Kinematic Attitudes of a Planar Robot and a Spherical Wrist

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Abstract: **--**The central issue in industrial robotic manipulation is the ability to position the robotic hand at a specified location with a specified orientation at a given time. The arm and wrist assemblies of a robot are primarily for positioning the end – effector (or robot hand) and any tool it may carry in a world space. The combination of the spatial location and orientation of the end – effector (kinematic attitudes) can be described mathematically by means of a homogeneous transformation (HT) matrix. These homogeneous transformations are used to solve both direct and inverse kinematics problems of robotic manipulation. In view of the importance attributed to the kinematic attitudes of the robotic manipulation in an automated factory of the future, the present work focuses on the kinematic analysis of a 2 – degrees of freedom (DOF) planar robot and the orientation aspects of a spherical wrist. This is to say that the work herein gives an insight into the kinematic modelling of a planar robot and a spherical wrist from the viewpoint of a geometric approach and a D – H convention – a more convenient form of HT matrix in robotic engineering. A computer based numerical tool called MATALB was used in developing the algorithms for analyzing kinematic attitudes of a robot and a wrist. The codes so developed were tested and verified with suitable numerical examples.

Keywords: **--HT matrix, Kinematic modelling, Planar robot, Robotic manipulation, Robot hand, Spherical wrist.**

Introduction

Robotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their applications requires knowledge of electrical engineering, mechanical engineering, systems and industrial engineering, computer science, economics, and mathematics. [1] An industrial robot is a general-purpose, reprogrammable machine. It possesses some anthropomorphic characteristics, i.e. human-like characteristics that resemble the human physical structure. The robots also respond to sensory signals in a manner that is similar to humans. Anthropomorphic characteristics such as mechanical arms are used for various industry tasks. Robots are essentially motion devices. So, kinematics is the fundamental aspect of the multidisciplinary area of robotics. The kinematic model of a manipulator describes the relationship between joint displacements and end effector motion. It is composed of position and velocity formulations. [2] The position kinematics relates joint positions and end effector pose. The forward kinematic analysis is the processes of calculating end effector pose from given joint positions, while the inverse kinematics analysis is the process of obtaining joint positions necessary to establish a desired end effector pose.

Differential kinematics deals with relationship between velocities. The direct velocity kinematics deals with the determination of linear and angular velocities of the end effector from the joint velocities. The inverse velocity analysis deals with the opposite relationship. [3] Spherical wrist robot arms are those having the axes of their hand joints intersecting at one common point. These joints must be of the revolute type. A spherical joint can be represented by three consecutive rotary joints with intersecting rotation axes. This paper aims to clarify some concepts related to the kinematic analysis of a 2-DOF planar robot using Geometric approach, DH Convention method and also discusses orientation aspects of a spherical wrist.

Kinematic Analysis By Geometric Approach

A. Forward Kinematics

Calculating the position and orientation of the end-effector in terms of the joint variables is called as forward kinematics

From the Fig.1, the position of the end effector can be written in terms of the joint coordinates in the following way

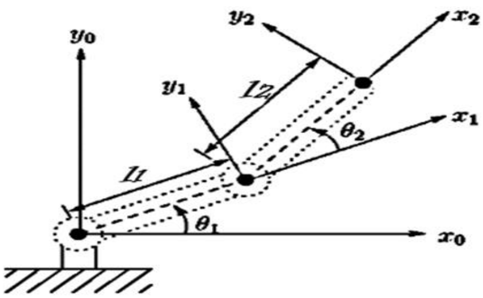

Fig. 1: Schematic representation of 2-DOF planar manipulator [12]

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \quad (1)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \quad (2)$$

where θ_1, θ_2 are joint angles and (x, y) is the final position of the hand in Cartesian space

B. Inverse Kinematics

The analysis or procedure that is used to compute the joint coordinates for a given set of end effector coordinates is called inverse kinematics.

If the given (x, y) coordinates are within the robot's reach there may be two solutions as shown in Fig. 2, the so-called elbow up and elbow down configurations, or there may be exactly one solution if the manipulator must be fully extended to reach the point. There may even be an infinite number of solutions in some cases.

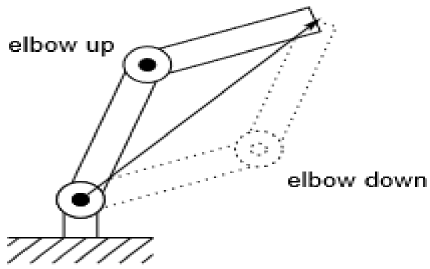

Fig. 2: Multiple inverse kinematic solutions [1]

From the Fig. 3, using the law of cosines we see that the angle θ_2 is given by

$$\cos\theta_2 = \frac{x^2 + y^2 - l_1^{\,2} - l_2^{\,2}}{2 l_1 l_2} = D$$

$$\sin\theta_2 = \pm\sqrt{1 - D^2}$$

hence, θ_2 can be found by

$$\theta_2 = a\tan 2(\pm\sqrt{1 - D^2}, D) \quad (3)$$

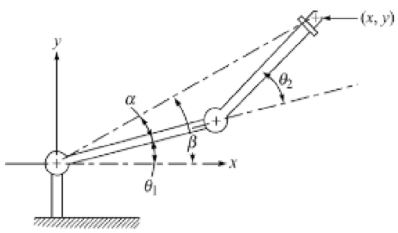

Fig. 3: Solving for the joint angles [12]

$$(\beta - \alpha) = \theta_1$$

$$\tan\alpha = \frac{l_2 \cos\theta_2}{l_2 \cos\theta_2 + l_1}$$

$$\tan\beta = \frac{y}{x}$$

$$\tan(\beta - \alpha) = \frac{y(l_2 \cos\theta_2 + l_1) - x(l_2 \sin\theta_2)}{x(l_2 \cos\theta_2 + l_1) + y(l_2 \sin\theta_2)} \qquad (4)$$

The orientation of a 2R robot is,

$$\phi = \theta_1 + \theta_2 \qquad (5)$$

Kinematic Analysis ByDenavit – Hartenberg Convention

The Denavit – Hartenberg (D-H) convention is consistent and concise description of the kinematic relations between the links of a kinematic chain with one degree of freedom lower pair joints. The systematic procedure to obtain the parameter set of an open kinematic chain is described in the following section.

The parameter identification procedure requires a specific definition of reference frames attached to the links of the kinematic chain. Fig. 4 illustrates this definition.

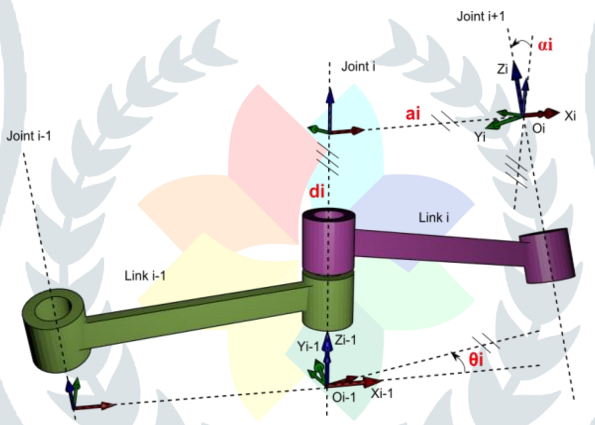

Fig. 4:Representation of four D-H parameters [11]

Matrix Ai representing the four movements is found by

(a) Rotation of θ_i about current z axis

(b) Translation of d_i along current z axis

(c) Translation of a_i along current x axis

(d) Rotation of α_i about current x axis

$$^{i-1}A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$^{i-1}A_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

The overall transformation matrix is represented as

$$^{0}A_n = {}^{0}A_1\,{}^{1}A_2 \ldots {}^{n-i}A_n = \begin{bmatrix} {}^{0}R_n & {}^{0}p_n \\ 0 & 1 \end{bmatrix} \quad (7)$$

It must be observed that for prismatic joints d_i is the joint variable and for rotational joints, θ_i is the joint variable. Matrix $^{0}A_n$ expresses the direct kinematics of the manipulator. $^{0}p_n$ is the end-effector position, while its orientation is given by the rotation matrix $^{0}R_n$, which can be used to obtain Euler angles or other representations.

A. Forward Kinematics

The following table describes the D-H parameters for the 2-DOF planar manipulator

Table I: D-H parameters for 2-DOF planar manipulator

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2

substitute the above parameters in A matrix, we get

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The successive homogeneous transformation would result in $T_2^0 = A_1 * A_2$

$$T_2^0 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

with,

$$c_1 = \cos\theta_1$$
$$s_1 = \sin\theta_1$$
$$c_{12} = c_1 + c_2$$

B. Inverse Kinematics

The inverse kinematics can be obtained by using the relation

$$T_2^0 = A_1 * A_2$$
$$A_1^{-1} * T_2^0 = A_2$$

$$\begin{bmatrix} c_1 & s_1 & 0 & -a_1 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

by comparing the elements of (LHS and RHS) the above matrices, we get

$$\begin{aligned} & n_x c_1 + n_y s_1 = \cos\theta_2 \\ & o_x c_1 + o_y s_1 = -\sin\theta_2 \\ & a_x c_1 + a_y s_1 = 0 \\ & p_x c_1 + p_y s_1 - a_1 = a_2 \cos\theta_2 \quad (10) \\ & -n_x s_1 + n_y c_1 = \sin\theta_2 \\ & -o_x s_1 + o_y c_1 = \cos\theta_2 \\ & -a_x s_1 + a_y c_1 = 0 \\ & -p_x s_1 + p_y c_1 = a_2 \sin\theta_2 \end{aligned}$$

Eq. (10), can be used to get $\theta_1 and \theta_2$

Orientation Aspects of A Spherical Wrist

Spherical wrist is having the axes of their joints intersecting at one common point. These joints must be of the revolute type. A spherical joint can be represented by three consecutive rotary joints with intersecting rotation axes.

There are many ways to represent rotation, including the following: Euler angles, Gibbs vector, Cayley-Klein parameters, Pauli spin matrices, axis and angle, orthonormal matrices, and Hamilton's quaternions.

Let $(i_x, j_y, k_z) and (i_u, j_v, k_w)$ be the unit vectors along the coordinate axes of the OXYZ and OUVW systems respectively. The 3x3 transformation matrix R that will transform the coordinates of p $_{uvw}$ to the coordinate expressed with respect to the OXYZ coordinate system, after the OUVW coordinate system has been rotated. That is,

$$P_{xyz} = RP_{uvw}$$

The general rotation matrix is written as

$$R = \begin{bmatrix} i_x.i_u & i_x.j_v & i_x.k_w \\ j_y.i_u & j_y.j_v & j_y.k_w \\ k_z.i_u & k_z.j_v & k_z.k_w \end{bmatrix} \quad (11)$$

where I, j, k are unit vectors

A. Euler Angles

Euler angles are a minimal representation of relative orientation. This set of three angles describes a sequence of rotations about the axes of a moving reference frame. There are however many (12, to be exact) sets that describe the same orientation: different combination of axes (e.g. ZXZ, ZYZ, and so on) lead to different Euler angles. Identical axes should not be in consecutive places (e.g. ZZX). In our study we consider ZYZ Euler angles

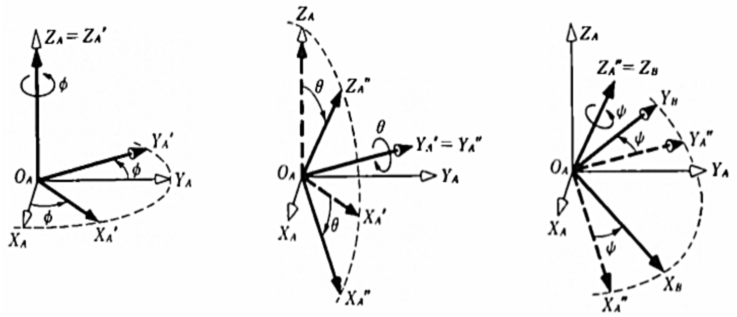

Fig. 5: Representation of Euler angles [5]

The final form of orientation matrix of a spherical wrist is

$$R_{zyz} = R_{z,\phi} R_{y,\theta} R_{z,\varphi}$$

$$R_{zyz} = \begin{bmatrix} c\phi c\theta c\varphi - s\phi s\varphi & -c\phi c\theta s\varphi - s\phi c\varphi & c\phi s\theta \\ s\phi c\theta c\varphi + c\phi s\varphi & -s\phi c\theta s\varphi + c\phi c\varphi & s\phi s\theta \\ -s\theta c\varphi & s\theta s\varphi & c\theta \end{bmatrix} \quad (12)$$

where I, j, k are unit vectors

(a)Inverse orientation

The inverse orientation values (i.e., Euler angles) are obtained by solving the Eq. (12)

The obtained angles are

$$\theta = a\tan 2(\pm\sqrt{(R_{13})^2 + (R_{23})^2}, R_{33})$$

If sin θ ≠0, we have

$$\phi = a\tan 2(\pm R_{23}, \pm R_{13}) \quad (13)$$

$$\varphi = a\tan 2(\pm R_{32}, \mp R_{31})$$

B. D-H Convention

Representation of orientation of a spherical wrist by D-H convention

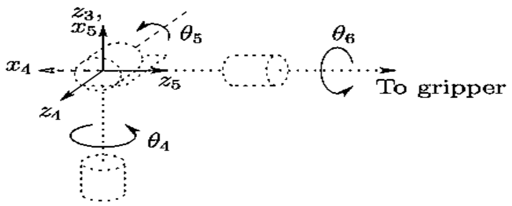

Fig. 6: The spherical wrist frame assignment [1]

The following table describes the D-H parameters for a spherical wrist

Table II: D-H parameters for the spherical wrist.

Link	a^i	α^i	d^i	θ^i
4	0	-90°	0	θ_4
5	0	90°	0	θ_5
6	0	0	d^6	θ_6

The final orientation matrix is given by

$$T_6^3 = A_4 * A_5 * A_6$$

$$T_6^3 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4s_5s_6 - s_4c_6 & c_4s_5 & c_4s_5d_6 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 & s_4s_5d_6 \\ -s_5c_6 & s_5s_6 & c_5 & c_5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

The above 4*4 homogeneous transformation matrix is in the form of eq. (6), so the rotation matrix from the above equation is

$$R_6^3 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4s_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix} \quad (15)$$

(a) Inverse orientation

$$\theta_5 = a\tan 2(\pm\sqrt{(R_{13})^2 + (R_{23})^2}, R_{33})$$

If sin θ ≠0, we have

$$\theta_4 = a\tan 2(\pm R_{23}, \pm R_{13}) \quad (16)$$

$$\theta_6 = a\tan 2(\pm R_{32}, \mp R_{31})$$

Results and Discussion

A. Result for 2-DOF planar robot by Geometric approach

Let us consider the link lengths and joint angles as follows

l_1 = 10 cm, l_2 = 8 cm

$\theta_1 = 30°$, $\theta_2 = 30°$

from eq. (1) and (2), we get forward kinematic solution as below

x = 12.6602 cm

y = 11.9282 cm

The inverse kinematic solution can be obtained from eq. (3), (4) is

For elbow up, $\theta_2 = -30°, \theta_1 = -30°$

For elbow down, $\theta_2 = 30°, \theta_1 = 30°$

The orientation of the 2-DOF planar robot is obtained by using eq. (5)

$$\phi = \theta_1 + \theta_2$$
$$\phi = 60° or - 60°$$

B. Result for 2-DOF planar robot by D-H Convention method

Let us consider the link lengths and joint angles are as follows

l_1 = 10 cm, l_2 = 8 cm

$\theta_1 = 30°$, $\theta_2 = 30°$

The forward kinematic result obtained from eq. (8) is

x = 12.6601 cm

y = 11.9281 cm

The inverse kinematic result obtained from eq. (10) is

$\theta_2 = 30°$; $\theta_1 = 30°$

C. Results for Orientation of a Spherical Wrist

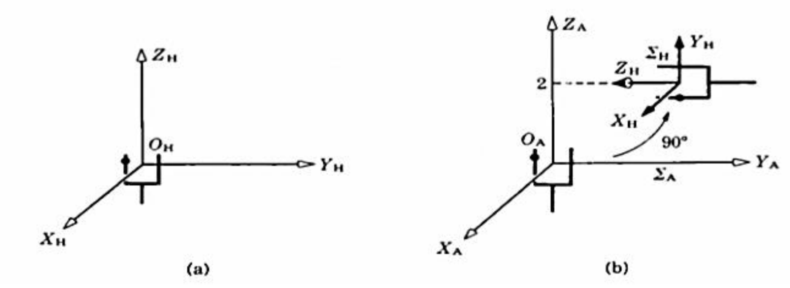

Fig. 7: Representation of position and orientation of a hand. (a) Hand frame $\sum_H$

(b) Relation between $\sum_A and \sum_H$ [5]

The hands orientation after rotation is given by

$$^{A}R_{H} = R_{zyz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

The joint angles of the spherical wrist are obtained by using Euler angles and D-H convention method.

Euler angles	D-H Convention
$\phi = -90°$ $\theta = 90°$ $\varphi = 90°$	$\theta_4 = -90°$ $\theta_5 = 90°$ $\theta_6 = 90°$

For the same rotation matrix, the joint angles obtained are equal for Euler angles and D-H convention method.

For the same joint angles, the rotation matrix obtained by using eq. (12) and eq. (15) is same.

From the above it is evident that D-H Convention method has yielded better results when compared to Geometric approach. This is due to the fact that the inverse kinematic solution of spatial robot (higher order DOF) can be obtained without much difficulty by D-H convention method.

Conclusions

In the present work mathematical modeling of the forward, inverse kinematic problem of 2-DOF planar manipulator has been carried out by using Geometric approach, D-H Convention methods and their significance for the kinematic analysis have been studied.Orientation aspects of the spherical wrist were also analysed by using Euler angles and D-H Convention

Among all the methods used, D-H Convention method is much better for analysis of a given robot manipulator and wrist combination.

D-H Convention method is also applicable to redundant robot arms

The algorithms were developed using MATLAB to verify the accuracy of the solutions obtained.

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