A STUDY ON APPLICATION OF NEWTON'S LAW OF COOLING IN SAPE HEAT TREATMENT ENGINEERS

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***Abstract*:** Differential Equation is used in our day to day life. An appropriate knowledge about the basic differential will go a long way in improving the process happened in the machines. The cooling of objects is often described by a law, attributed to Newton, which states that the temperature difference of a cooling body with respect to the surroundings decreases exponentially with time. However, the heat transfer from any object to its surrounding is not only due to conduction and convection but also due to its radiation. This paper presents a theoretical analysis of the cooling of objects.

Keywords: Newton's law of Cooling, Temperature, Wear Plate, Case Hardening

INTRODUCTION

Newton's law of cooling is one of the basic laws of physics with wide applications. The mathematics of this law helps us to understand many other phenomena for instance radioactive decay of isotopes. This relationship was derived from an empirical observation of convective cooling of hot bodies made by Isaac Newton in 1701, who stated that "the rate of loss of heat by a body is directly proportional to the excess temperature of the body above that of its surroundings".

MAIN WORK

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings).

$$\frac{\mathbf{dT}}{\mathbf{dx}} \text{is proportional to } (\mathbf{T}-\mathbf{T_a})$$

$$\frac{\mathbf{dT}}{\mathbf{dx}} = -\mathbf{k}\,(\mathbf{T}-\mathbf{T_a})$$

Where k is known as positive constant

This is an example of first order differential equation.

The independent variable is t for time, the function we want to find is T (t) ,and the quantities T_a, k are constants

We assume that,

$y(t) = T(t) - T_a$ …… (1)

$y_0 = T(0) - T_a$ …… (2)

Now that if we take a derivative of y (t) and Newton's law of cooling becomes

$$\frac{dy}{dt} = \frac{d}{dx}(T(t) - T_a)$$

$$\frac{dy}{dt} = \frac{dT}{dt} - \frac{dT_a}{dt}$$

$$\frac{dT}{dt} = -k(T - T_a)$$

$$\frac{dT}{dt} = -ky$$

Whose solution is well known to us, namely

$y(t) = y_0 e^{-kt}$ …… (3)

Using (1) and (2) in (3)

$$T(t) - T_a = (T_0 - T_a)e^{-kt}$$

$$\mathbf{T(t) = T_a + (T_0 - T_a)e^{-kt}}$$

Hence this is the solution of Newton's law of cooling.

PROBLEM USING NEWTON'S LAW OF COOLING

Question

A Wear Plate is cooled after the process of case hardening in Bottom Drop Furnace at a temperature of 550ºC. Temperature of the wear plate dropped from 550ºC to 470ºC for the first hour. Determine how many degrees the wear plate is cooled in another one hour if the environment temperature is 30ºC.

Solution

Let the initial temperature of the heated material (wear plate) be $T_0 = 550$

Let the surrounding environment temperature be $T_s = 30$

So the solution of the newton's law of cooling is

$$\mathbf{T(t) = T_a + (T_0 - T_a)e^{-kt}}$$

Where k is the positive constant

Since $T_0 = 550$

$$T(t) = T_s + (550 - T_s)e^{-kt}$$

At the end of the first hour the wear plate has cooled 470ºC. Therefore, we can write the following relationship is

t=1 ➡ $T(1) = 470$

$$470 = T_s + (550 - T_s)e^{-k.1}$$

After second hour the wear plate's temperature becomes equal to X degree

t=2 ➡ $T(2) = X$

$$X = T_s + (550 - T_s)e^{-k.2}$$

Thus we obtain the system of two equations with three unknown T_s , k , X

$$470 = T_s + (550 - T_s)e^{-k.1}$$

$$X = T_s + (550 - T_s)e^{-k.2}$$

We cannot determine uniquely the wear plate's temperature X after the second hour from this system

However, we can derive the dependence of X on the environment temperature T_s. Express the function e^{-k} from the first equation

$$470 = T_s + (550 - T_s)e^{-k}$$

$$470 - T_s = (550 - T_s)e^{-k}$$

$$e^{-k} = \frac{470 - T_s}{550 - T_s}$$

Hence, $e^{-2k} = \left(e^{-k}\right)^2$

$$\left(e^{-k}\right)^2 = \left(\frac{470 - T_s}{550 - T_s}\right)^2$$

Then the dependence $X(T_s)$ has the form

$$X(T_s) = T_s + (550 - T_s)\left(\frac{470 - T_s}{550 - T_s}\right)^2$$

$$X(T_s) = T_s + \frac{(470 - T_s)^2}{550 - T_s}$$

If the surrounding environment temperature is 30ºC, the wear plate temperatures X in 2 hour will be

$$X(T_s = 30) = 30 + \frac{(470 - 30)^2}{550 - 30}$$

$$X(30) = 30 + \frac{(440)^2}{520}$$

$$X(30) = 30 + \frac{193600}{520}$$

$$X(30) = \frac{15600 + 193600}{520}$$

$$X(30) = \frac{209200}{520}$$

$$X = 402.31$$

$$X = 402°C$$

At the end of 2nd hour it dropped to 402°C.

CONCLUSION

Temperature of the wear plate is decreased from 550°C to 402°C at the end of second hour at the room temperature of 30ºC.

Thus the problem on differential equation concludes that time required for the material (wear plate) of the temperature to cool down can be determined.

From this application of Newton law of cooling I came to know that the cooling temperature of products will vary according to their state.

In general, the company will allow every product for maximum of 10 hours in cooling state. Allowing every product to 10 hours is not necessary, by using Newton law of cooling we can find the time consumption and we can reduce the time for each and every product. This time consumption may help the company to produce more products than now.

REFERENCES

[1] Shepley L. Ross, Differential Equations [third edition]
[2] ZafarAhsan, Differential Equations and their applications [second edition]
[3] N.P.Bali, Differential Equations
[4] S.Priyadharsini, P.Gowri and T.Aparna, Solution of fuzzy integro-differential equations by fuzzy Laplace transform method, International Journal of Advanced Mathematics. 2017
[5] S.Priyadharsini, Shanjeena M Basheer, R.Kaushalya and M.Sneha, A Study on Motion of a free falling body in Kinematic Equation, International Journal for Research in Applied Science and Engineering Technology. 2017