ON #REGULAR GENERALIZED CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce a new class of sets called $\tau_1\tau_2$#regular generalized closed (briefly, $\tau_1\tau_2$#rg-closed) sets intopological space. We prove that this class lies between closed sets and$\tau_1\tau_2$ rg-closed sets. We discuss some basicproperties of $\tau_1\tau_2$#regular generalized closed sets. Applying this we introduce a new space called T$\tau_1\tau_2$#rg space.

***Keywords**: $\tau_1\tau_2$rw-open sets, $\tau_1\tau_2$#rg-closed sets, T$\tau_1\tau_2$#rg-space.*

NTRODUCTION:

A triple (X, τ_1, τ_2), where X is a non empty set and τ_1 and τ_2 are topologies on X is called a bitopological space. In 1963, Kelly initiated the study of bitopological spaces. In 1985, Fukutake introduce the concept of g-closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. The aim of this paper is to extend the same concept in bitopological spaces.We introduce a new class of sets called $\tau_1\tau_2$#regular generalized closed setswhich is properly placed in between the class of closed sets and the class of $\tau_1\tau_2$rg-closed sets.

Throughout this paper (X,τ_1, τ_2) represents a bitopological space on which no separation axiom is assumed unless otherwisementioned. For a subset A of a bitopological space (X,τ_1, τ_2), $\tau_2$cl(A) and $\tau_1$int(A) denote the $\tau_2$closure of A and the $\tau_1$interior of Arespectively. X\A denotes the complement of A in X. We recall the following definitions and results.

Definition: **1.1**A subset A of a bitopologicalspace (X, τ_1, τ_2) is called

(1) $\tau_1\tau_2$ preopen set[9] if A ⊆ $\tau_1$int$\tau_2$cl (A) and a $\tau_1\tau_2$preclosed set if $\tau_2$cl$\tau_1$int (A) ⊆ A.
(2) $\tau_1\tau_2$semiopen set[1] if A ⊆$\tau_2$cl$\tau_1$int (A) and a $\tau_1\tau_2$semiclosed set if $\tau_1$int$\tau_2$cl (A) ⊆A.
(3) $\tau_1\tau_2$ regular open set[14]if A = $\tau_1$int$\tau_2$cl (A) and a $\tau_2$regular closed set if A = $\tau_2$cl$\tau_1$int (A).
(4) $\tau_1\tau_2\pi$- open set[25]if A is a finite union of regular open sets.
(5) $\tau_1\tau_2$regular semi open if there is a $\tau_1$regular open U such U ⊆A ⊆$\tau_2$cl(U).

Definition: 1.2A subset A of (X,τ_1, τ_2) is called

(1) $\tau_1\tau_2$generalized closed set (briefly, $\tau_1\tau_2$g-closed)[5] if $\tau_2$cl (A)⊆ U whenever A⊆U and U is open in X.
(2) $\tau_1\tau_2$regular generalized closed set (briefly, $\tau_1\tau_2$rg-closed)[7] if $\tau_2$cl (A)⊆U whenever A⊆U and U is τ_1-regular open in X.
(3) $\tau_1\tau_2$generalized preregular closed set (briefly,$\tau_1\tau_2$gpr-closed) if $\tau_2$pcl (A)⊆U whenever A ⊆ U and U is τ_1-regular open in X.
(4) $\tau_1\tau_2$weakely generalized closed set (briefly,$\tau_1\tau_2$wg-closed) if $\tau_2$cl$\tau_1$int (A) ⊆U whenever A⊆ U and U is τ_1-open in X.
(5)$\tau_1\tau_2\pi$-generalized closed set (briefly,$\tau_1\tau_2\pi$g-closed) if $\tau_2$cl(A)⊆U whenever A⊆U and U is$\tau_1\pi$-open in X.
(6) $\tau_1\tau_2$weakely closed set (briefly, $\tau_1\tau_2$w-closed) if $\tau_2$cl(A) ⊆U whenever A⊆U and U is $\tau_1$semi open in X.
(7) $\tau_1\tau_2$regular weakly generalized closed set (briefly, $\tau_1\tau_2$rwg-closed) if $\tau_2$cl$\tau_1$int(A)⊆U whenever A ⊆ U and U is $\tau_1$regularopen in X.
(8)$\tau_1\tau_2$ rw-closed if $\tau_2$cl(A)⊆U whenever A⊆U and U is $\tau_1$regular semi open.
(9) $\tau_1\tau_2$*g-closed if $\tau_2$cl(A)⊆U whenever A⊆U and U is τ_1w-open.

The complements of the above mentioned closed sets are their respective open sets.

Definition: 1.3 A space (X,τ_1 ,τ_2) is called T1/2- space [9] if every g-closed set is closed.

2. $\tau_1\tau_2$#REGULAR GENERALIZED CLOSED SETS AND THEIR BASIC PROPERTIES

We introduce the following definition

Definition: 2.1A subset A of a space X is called $\tau_1\tau_2$#regular generalized closed (briefly $\tau_1\tau_2$#rg-closed) set if $\tau_2$cl(A)⊆Uwhenever A ⊆U and U is$\tau_1\tau_2$ rw-open. We denote the set of all $\tau_1\tau_2$#rg– closed sets in X by $\tau_1\tau_2$#RGC(X).

First we prove that the class of $\tau_1\tau_2$#regular generalized closed sets properly lies between the class of closed sets and theclass of $\tau_1\tau_2$rg-closed sets.

Example: 2.2 Let X= {a, b, c} be with topologies τ_1={ ϕ, X, {a}, {a, c}} and τ_2={ ϕ, X, {a}, {b, c}} then $\tau_1\tau_2$#rg-closed sets are { ϕ, X, {a}, {b}, {b, c}}.

Theorem: 2.3 Every closed sets are $\tau_1\tau_2$#rg-closed sets, but not conversely.

Proof: The proof follows from the definition.

The converse of the above theorem need not be true as seen from the following example.

Example: 2.4 Let X= {a, b, c} be with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}} and τ_2={ ϕ, X, {a}, {b, c}} τ_2 closed sets are { ϕ, X, {a}, {b, c}} and $\tau_1\tau_2$#rg-closed sets are { ϕ, X, {a}, {a, c}, {b, c}} then {a, c} is $\tau_1\tau_2$#rg-closed sets but it is not τ_2 closed sets.

Theorem: 2.5.Every $\tau_1\tau_2$#rg-closed sets are $\tau_1\tau_2$rg-closed but not conversely.

Proof: Let A be a $\tau_1\tau_2$#rg- closed. Let A⊆U and U is regular open. Since U is regular open, U is rw-open. Now A ⊆ U andA is $\tau_1\tau_2$#rg– closed then $\tau_2$cl(A) ⊆ U. Therefore A is$\tau_1\tau_2$rg – closed.
The converse of the above theorem need not be true as seen from the following example.

Example: 2.6 Let X= {a, b, c} be with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}} and τ_2={ ϕ, X, {a}, {b, c}} then {a, b} is $\tau_1\tau_2$rg-closed sets but not $\tau_1\tau_2$ #rg-closed sets .

Theorem: 2.7 Every $\tau_1\tau_2$#rg-closed sets are g-closed but not conversely.

Proof: Let A be a $\tau_1\tau_2$#rg-closed. Let A⊆U and U is open. Since U is open, U is $\tau_1\tau_2$ rw-open. Now A ⊆ U and A is$\tau_1\tau_2$#rg– closed then $\tau_2$cl(A) ⊆ U. Therefore A is $\tau_1\tau_2$g – closed.

The converse of the above theorem need not be true as seen from the following example.

Example: 2.8 Let X= {a, b, c, d} be with topologies τ_1={ ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and τ_2={ ϕ, X, {a}, {b}, {a, b}} , then the set A={d} is g-closed but not$\tau_1\tau_2$#rg-closed set in X.

Theorem: 2.9 The union of two $\tau_1\tau_2$#rg-closed subsets of X is also $\tau_1\tau_2$#rg-closed subset of X.

Proof: Assume that A and B are $\tau_1\tau_2$#rg - closed set in X. Let U be $\tau_1\tau_2$rw-open in X such that A∩B ⊆ U. Then A ⊆ U andB⊆U. Since A and B are $\tau_1\tau_2$#rg-closed, $\tau_2$cl(A)⊆U and $\tau_2$cl(B)⊆U. Hence $\tau_2$cl(A∩B)=τ_2(cl(A))∩τ_2(cl(B))⊆U. Thus $\tau_2$cl(A∩B)⊆ U.Therefore A∩B is $\tau_1\tau_2$#rg-closed set in X.

Remark: 2.10 The intersection of two $\tau_1\tau_2$#rg-closed sets in X is generally not $\tau_1\tau_2$#rg-closed set in X.

Example: 2.11 Let X= {a, b, c, d} be with topologiesτ_1={ ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and τ_2={ ϕ, X, {a}, {b}, {a, b}} , $\tau_1\tau_2$#rg-closed sets are { ϕ, X, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {b, c, d}, {a, c, d}} then {b, c}∩{b, d}={b} but it is not $\tau_1\tau_2$#rg-closed set.

Theorem: 2.12 Let A be a $\tau_1\tau_2$#rg – closed set in X then $\tau_2$cl(A)\A does not contain any non empty $\tau_1\tau_2$rw- closed set.

Proof: Let F be a non empty$\tau_1\tau_2$rw- closed subset of $\tau_2$cl (A)\A. Then A⊆X\F where A is $\tau_1\tau_2$#rg- closed and X\F is rw-open.Then $\tau_2$cl(A)⊆X . For equivalently F⊆X\$\tau_2$cl(A). Since by assumption F⊆$\tau_2$cl(A), we get a contradiction.

The converse of the above theorem need not be true seen from the following example.

Example: 2.13 Let X= {a, b, c, d} be with topologies τ_1={ ϕ, X, {a}, {a, b}} and τ_2={ ϕ, X, {a}, {b}, {a, b}, {b, c}, {a, b, c} }. Let A= {c, d} then $\tau_2$cl(A)\A= {a, c, d}\{a, d} ={c} does not contain nonempty $\tau_1\tau_2$rw-closed set in X, but A is not $\tau_1\tau_2$#rg-closed set.

Corollary: 2.14 If a subset A of X is an $\tau_1\tau_2$#rg-closed set in X, then $\tau_2$cl(A)\A does not contain any non empty regular closedset in X, but not conversely.

Proof: Follows from theorem 2.7 and the fact that every regular closed set is $\tau_1\tau_2$rw-closed.

Corollary: 2.15 Let A be $\tau_1\tau_2$#rg – closed in (X,τ_1,τ_2), Then A is closed if and only if $\tau_2$cl(A)\A is $\tau_1\tau_2$rw- closed.

Proof: Necessity: Let A be $\tau_1\tau_2$#rg– closed. By hypothesis $\tau_2$cl(A)=A and so $\tau_2$cl(A)\A = f which is $\tau_1\tau_2$rw-closed.
Sufficiency: Suppose $\tau_2$cl(A)\ A is rw-closed. Then by Theorem 2.7, $\tau_2$cl(A)\A = f. That is $\tau_2$cl(A)=A. Hence A is closed.

Theorem: 2.16 For every point x of a space X, X\{x} is $\tau_1\tau_2$#rg– closed (or) $\tau_1\tau_2$rw- open.

Proof: Suppose X \{x} is not rw-open. Then X is the only$\tau_1\tau_2$rw-open set containing X\{x}.This implies $\tau_2$cl(X\{x}) ⊆ X.Hence X\{x} is $\tau_1\tau_2$#rg– closed.

Theorem: 2.17 If A is a $\tau_1\tau_2$#rg – closed subset of (X,τ_1,τ_2) such that A⊆B⊆$\tau_2$cl(A) then B is also $\tau_1\tau_2$#rg- closed subset of (X,τ_1,τ_2).

Proof: Let U be a $\tau_1\tau_2$rw-open set in (X,τ_1,τ_2) such that B ⊆U. Then A⊆U. Since, A is $\tau_1\tau_2$#rg – closed, then $\tau_2$cl(A)⊆U. Now, since$\tau_2$cl(A) is closed, $\tau_2$cl(B)⊆$\tau_2$cl($\tau_2$cl(A))=$\tau_2$cl(A)⊆U. Therefore B is also $\tau_1\tau_2$#rg – closed.

Remark: The converse of the above theorem need not be true in general.
Consider the topological space (X,τ_1,τ_2), Let X= {a, b, c, d} be with topologies τ_1={ ϕ, X, {a}, {b}, {a, c}, {a, b}, {b, c}, {a, b, c} } and τ_2={ ϕ, X, {a}, {b}, {a, b}, {a, b, c} }Let A= {c, d} and B= {b, c, d}. Then A and B are $\tau_1\tau_2$#rg-closed setsbut A⊆B and B is not a subset in $\tau_2$cl(A).

Theorem: 2.18 If a subset A of a bitopological space (X,τ_1,τ_2) is both$\tau_1\tau_2$ rw-open and $\tau_1\tau_2$#rg- closed then it is closed.

Proof: Suppose a subset A of a bitopological space (X,τ_1,τ_2) is both $\tau_1\tau_2$rw-open and $\tau_1\tau_2$#rg-closed. Now A⊆A. Then $\tau_2$cl(A)⊆A. ThusA is closed.

Corollary: 2.19 Let A be $\tau_1\tau_2$rw-open and $\tau_1\tau_2$#rg-closed in (X,τ_1,τ_2). Suppose that F is closed in (X,τ_1,τ_2). Then A∩F is a $\tau_1\tau_2$#rg-closed set inX.

Proof: Let A be $\tau_1\tau_2$rw-open and $\tau_1\tau_2$#rg-closed in (X,τ_1,τ_2) and F be closed. By theorem 2.10, A is closed. So A∩F is closed andhence A∩F is $\tau_1\tau_2$#rg-closed set in (X,τ_1,τ_2).

Theorem: 2.20 If A is open and g-closed then A is $\tau_1\tau_2$#rg-closed.

Proof: Let A be an open and $\tau_1\tau_2$g-closed set in X. Let A⊆U and U be rw-open in X. Now A⊆A. By hypothesis $\tau_2$cl(A)⊆A.That is $\tau_2$cl(A)⊆U. Thus A is $\tau_1\tau_2$#rg-closed.

Remark:
1. If A is $\tau_1\tau_2$regular open and $\tau_1\tau_2$rg-closed then A is $\tau_1\tau_2$#rg-closed set.
2. If A is $\tau_1\tau_2$g-open open and $\tau_1\tau_2$g*-closed then A is $\tau_1\tau_2$#rg-closed set.
3. If A is $\tau_1\tau_2$ π- open and $\tau_1\tau_2$pg-closed then A is $\tau_1\tau_2$#rg-closed set.
4. If A is $\tau_1\tau_2$semi open and $\tau_1\tau_2$w-closed then A is $\tau_1\tau_2$#rg-closed set.
5. If A is $\tau_1\tau_2$regular semi open and $\tau_1\tau_2$rw-closed then A is $\tau_1\tau_2$#rg-closed set.

Theorem: 2.21 If a subset A of a bitopological space(X,τ_1,τ_2) is both open and $\tau_1\tau_2$wg-closed then it is $\tau_1\tau_2$#rg-closed.

Proof: Suppose a subset A of X is both open and wg-closed. Let A⊆U with U is rw-open in X. Now $\tau_2$cl(int(A))⊆A.Since A is open, int(A)=A .Then $\tau_2$cl (int(A))= $\tau_2$cl(A)⊆A⊆U. Hence A is $\tau_1\tau_2$#rg closed set.

Definition: 2.22 A space (X,τ_1,τ_2) is called a T$\tau_1\tau_2$#rg-space if every $\tau_1\tau_2$#rg-closed set in it is closed.

Theorem: 2.23. A space (X,τ_1,τ_2) is T$\tau_1\tau_2$#rg-space if and only if every singleton of x is either $\tau_1\tau_2$rw-closed or open.

Proof: **Necessity**: Let x∈X be such that {x} is not $\tau_1\tau_2$rw-closed. Then X\{x} is not $\tau_1\tau_2$rw-open. So X is the only rw-open setcontaining X\{x}. This implies that X\{x} is $\tau_1\tau_2$#rg-closed. By assumption, X\{x} is closed. Then {x} is open.
Suffiency: Let A be a $\tau_1\tau_2$#rg-closed subset of X and let x∈$\tau_2$cl(A). By assumption, we have following cases.
(1) x is open, but x∈$\tau_2$cl(A), so x∈A
(2) x is rw-closed. By theorem 2.7, x ∉ $\tau_2$cl (A) \ A, but x ∈$\tau_2$cl (A) so x ∈ A.
Hence $\tau_2$cl (A) = A. That is A is closed.

Theorem: 2.24 Every T1\2-space is T$\tau_1\tau_2$#rg-space, but not conversely.

Proof: Let X be a T1\2-space and A be a $\tau_1\tau_2$#rg-closed set in X. We have, every $\tau_1\tau_2$#rg-closed set is g-closed. Hence A isg-closed. Since it is a T1\2-space, A is closed in X. Hence X is T$\tau_1\tau_2$#rg-space.

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