Deteriorating Items Supply Chain Inventory Model for Single Vendor Single Buyer under Exponential Demand

Jayshree J. Pandey and R.D. Patel
Department of Statistics
Veer Narmad South Gujarat University
Surat, Gujarat, India

Abstract: The optimal strategy for single vendor single buyer studied when components of integrated inventory models are based on deterioration and demand is exponential function of time under supply chain. The model introduced for single vendor single buyer system as profit maximized to determine the optimal cycle time without considering collaboration between vendor and buyer. We also determine the total profit when buyer and vendor take joint decision under supply chain policy. The model is illustrated with numerical examples and observed that both buyer and vendor earn significant profit in supply chain inventory system.

Keywords: Supply chain, optimal strategy, Deterioration, Exponential demand.

I. NTRODUCTION

Supply chain is the integration of manufacture, transportation and inventory among the contributors. Supply chain is a strategy for acquire the significant collaboration for responsive and efficient business for the market being served. A supply chain concerned directly or indirectly by considering all parties to satisfy the customer's demand. The supply chain inventory involves producers, sellers, vendors, buyers, shippers, warehouses together for significant business strategy. Supply chain is the policy for effective integration among suppliers, manufactures, distributors and inventory and so on, so that goods can be in the optimum quantity and be sent to the exact place at the optimum time, making the system-wide profit to a maximum and meeting the required service level at the same time. Supply chain works under the principle objectives such as minimize total costs and maximum total profits after collaboration between two more systems or individuals.

Goyal (1995) used the scheme of geometric consignment bulk which represent that the product of the preceding consignment bulk in relation to the ratio of manufacturing and demand rate is consecutive delivery amount under integrated inventory. Lu (1995) introduced the optimal policy when the delivered quantity sent to the buyer is identical and the stock was replenished for every time. Hill (1997) developed a single supplier who manufacture a product at a fixed rate and deliveries the product in sets of unequal amount to buyer by taking geometric expansion feature as a decision variable within a certain range and assumed the geometric consignment strategy.

Hill (1999) also derived supply chain model for global most favorable batching and delivery plan under manufacturing inventory policy for single vendor and single buyer. Goyal and Nebebe (2000) also expanded the model to study the proposed a simple geometric consignment strategy for same amount where the first delivery amount increases by the product of the first consignment bulk and the ratio of manufacturing rate to demand. Hoque and Goyal (2000) considered equal as well as unequal shipment size and limited capacity of transportation to present an optimal policy. Integrated inventory model explained by Goyal (2000) was the expansion of Hill's (1997) proposed model and designed that the consignment bulk would be estimated by first delivery amount. Hill and Omar (2006) introduced integrated inventory model for manufacture and consignment plan subject to minimize average total cost under the assumption that the component of supply caring costs increases as stock shifts down where shipments are not essentially equal in size. Yu (2010) derived collaborative inventory system under deterioration for imperfect quality and shortage backordering. Rad et al. (2014) discussed the collaborative inventory model for single buyer and single vendor for price dependent demand. Wakhid et al. (2014) derived joint inventory under supply chain policy for single vendor single buyer by considering defective quality and assessment errors.

II. NOTATIONS

$D(t)$ = Demand is exponential function of time, where $a > 0$, $0<b<1$
$I_b(t)$ = Inventory level for buyer at any instant of time t
$I_v(t)$ =Inventory level for vendor at any instant of time t
A_b = Ordering cost per order for buyer
A_v = Ordering cost per order for vendor
C_b = Purchase cost per unit for buyer
θ = Deterioration rate of items for buyer
x_b = Fixed holding cost for buyer
y_b = Varying holding cost for buyer
x_v = Fixed holding cost for vendor

y_v = varying holding cost for vendor
p = Selling price of buyer's unit
n = Buyer's number of orders placed during cycle time.
TP_b = Total profit for buyer per unit time
TP_v = Total profit for vendor per unit time
TP = Total profit for both vendor and buyer per unit time
$t_1 = v_1 * T/n$
T = Vendor's cycle time (a decision variable)

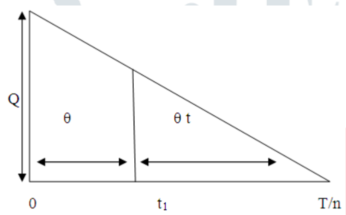
III. ASSUMPTIONS

The assumptions for the development of the model are as follows:

1. The demand of the product is declining as an exponential function of time.
2. Single vendor - single buyer is considered.
3. Shortages are not allowed.
4. Lead-time is zero.
5. Deteriorated units can neither be repaired nor replaced during the cycle time and deterioration is dependent on time for buyer's inventory.
6. Time varying holding cost is considered for buyer and vendor.

IV. MATHEMATICAL MODEL

Let $I_b(t)$ be the inventory level at time t ($0 \le t \le T/n$) as shown in figure.
Buyer's Inventory



We discuses two situations, in the first situation the inventory model is develop without collaboration between vendor and buyer, while the second situation considers the vendor buyer collaboration.

The inventory level is depleted by exponential demand for both vendor and buyer. The differential equations are given for rate of change of inventory for the vendor and the buyer:

$$\frac{dI_b(t)}{dt} + \theta I_b(t) = -a e^{bt} \quad ; \quad 0 < t < t_1 \tag{1}$$

$$\frac{dI_b(t)}{dt} + \theta t I_b(t) = -a e^{bt} \quad ; \quad t_1 < t < \frac{T}{n} \tag{2}$$

$$\frac{dI_v(t)}{dt} = -a e^{bt} \quad ; \quad 0 < t < T \tag{3}$$

with the boundary conditions:

$$I_b(0) = Q, \quad I_b\left(\frac{T}{n}\right) = 0, \quad and \quad I_v(T) = 0$$

Their solutions are given by

$$I_b(t) = Q(1-\theta t) - a\left(t + \frac{b}{2}t^2 + \frac{\theta}{2}t^2 + \frac{b\theta}{3}t^3\right) + a\theta t\left(t + \frac{b}{2}t^2\right) \tag{4}$$

$$I_b(t) = a\left\{\begin{array}{l} \left(\frac{T}{n} - t\right) + \frac{b}{2}\left(\frac{T^2}{n^2} - t^2\right) + \frac{\theta}{6}\left(\frac{T^3}{n^3} - t^3\right) \\ + \frac{b\theta}{8}\left(\frac{T^4}{n^4} - t^4\right) - \frac{\theta t^2}{2}\left(\frac{T}{n} - t\right) - \frac{b\theta t^2}{4}\left(\frac{T^2}{n^2} - t^2\right) \end{array}\right\} \tag{5}$$

$$I_v(t) = a\left[(T - t) + \frac{b}{2}\left(T^2 - t^2\right)\right] \tag{6}$$

Substituting $t=\frac{t_1}{n}$ in equations (4) and (5) and simplifying, we get

$$Q=\frac{1}{\left(1-\frac{\theta t_1}{n}\right)}\left\{\begin{array}{l} a\left(\frac{\theta}{2}\frac{t_1^2}{n^2}+\frac{b\theta}{3}\frac{t_1^3}{n^3}\right)-a\theta\frac{t_1}{n}\left(\frac{t_1}{n}+\frac{b}{2}\frac{t_1^2}{n^2}\right) \\ +a\left(\begin{array}{l} \frac{T}{n}+\frac{b}{2}\frac{T^2}{n^2}+\frac{\theta}{6}\left(\frac{T^3}{n^3}-\frac{t_1^3}{n^3}\right) \\ +\frac{b\theta}{8}\left(\frac{T^4}{n^4}-\frac{t_1^4}{n^4}\right)-\frac{\theta}{2}\frac{t_1^2}{n^2}\left(\frac{T}{n}-\frac{t_1}{n}\right) \\ -\frac{b\theta}{4}\frac{t_1^2}{n^2}\left(\frac{T^2}{n^2}-\frac{t_1^2}{n^2}\right) \end{array}\right) \end{array}\right\} \quad (7)$$

Putting the value of Q in equation (4) we get

$$I_b(t)=\frac{(1-\theta t)}{\left(1-\frac{\theta t_1}{n}\right)}\left\{\begin{array}{l} a\left(\frac{\theta}{2}\frac{t_1^2}{n^2}+\frac{b\theta}{3}\frac{t_1^3}{n^3}\right)-a\theta\frac{t_1}{n}\left(\frac{t_1}{n}+\frac{b}{2}\frac{t_1^2}{n^2}\right) \\ +a\left(\begin{array}{l} \frac{T}{n}+\frac{b}{2}\frac{T^2}{n^2}+\frac{\theta}{6}\left(\frac{T^3}{n^3}-\frac{t_1^3}{n^3}\right) \\ +\frac{b\theta}{8}\left(\frac{T^4}{n^4}-\frac{t_1^4}{n^4}\right)-\frac{\theta}{2}\frac{t_1^2}{n^2}\left(\frac{T}{n}-\frac{t_1}{n}\right) \\ -\frac{b\theta}{4}\frac{t_1^2}{n^2}\left(\frac{T^2}{n^2}-\frac{t_1^2}{n^2}\right) \end{array}\right) \end{array}\right\}$$

$$-a\left(t+\frac{b}{2}t^2+\frac{\theta}{2}t^2+\frac{b\theta}{3}t^3\right)+a\theta t\left(t+\frac{b}{2}t^2\right) \quad (8)$$

V. BUYER'S RELEVANT COSTS

Holding Cost:

$$HC_b=n\left\{x_b\left(\int_0^{t_1}I_b(t)dt+\int_{t1}^{\frac{T}{n}}I_b(t)dt\right)+y_b\left(\int_0^{t_1}t\,I_b(t)dt+\int_{t_1}^{\frac{T}{n}}t\,I_b(t)dt\right)\right\} \quad (9)$$

Deterioration Cost:

$$DC_b=nC_b\theta\left\{\int_0^{t_1}I_b(t)dt+\int_{t_1}^{\frac{T}{n}}tI_b(t)dt\right\} \quad (10)$$

Ordering Cost:

$$OC_b=nA_b \quad (11)$$

Sales Revenue:

$$SR_b = n\,p\int_0^{\frac{T}{n}} D(t)dt$$
$$= n\,p\int_0^{\frac{T}{n}} a(1+bt)dt$$
$$= n\,p\,a\left(\frac{T}{n}+\frac{b}{2}\frac{T^2}{n^2}\right) \tag{12}$$

Total Profit:

$$TP_b = \frac{1}{T}\left[SR_b - HC_b - DC_b - OC_b\right] \tag{13}$$

VI. VENDOR'S RELEVANT COSTS

Holding Cost:

$$HC_v = x_v\left\{\int_0^T I_v(t)dt - n\left(\int_0^{t_1} I_b(t)dt + \int_{t_1}^{\frac{T}{n}} I_b(t)dt\right)\right\} + y_b\left\{\int_0^T t\,I_v(t)dt - n\left(\int_0^{t_1} t\,I_b(t)dt + \int_{t_1}^{\frac{T}{n}} t\,I_b(t)dt\right)\right\} \tag{14}$$

Ordering cost:

$$OC_v = A_v \tag{15}$$

Sales Revenue:

$$SR_v = C_b\int_0^T D(t)\,dt$$
$$= C_b\int_0^T a(1+bt)\,dt$$
$$= C_b\,a\left(T+\frac{b}{2}T^2\right) \tag{16}$$

Total Profit for Vendor:

$$TP_v = \frac{1}{T}\left[SR_v - HC_v - OC_v\right] \tag{17}$$

VII. SITUATION-I: BUYER AND VENDOR TAKE DECISIONS WITHOUT COLLABORATION

Here the buyer and vendor take decisions without collaboration

Buyer's maximum profit TP_b can be determined by following conditions:

$$\frac{dTP_b}{dT_b} = 0 \; where \; T_b = \frac{T}{n} \tag{18}$$

It satisfies the condition

$$\frac{d^2TP_b}{dT_b^2} < 0 \tag{19}$$

This solutions (n, T) maximizes vendor's profit TP_v

Then the total profit without collaboration is given by;

$TP = \max(TP_b + TP_v)$ (20)

VIII. SITUATION-II: BUYER AND VENDOR TAKE DECISIONS WITH COLLABORATION

Here the buyer and the vendor jointly take decisions:

Simultaneously, the optimum value of T must satisfy the following conditions, which maximize total profit (TP) when buyer and vendor take joint decision:

$$\frac{dTP}{dT} = 0 \tag{21}$$

It satisfies the condition

$$\frac{d^2TP}{dT^2} < 0 \quad (22)$$

Where total profit (TP) with collaboration is given by;

$$TP = TP_b + TP_v \quad (23)$$

IX. NUMERICAL EXAMPLE

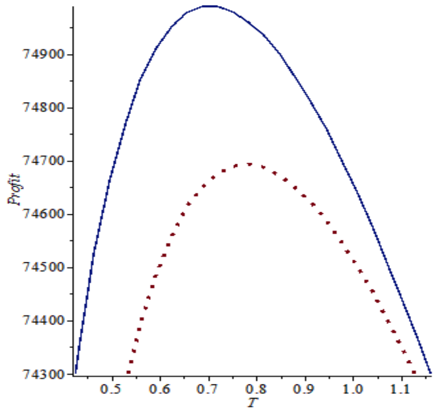
In order to illustrate our proposed model, we considering a = 1000, b = 0.05, x_b = 10, y_b = 0.03, x_v = 8, y_v = 0.01, A_b = 150, A_v = 1500, C_b = 35, p = 45, v_1 = 0.4 in appropriate units. The optimal values of T and profits for buyer and vendor are given in Table-1. The second order conditions given in equation (19) and equation (22) are also satisfied. The graphical representations of the concavity of the profits for independent and joint profits are also shown.

The optimal total profit TP = Rs. 74678.9 at n = 4 for buyer's profit TP_b* = Rs. 43355.17, T* = 0.7235 T_b^* = 0.1809 and TP_v = 31323.73 when buyer and vendor take decisions without collaboration. While when buyer and vendor take joint decision then the optimal total profit TP^* = Rs. 74991.4 at n = 2 and T^* = 0.7047 with buyer's profit TP_b = Rs. 42959.88 and TP_v= 32031.51.

Table-1

The optimal solutions for with collaboration and without collaboration

	Without collaboration	with collaboration
n	4	2
T_b	0.1809	0.3524
T	0.7236	0.7047
Buyer's Profit	43355.2	42959.9
Vendor's Profit	31323.7	32031.5
Total Profit	74678.9	74991.4



		Independent Decision			Joint Decision		
%	Parameters	TP_b	TP_v	TP	TP_b	TP_v	TP
20%	a	52199.5	37979.8	90179.2	51765.8	38757.1	90522.9
10%		47775.6	34647.8	82423.4	47360.6	35391.2	82751.8
-10%		38938.9	28008.7	66947.6	38564.2	28679.1	67243.3
-20%		34527.4	24704.3	59231.6	34174.5	25335.2	59509.7
20%	A_b	43196.8	31367.9	74564.6	33238.3	41695.7	74933.9
10%		43274.2	31338.4	74612.6	41731.4	33225.9	74957.4
-10%		43440.2	31297.6	74737.8	43012.8	32021.4	75034.1
-20%		43530.1	31255.9	74786.0	43066.4	32010.8	75077.2
20%	x_b	43182.1	31265.1	74447.1	42700.3	31945.5	74645.7
10%		43266.5	31297.7	74564.1	42827.0	31988.7	74815.8
-10%		43448.9	31340.6	74789.5	41917.8	33400.6	75318.4
-20%		43548.6	31411.3	74959.8	42074.2	33606.6	75680.9
20%	θ	43333.5	31318.0	74651.4	42926.4	32020.2	74946.6
10%		43344.3	31320.9	74665.2	42943.1	32025.9	74968.9

-10%		43366.1	31326.4	74692.5	41784.5	33241.7	75026.2
-20%		43377.2	31328.9	74706.1	41801.6	33270.4	75072.0
20%	x_v	43355.2	30932.6	74287.8	41763.7	33219.9	74983.6
10%		43355.2	31156.1	74511.2	41766.0	33215.9	74981.9
-10%		43355.2	31547.3	74902.4	42922.8	32216.4	75139.2
-20%		43355.2	31770.8	75126.0	42881.3	32410.3	75291.6
20%	A_v	43355.2	30909.1	74264.3	42861.7	31719.8	74581.5
10%		43355.2	31116.4	74471.6	42910.4	31872.2	74782.6
-10%		43355.2	31116.4	74471.6	41895.6	33326.2	75221.8
-20%		43355.2	31781.7	75136.9	42029.2	33445.2	75474.4

Sensitive analysis is carry out by changing the values of given parameters a, A_b, A_v, x_b, x_v and θ respectively, one parameter at a time and the reaming parameters are kept constant. Based on the results of Table-2 we can observe that total profit increases when buyer and vendor take joint decision instead as compared to independent decision. When a increases/decreases then total profit will increase/decrease, while if A_b, x_b, x_v, A_v and θ increase/decrease then total profit will decrease/increase in independent and joint decision.

X. CONCLUSION

The result shows that the optimal cycle time is significantly decreased and total profit significantly increased when buyer and vendor consider joint decision policy under supply chain as compared to independent decision taken by buyer and vendor. We can also observe that the vendor's profit is increased and number of times order placed by buyer during cycle time is also decreased when buyer and vendor take joint decision.

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