \frac{h^2}{2}\right]} \qquad (15)$$

When the Jeffrey fluid parameter $\lambda_1 \to 0$, the fluid becomes Newtonian.

The pressure drop $\Delta p$ across the stenosis between $z = 0$ to $z = 1$ is obtained by integrating Eq. (15), as

$$\Delta p = \int_0^1 \frac{dp}{dz}\, dz \qquad (16)$$

$$\Delta p = \int_0^1 \frac{M^2Q}{2\pi\left[\frac{hI_1(M\sqrt{1+\lambda_1}h)}{M\sqrt{1+\lambda_1}I_0(M\sqrt{1+\lambda_1}h)} - \frac{h^2}{2}\right]}\, dz \qquad (17)$$

the resistance to the flow, λ, is defined by

$$\lambda = \frac{\Delta p}{Q} = \frac{1}{Q}\int_0^1 \frac{M^2Q}{2\pi\left[\frac{hI_1(M\sqrt{1+\lambda_1}h)}{\sqrt{1+\lambda_1}MI_0(M\sqrt{1+\lambda_1}h)} - \frac{h^2}{2}\right]}\, dz \qquad (18)$$

The pressure drop in the normal artery $(h = 1)$ is denoted by $\Delta p_N$, is obtained from Eq. (15).

$$\Delta P_N = \int_0^1 \frac{M^2Q}{2\pi\left[\frac{I_1(M\sqrt{1+\lambda_1})}{\sqrt{1+\lambda_1}MI_0(M\sqrt{1+\lambda_1})} - \frac{1}{2}\right]}\, dz \qquad (19)$$

The resistance to the flow in the normal artery is denoted by $\lambda_N$ is obtained from Eq. (19), as

$$\lambda_N = \frac{\Delta P_N}{Q} = \frac{1}{Q}\int_0^1 \frac{M^2Q}{2\pi\left[\frac{I_1(M\sqrt{1+\lambda_1})}{\sqrt{1+\lambda_1}MI_0(M\sqrt{1+\lambda_1})} - \frac{1}{2}\right]}\, dz \qquad (20)$$

The normalized resistance to the flow denoted by

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} \qquad (21)$$

The expression for the wall shear stress is given by

$$S_{rz} = -\frac{r}{2}\frac{dp}{dz}\bigg|_{r=h} \qquad (22)$$

## IV. RESULTS AND ANALYSIS:

In this section, the effects of various parameters on resistance to the flow $(\bar{\lambda})$ and wall shear stress$(\tau_h)$ are analyzed through the graphs [Figs. 2-8]. Variation of flow resistance for different values of stenoses height, Jeffrey parameter and magnetic parameter (M) are shown in Figs. 2-4. It is observed that, the resistance to the flow increases with height of primary and secondary stenoses, Jeffrey fluid parameter and decreases with the magnetic parameter.

Figs.5-8 illustrates the variation of wall shear stress for different values of height of stenoses, Jeffrey fluid parameter and magnetic parameter. It is noticed that the wall shear stress increases with the height of the stenoses, magnetic parameter but decreases with Jeffrey fluid parameter.

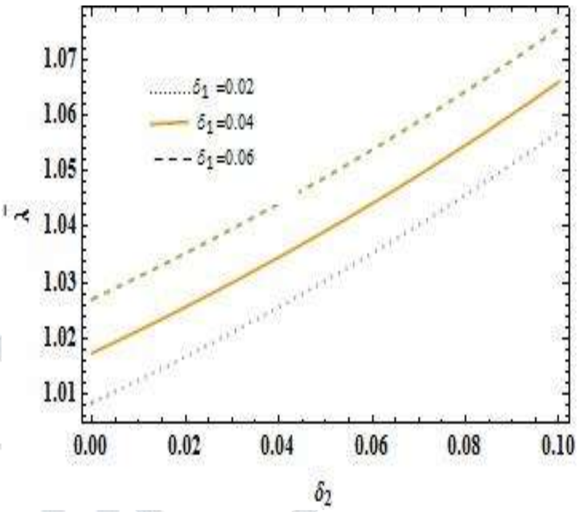


Fig 2: Effect of $\delta_2$ on $\bar{\lambda}$ for different $\delta_1$
$(d_1 = 0.2, d_2 = 0.6, L_1 = L_2 = 0.2, \lambda_1 = 0.2, M = 0.1, L = 1)$

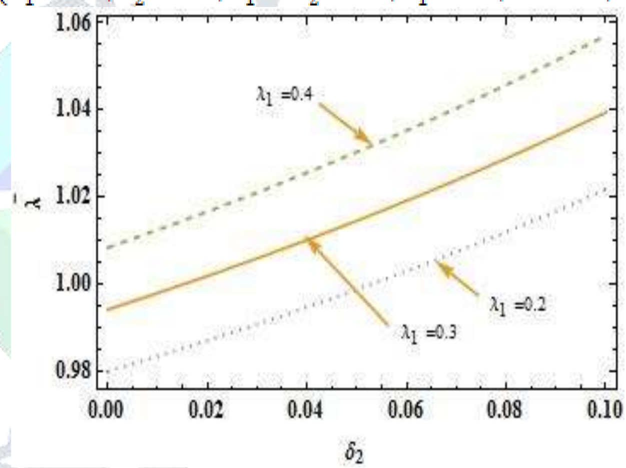


Fig 3: Effect of $\delta_2$ on $\bar{\lambda}$ for different $\lambda_1$
$(d_1 = 0.2, d_2 = 0.6, L_1 = L_2 = 0.2, \delta_1 = 0.02, M = 0.1, L = 1)$

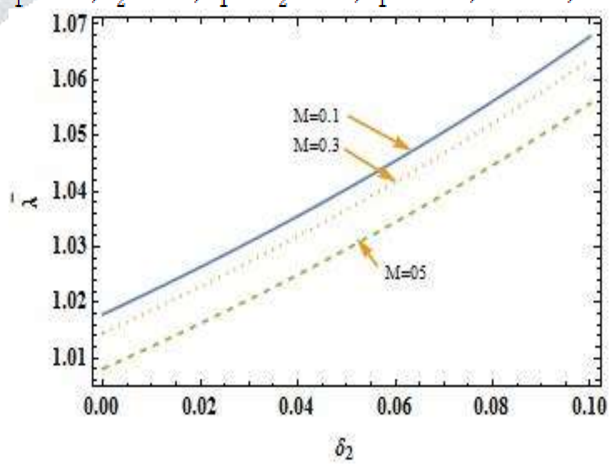


Fig 4: Effect of $\delta_2$ on $\bar{\lambda}$ for different $M$
$(d_1 = 0.2, d_2 = 0.6, L_1 = L_2 = 0.2, \delta_1 = 0.02, \lambda_1 = 0.2, L = 1)$

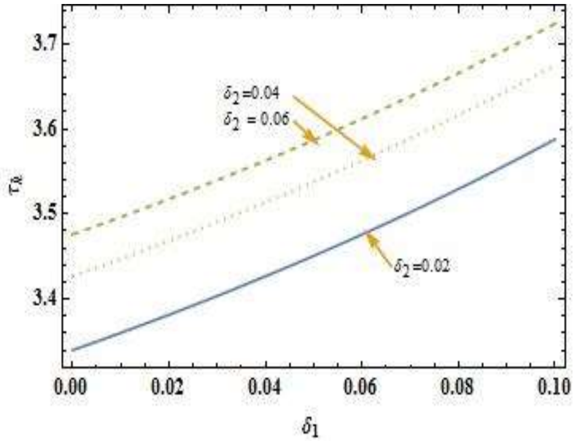


Fig-5 : Effect of $\delta_1$ on $\tau_h$ for different $\delta_2$
$(d_1 = 0.2, d_2 = 0.6, L_1 = L_2 = 0.2, Q = 0.1, M = 0.1, \lambda_1 = 0.2, L = 1)$

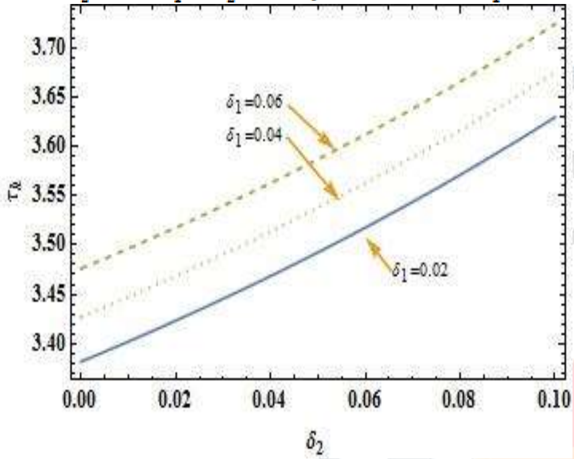


Fig-6 : Effect of $\delta_2$ on $\tau_h$ for different $\delta_1$
$(d_1 = 0.2, d_2 = 0.6, L_1 = L_2 = 0.2, Q = 0.1, M = 0.1, \lambda_1 = 0.2, L = 1)$

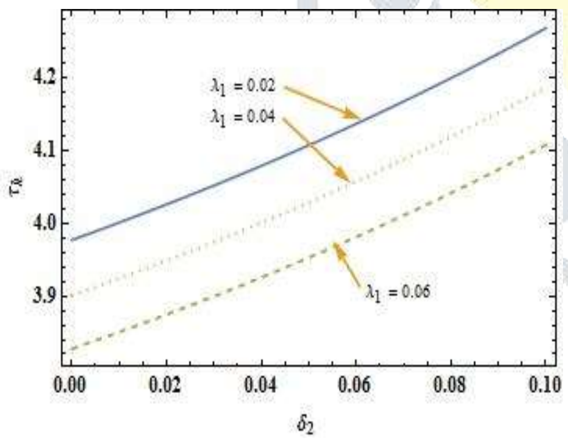


Fig-7 : Effect of $\delta_2$ on $\tau_h$ for different $\lambda_1$
$(d_1 = 0.2, d_2 = 0.6, L_1 = L_2 = 0.2, Q = 0.1, M = 0.1, \delta_2 = 0.02, L = 1)$

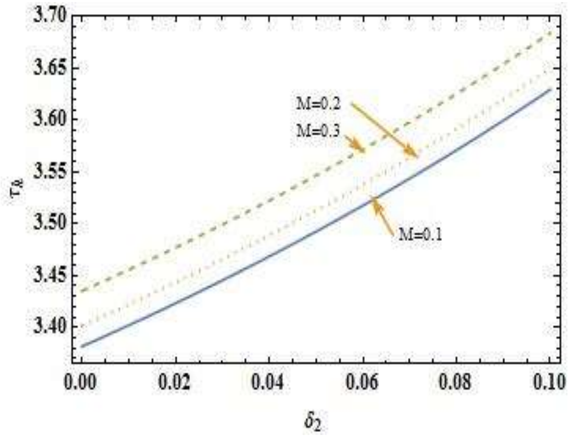


Fig-8 : Effect of $\delta_2$ on $\tau_h$ for different $M$
$(d_1 = 0.2, d_2 = 0.6, L_1 = L_2 = 0.2, Q = 0.1, \lambda_1 = 0.2, \delta_2 = 0.02, L = 1)$

**V. CONCLUSION:** A mathematical model of Jeffrey fluid through an artery with multiple stenoses in the presence of magnetic field has been presented. It is found that the resistance to the flow increases with height of the stenosis, Jeffrey fluid parameter but decreases with magnetic parameter. The wall shear stress increases with stenosis height, magnetic parameter but decreases with Jeffrey parameter.

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