FEM for Stress Reduction by Optimal Auxiliary Holes in a Loaded Plate with Elliptical Hole

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***Abstract:* --**Steel is widely used in machine parts, structural equipment and many other applications. In many steel structural elements, holes of different shapes and orientations are made with a view to satisfy the design requirements. The presence of holes in steel elements creates stress concentration, which eventually reduce the mechanical strength of the structure. Therefore, it is of great importance to investigate the state of stress around the holes for the safety and properties design of such elements. By literature survey, it is known that till date, there is no analytical solution to reduce the stress concentration by providing auxiliary holes at a definite location and radii in a steel plate. The numerical method can be used to determine the optimum location and radii of auxiliary holes. In the present work plate with an elliptical hole, for a steel material subjected to uniaxial load is analyzed and the effect of stress concentration is graphically represented .The introduction of auxiliary holes at a optimum location and radii with its effect on stress concentration is also represented graphically.The finite element analysis package ANSYS 11.0 is used to analyse the steel plate. The analysis is carried out using a plane 42 element. Further the ANSYS optimization model is used to determine the location and radii for optimum values of auxiliary hole to reduce stress concentration. All the results for different diameter to plate width ratio are presented graphically.The results of this study are in the form of the graphs for determining the locations and diameter of optimal auxiliary holes .The graph of stress concentration v/s central hole diameter to plate width ratio.The Finite Elements results of the study indicates that the stress concentration effect of central elliptical hole in an uniaxial loaded plate can be reduced by introducing auxiliary holes on either side of the central circular hole.

***Keywords:* --Finite Element Method, Optimization, Stress concentration factor, auxiliary holes.**

Introduction

The failure of structures due to stress concentration at any discontinuity has been baffling engineers for long. It is found that structure failures in ships, offshore structures, boilers or high rise buildings subjected to natural calamities is due to stress concentration. problem of determining stress distribution around holes in a plate has been treated extensively in the literature. T. Hasan [2] has reported stress analysis of steel plate having holes of various shapes, sizes and orientations using finite element method. Finite element analysis is carried out by using the commercial software. Effects of hole shapes are critically analyzed and results are compared with analytical method which showed good agreement with the results.

A.J.Durelli, V.J.Parks and H.C.Feng[3] presented Photoelastic solution of the distribution of stresses around a centrally located elliptical hole in a plate of finite width subjected to uniform axial loading and reported stress distribution at the boundaries for a wide range of the parameters, and stress concentration factors have been computed for the points of maximum tensile and maximum compressive stress. Ukadgaonkar [4] has given a new approach of stress analysis of an infinite plate with elliptical hole or crack with tensile stress .The closed form equations for SCF are given for anisotropic plate. According to the study SCF in flat plate with different geometry can be reduced by introducing auxiliary holes. The location of auxiliary holes from the center of ellipse has to be reported. G. V. Kolosoff and C. E. Inglis [5]: proposed boundary element method to the problems of anisotropic plates containing multiple various shaped holes, this research solves the stress and strain fields numerically by embedding the fundamental solutions of polygonal holes into boundary element method (BEM), and using the technique of sub region. In the literature, most of the examples of multi-holes consider elliptical holes due to the geometry restraints; it limits the range of the problems. Since the fundamental solutions have satisfied the boundary condition for holes, it's unnecessary to descretize the hole boundary. Since, the fundamental solutions are for the polygon-like holes, their associated BEM would be able to solve multi-holes much more effectively.

There is extensive literature which uses finite element method (FEM) to optimize the stress concentration but not used for auxillaryhole optimization for stress reduction. In the present work FEM optimization was carried to determine the optimum sizes and location for the auxiliary holes for different center elliptical hole diameter to plate width ratios were minimization was achieved for all possible ratios.

Description of the Problem

To examine the stress distribution around the original hole and to study the effect of auxiliary hole on stress concentration following model with given dimension and notations are used for the study as shown in Fig.1.

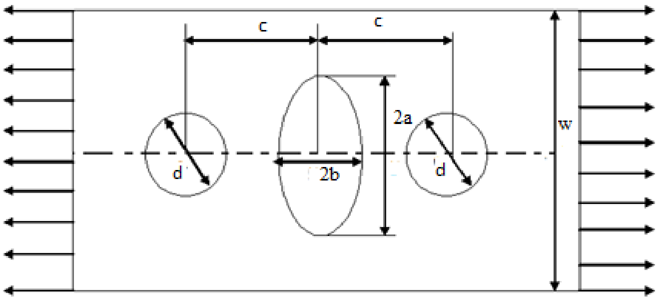

Fig. 1. Model of full plate.

Initial the model is analyzed without auxillary holes to determine stress distribution for different c/w ratios of 0.22, 0.25 and 0.285. For each ratios optimum auxillary hole location and radii are determine and its effect on stress concentration is analysed.

Modelling With Fem

Mathematically, the structure to be analyzed is subdivided into mesh of finite sized elements of simple shapes .Within each element, the variation of displacement is assumed to be determined by simple polynomial shape functions and nodal displacement. After the boundary conditions are applied, the nodal displacements are found by solving the matrix stiffness equation. Once nodal displacements are known, element stresses and strains can be calculated.Quarter plate modeling with symmetrical boundary condition as in Fig. 2 yields same results that of full plate and half plate modeling. Hence, to take the advantage of symmetrical boundary condition and to reduce computational time, the present work is carried out using quarter plate.

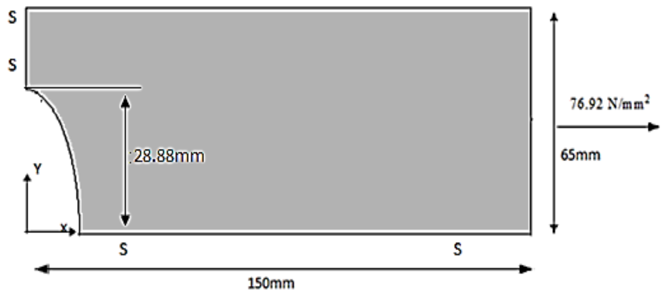

Fig. 2 Model of quarter plate

The analysis is carried out using ANSYS software version 8.0 using plane 42 element types for steel material and the stress distribution along the edge of the hole to plate is determined and represented graphically as in Fig. 3. The FEM results are compared with analytical method to validate the boundary conditions and results with that of FEM .

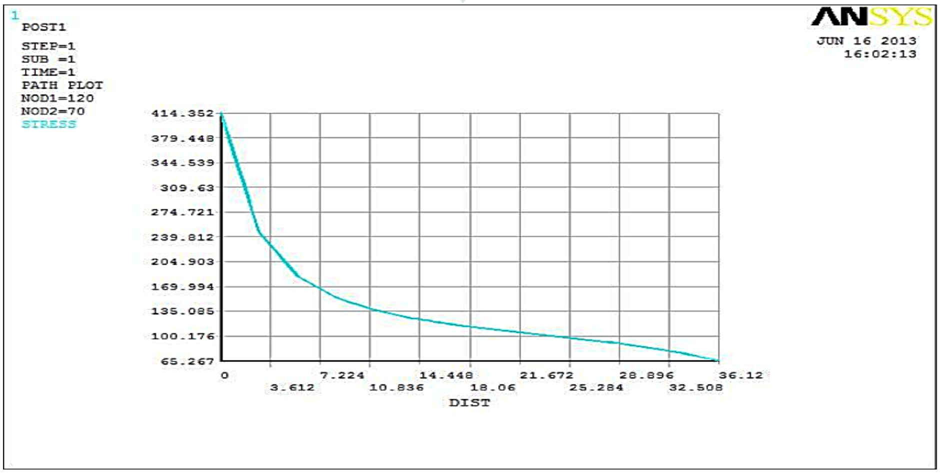

Fig 3. Finite plate width to hole Stress Gradient

A. Analytical Solution

From the analytical solution for a finite plate with central hole under uniaxial loading conditions,

Table IAnalytical Method For a/w =0.22

Elliptical major axis(mm)	a= 28.88
Elliptical minor axis(mm)	b=14.44
Area (mm^2)	A= 130
Force (N)	P= 10000
Plate Width (mm)	W= 130
a/w ratio	0.22
K_t	5.353
$\sigma_{max} = K_t \times \sigma$ (N/mm^2)	411.85

By using above formula the variation of stress is calculated for different a/w ratios.
It is found from the result that FEM results are in good agreement with that of analytical method. The analysis is performed for different c/w ratios

B. Optimization using fem.

Optimization is a technique that seeks to determine an optimum design. By "optimum design," we mean one that meets all specified requirements but with a minimum expense of certain factors such as weight, surface area, volume, stress, cost, etc. In other words, the optimum design is usually one that is as effective as possible.In the present work introduction of auxiliary holes on either side of parent holes smoothens stress gradients around the holes but at what location and of what radii is still in trial and error approach or possible by experimental approach[1].FEM is used to locate optimum auxiliary holes for the stress reduction using ANSYS optimization tool to locate auxiliary holes for different c/w ratios and reduction is compared with that of plate without auxiliary holes.

B.1. Terminology used in optimization.

Design Variables (DVs) are independent quantities that are varied in order to achieve the optimum design. Here “a” and “d” parameters are chosen as design variables.

State Variables(SVs) are quantities that constrain the design. They are also known as "dependent variables," and are typically response quantities that are functions of the design variables. A state variable may have a maximum and minimum limit, here stress without auxillary holes is chosen as SV.

The **Objective Function** is the dependent variable that you are attempting to minimize the stress

The design variables, state variables, and objective function are collectively referred to as the optimization variables

A design set, feasible design, infeasible and best set are the possible outcome of optimization process.

B.2. Optimization For C/W Ratio 0.22.

For determining optimum auxiliary hole diameter and location for the c/w=0.22 ratio above optimization procedure is used and feasible results are tabulated as shown in Table II and Table III.

Table II

Possible set of optimization for c/w =0.22

Parameters	R (DV)	X (DV)	SMAX(OBJ)
Set 1	9.000	2.000	425.89
Set 2	16.947	37.111	380.49
Set 3	7.772	55.619	414.19
Set 4	10.760	49.816	406.45
Set 5	19.101	71.80	392.99
Set 6	19.0215	9.5017	367.62
Set 7	19.752	18.521	359.85
Set 8	19.904	23.680	355.12

From the above set of optimal results, it is found that auxiliary hole nearer(x=2mm) to the parent hole with radius r=9mm stress level increases with varying radius from r=0 to 20mm.On the other hand as we move away from the parent hole(x=71.80mm) with radius r=19.10mm stress level increases with varying radius for r=0 to 20mm. Observing all the set of optimal result it is found that auxiliary hole at x=23.680mm with radius 19.904mm yields optimal value for the ratio a/w=0.22.

Table III

Best set of optimization for c/w =0.22

a/w	c (in mm)	d (in mm)	σmax (N/mm2)	% of stress reduction
0.22	39.808	23.680	355.12	18.36%

Result and Discussions

The uniaxially loaded rectangular plate with a elliptical hole was modeled for different Central hole diameter-to-plate width ratios using ANSYS 11.0. The loading conditions were maintained constant for all models. Optimization for finding the location of optimum auxiliary holes and its diameter was carried out using Optimization tool available in ANSYS 11.0.
The results have been tabulated in the Table IV.

Table IV

Optimization results for different c/w ratios

a/w	d (in mm)	C (in mm)	σ_{max}(N/mm^2)	% of Stress reduction
0.22	39.992	23.680	358.05	18.36%
0.25	45.778	24.539	387.17	19.35%
0.285	49.76	26.55	452.17	15.05%

For plate with only central hole, the Nominal stress (σ_{nom}) for different c/w ratios is calculated as follows.It is known that,
$K_t = \sigma_{max} / \sigma_{nom}$

For the present loading condition σ_{max} N/mm^2 and K_t for different c/w ratios is obtained.
By using above equation, resulting σ_{nom} is tabulated in Table IVand Table V

Table IV
Scf for different a/w ratios

a/w	σ_{max}(N/mm^2)	σ_{max}(nom)	K_t
0 .22	438.579	76.92	5.70
0.25	480.045	76.92	6.24
0.285	532.28	76.92	6.9

Table IV
Optimum scf for different a/w ratios

a/w	σ_{max}(N/mm^2)	σ_{nom} (N/mm^2)	K_t (optimum)
0.22	358.05	76.92	4.65
0.25	387.17	76.92	5.03
0.285	452.17	76.92	5.87

The results of the study are presented in the form of three design curves. In Fig. 7.2 the optimum hole spacing to plate width ratio, c/w, is presented as function of central elliptical half the major axis to plate width ratio, a/w. This curve can be used to locate the auxiliary holes once the a/w ratio for the plate has been determined.

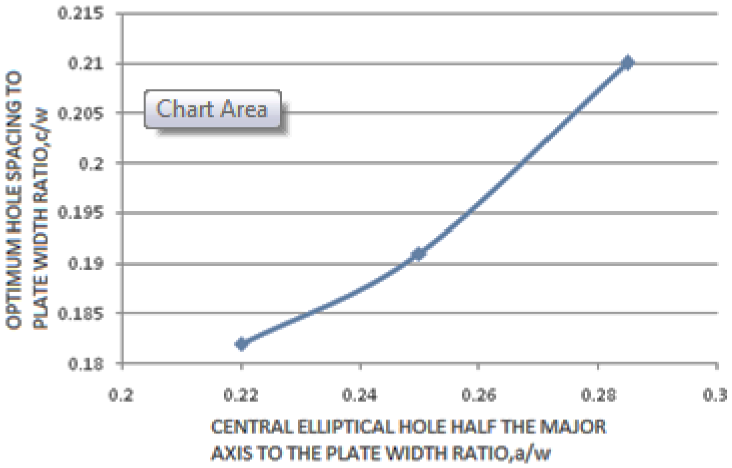

Fig 7.2(a) Optimum hole spacing to plate width ratio, c/w, as a function of central elliptical hole half the major axis to the plate width ratio, a/w

In Fig.7.2 (a) optimum auxiliary hole diameter to plate width ratio, d/w, is plotted as a function of central elliptical hole half the major axis to the plate width ratio, a/w. From this curve, the diameter of the auxiliary needed to produce the maximum reduction in stress level can be selected.

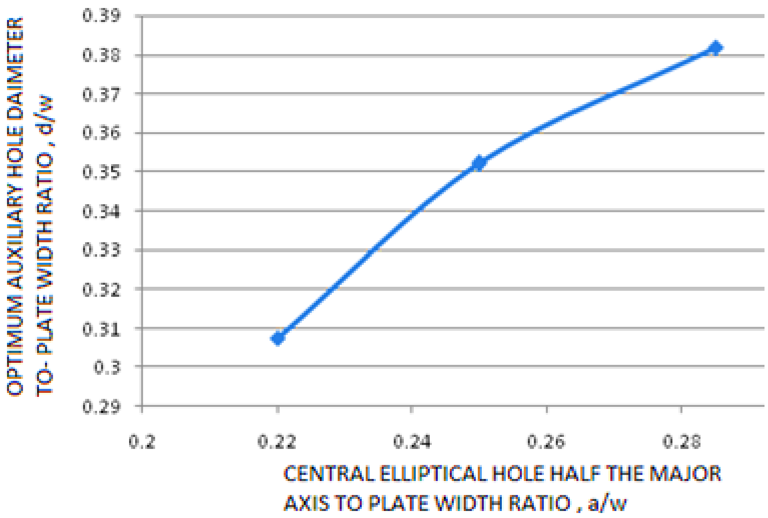

Fig 7.2(a) Optimum hole spacing to plate width ratio, c/w, as a function of central elliptical hole half the major axis to the plate width ratio, a/w.

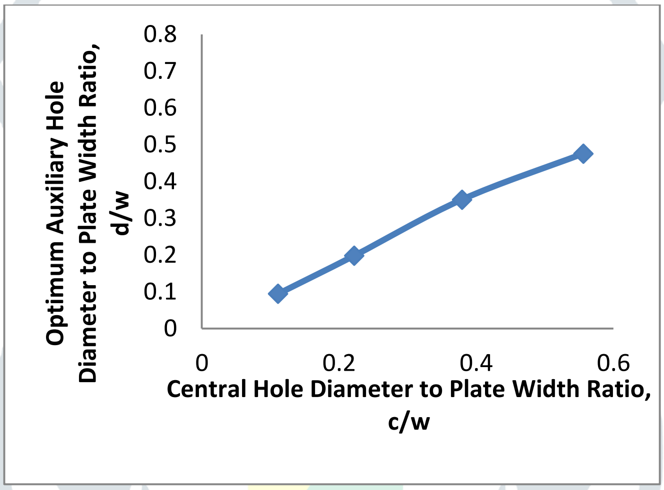

Fig 7.2(b) Optimum auxiliary hole diameter to plate width ratio, d/w, as a function of central elliptical hole half the major axis to plate width ratio, a/w.

Finally in Fig.7.2 stress concentration factor K_t, is plotted as a function of central elliptical hole half the major axis to plate width ratio, a/w. Curves are presented for plate without auxiliary hole and plates with optimum spaced and sized auxiliary holes. The results presented in Fig. 7.2(b) indicate that stress concentration effects of the central elliptical hole can be reduced by 15 to 20 percent depending on the A/W ratio of the plate. Further Table VI and Fig. 8 represents stress concentration factor with auxillary holes and without auxillary holes.

Table VI
kt with and without auxiliary hole

a/w	K_t without auxiliary hole	K_t with auxiliary hole
0.22	5.70	4.65
0.25	6.24	5.03
0.285	6.9	5.87

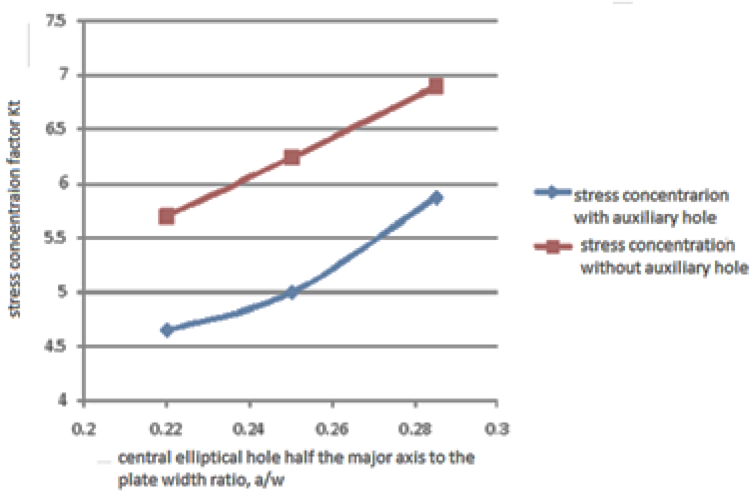

Fig. 8 stress concentration K_t as a function of central elliptical hole halfthe major axis to plate width ratio, a/w, for uniaxially loaded plate.

Conclusions

In this study, the location and diameter of optimal auxiliary holes is determined by using Finite Element Method with the software package ANSYS 11.0. The graphs are provided as a result of this study, which will help in determining the location and diameter of optimal auxiliary hole for any plate with a hole.The method of dividing the model into number of finite elements (FEM) is adopted for the study. Location and diameter of optimal auxiliary hole were determined by using the optimization tool available in the ANSYS 11.0.The results of this study are in the form of the graphs for determining the location and diameter of optimal auxiliary holes. The graph of stress concentration factor v/s Central hole diameter-to-plate width ratio.The Finite Element results of the study indicate that the stress concentration effects of a central circular hole in an uniaxially loaded plate can be reduced by 15 to 20 percent by introducing auxiliary holes on either side of the central elliptical hole. Such a reduction in stress amplitude could have a significant effect on the fatigue life of a part.

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