A STUDY ON SURFACE INSTABILITIES IN NEWTONIAN AND NON-NEWTONIAN FLUIDS

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Abstract

A large number of constitutive equations have been proposed to characterize the relationship between the stress and rate of strain in Newtonian/non Newtonian fluids. A Newtonian fluid is a power-law fluid with a behaviour index (n=1), where the shear stress is directly proportional to the shear rate. These fluids have a constant viscosity, across all the shear rates and include many of the most common fluids, such as water, most aqueous solutions, oil, corn syrup, glycerine, air and other gases. While this holds true for relatively low shear rates, at high rates most oils in reality also behave in a non-Newtonian fashion and thin. Typical examples include oil films in automotive engine shell bearings and to a lesser extent in gear tooth contacts.

Keyword: - Newtonian fluids, Boundary Value, Droplet

Introduction

The Newtonian fluids are true fluids that tend to exhibit constant viscosity at all shear rates (Karol, 2003; Krizek and Pepper, 2004). The relationship between the shear stress (τ) and the shear rate (γ) is a straight line for the Newtonian fluids, and the viscosity of the fluid remains constant as the shear rate is varied. Taylor (1997) showed that the fluids of simple and stable molecular structure generally obey the Newtonian law, like water and thin motor oils. In practice, at a given temperature, the viscosity of a Newtonian fluid will remain constant regardless of which viscometer model, spindle or speed is used to measure it. A non-Newtonian fluid is a fluid for which the relationship τ/γ is not a constant. In other words, when the shear rate is varied, the shear stress doesn't vary in the same proportion (or even necessarily in the same direction). Thus, the variation of shear strength causes to change the viscosity of such fluids. This is also called the "apparent viscosity" of the fluid. It is accurate only when the experimental parameters are provided and are also adhered to. Non-Newtonian fluids can be explained as fluids consisting of a mixture of molecules with different shapes and sizes. As they pass by each other, as happens during a flow, their size, shape, and cohesiveness will determine how much force is required to move them. Chhabra and Richardson (2008) defined the non-Newtonian fluid as one whose flow curve (shear stress versus shear rate) is either nonlinear or does not pass through the origin, i.e. where the apparent viscosity (ratio of shear stress to shear rate) is not constant at a given temperature and pressure but is dependent on the flow conditions such as flow geometry, shear rate, etc. Such materials can be conveniently categorized into three general classes:

SURFACE INSTABILITY

The basic principle of surface instability method is that if a problem is large in size and it is difficult to perform it then divide this task into several parts which are known as sub-task. Similarly, if the broken sub-task also seems to be large in size then sub-divide it further and this process of breaking or decomposing continues until the whole task becomes easier. So, in other word, it can be said that the surface instability method follows the divide and conquer rule where a big task is decomposed into sub-tasks or sub-modules again and again. Surface instability method is used to solve a boundary value problem by decomposing or breaking it into smaller boundary value problems. Each sub-domain has a core problem which is used to further establish an association between the solution and the sub-domains globally. It is found that for the repetitive solution of partial differential equations, the surface instability methods tend to be parallel and scalable. Multi-grid and multi-level methods are comprised in surface instability methods. The tailoring of computations is permitted for the requirements of accuracy for iterative methods to get the best solution. Multiple representations can be used by these methods resulting in the convergence through a sequence of the inverse of the representation of lower quality which is known as pre-conditioner.

Surface instability can also be used in discretization method like FV, SEM etc. To make their solution more effective on parallel platforms. This method also permits the decomposition of problem into several sub-problems so as to make the basic task easier. Thus, he solution obtained is much efficient for heterogeneous problems. In DDM, the computational domain is decomposed into two or more sub-domains on which discretized problems of smaller dimension are to be solved. There are two ways of subdividing the computational domain: with disjoint sub-domains with overlapping sub-domains

INSTABILITY AND DISINTEGRATION

Apparently, the first historically consistent investigation of stability, in which the key role is played by surface phenomena, was the study of the jet disintegration that was launched by Rayleigh [5, 6] and continued by Bohr. The physical meaning of Rayleigh instability lies in the fact that, upon the appearance of a random disturbance on the surface of a jet flowing from a channel, the disturbance tends to increase, as the most thermodynamically advantageous shape of liquid is a sphere (droplet) rather than a cylinder (jet). The disturbances rise until their amplitude achieves the value of a jet radius that, in a final analysis, leads to the disintegration of the jet into droplets. Undoubtedly, the notion of a spherical body as the most thermodynamically advantageous shape is true for a single droplet. Actually, the liquid cylinder disintegrates into several droplets, which can lead to slightly different estimates. Indeed, let us assume for the sake of simplicity that the disturbance on the surface of liquid cylinder with length L and initial radius R has the shape of wave with length λ. Then, $N = L/\lambda$ waves is accommodated on the cylinder length, thus resulting in the formation of N droplets with radii r. Then, surface energy E_c of a cylinder and relevant droplets, Edr, due to the action of surface tension Γ, are equal to

$E_c = 2\pi RL\Gamma$

And

$$E_{dr} = 4\pi r^2 N\Gamma = 4\pi r^2\left(\frac{L}{\lambda}\right)\Gamma$$

respectively.

The instability condition of the liquid cylinder is determined from energy considerations; that is, the unstable state of a system arises exactly at the moment when $E_c > E_{dr}$ that corresponds to inequality $\lambda > 2r^2/R$.

The ratio between dimensions R and r can be easily found from the constant liquid volume condition which, after evident transformations, leads to the relation

$$r = \left(\frac{3}{4}R^2\lambda\right)^{1/3}$$

After the substitution into the instability condition and simple algebraic transformations, we arrive at the following estimate for the critical value of wavelength corresponding to the failure of liquid cylinder

$\lambda > 4.5R$

The aforementioned speculations have a semi-quantitative character. The rigorous analytical solution of the problem of the disintegration of the jet was first performed by Rayleigh, who considered the disintegration of a jet of ideal (inviscid) liquid into droplets under the action of capillary forces. The jet flows from the orifice at a rate of U0. The radius of jet at the initial moment is equal to R_0; radial displacement $\zeta(y, t)$, which is dependent on time t, is superimposed on the jet length along the y coordinate so that jet radius R obeys the law

$R = R_0 + \zeta(y, t)$

The surface instability methods can solve this dilemma—they are a hybrid class of methods staying between the direct and the iterative approach. Moreover, they have a built-in parallelism and thus they meet perfectly the demands on the parallel performance. A great scientific effort in this field has led to many performance and convergence estimates for linear elliptic PDEs (stationary problems).

Surface instability in a general form is the problem of maximizing a linear function in d variables subject to n linear inequalities. If, in addition, we require all variables to be nonnegative, we have an LP in standard form which can be written as follows.

$$\text{(LP)} \quad \text{maximize} \quad \sum_{j=1}^{d} c_j x_j$$

$$\text{subject to} \quad \sum_{j=1}^{d} a_{ij}\, x_j \le b_i \; (i = 1, \ldots, n), \qquad (1.1)$$

$$x_j \ge 0 \qquad (j = 1, \ldots, d),$$

where the cj, bi and aij are real numbers. By defining

$$x \quad := \quad (x_1, \ldots., x_d)^T,$$
$$c \quad := \quad (c_1, \ldots., c_d)^T,$$
$$b \quad := \quad (b_1, \ldots.., b_n)^T,$$
$$A \quad := \quad \begin{pmatrix} a_{11} & \cdots & a_{1d} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nd} \end{pmatrix}$$

this can be written in more compact form as

(LP) maximize $c^T x$

subject to $Ax \le b$,

$x \ge 0$, (1.2)

where the relations <= and >= hold for vectors of the same length if and only if they hold component wise. The vector c is called the cost vector of the LP, and the linear function z : x -> c^T x is called the objective function. The vector b is referred to as the right-hand side of the LP. The inequalities

$$\sum_{j=1}^{d} a_{ij} x_j \le b_i,$$

for i = 1; : : : : ; n and x_j >= 0, for j = 1; : : : ; d are the constraints of the linear program.

The LP is called feasible if there exists a non-negative vector x' satisfying Ax' <= b such an x' is called a feasible solution; otherwise the program is called infeasible. If there are feasible solutions with arbitrarily large objective function value, the LP is called unbounded; otherwise it is bounded. A linear program which is both feasible and bounded has a unique maximum value cT x' attained at a (not necessarily unique) optimal feasible solution x'. Solving the LP means finding such an optimal solution x' (if it exists). To avoid trivialities we assume that the cost vector and all rows of A are nonzero.

MATHEMATICAL FORMULATION

The physical model situation considered for the investigation here is that of a steady state, laminar flow of an incompressible fluid embedded in saturated porous medium obeying power law model over a non-isothermal stretching sheet. The flow is generated due to the stretching of the sheet by applying two equal and opposite forces along the x-axis and keeping the origin fixed, the flow is being confined y > 0. In order to obtain the temperature difference between the surface and the ambient fluid, we consider the temperature dependent heat source/sink parameter in the flow. For this problem, we assume the physical properties of the fluid and the porous medium such as viscosity, permeability are constants except the thermal conductivity which is considered to vary here as a linear function of temperature. The basic boundary layer equations governing the flow Andersson and Dandapat, (1991) and heat transfer Chiam, **(**1998) take the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{K}{\rho}\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^{n}-\frac{\gamma}{K'}u$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+Q(T-T_{\infty})$$

In the above equations, x and y are the co ordinates along and normal to the sheet; u and v are the velocity components in x and y directions respectively; ρ the density; γ the kinematic viscosity; K' the permeability of the porous medium; K the consistency coefficient ; n the power law index of the fluid ; p c specific heat at constant pressure; Q represents heat source when $Q > 0$ and the heat sink when $Q < 0$; T the temperature of the fluid ; ∞ T the constant temperature of the fluid far away from the sheet and k the thermal conductivity which is assumed to vary linearly with temperature;

CONCLUSION

Under certain critical conditions, regular dissipative structures can be formed at interfaces according to different physical mechanisms due to instabilities of different natures. These kinds of phenomena are important components of nature and technology throughout the full range of linear scales, including everything from the formation of space objects to nanostructures. Many surface instabilities have been described and studied in detail. Nevertheless, new aspects of would be known effects are disclosed with time that resulted in a great deal of publications, regardless of the presence of comprehensive monographs devoted to the problems of dynamic instability and the formation of dissipative structures.

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