ANTI Q-FUZZY BI-IDEALS IN NEAR-RINGS

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Abstract:

In this paper, we introduce the notion of an anti Q-fuzzy bi-ideals in near-rings. Also we investigate some algebraic nature of an anti Q-fuzzy bi-ideals in near-rings. Further, we discuss the properties of an anti Q-fuzzy bi-ideals in near-rings.

Keywords:

Ideal, bi-ideal, fuzzy bi-ideal, anti fuzzy bi-ideal.

1. Introduction:

In 2009, T.Manikandan[6], introduced the concept of fuzzy bi-ideal in near-ring and S.K.Datta [2] introduced the concept of Anti fuzzy bi-ideals in rings and the notion of anti fuzzy subgroups and fuzzy subgroup introduced by R.Biswas [1]. In 2017, S.Usha Devi, Jeyalakshmi and T.Tamizh Chelvam [8] are introduced by Q-fuzzy bi-ideal in near-ring and some its properties. Motivated by this concept, we introduced anti Q-fuzzy bi-ideals in near-rings and some its properties.

2. Preliminaries:

In this section, we collect all basic concepts in near-rings, which are used in this paper. We also furnish certain results which are used in our work.

Definition 2.1:

A non-empty set N with two binary operations "+" (addition) and "." (multiplication) is called a right near-ring, if it satisfies the following conditions:

(i) $(N,+)$ is a group (not necessarily abelian)
(ii) $(N,.)$ is a semi group,
(iii) For all $x,y,z \in N, (x+y).z = x.z + y.z$

Remark 2.2:

Throughout this paper, by a near-ring, we mean only a zero symmetric right near-ring. The symbol N stands for a near-ring $(N,+,.)$ with atleast two elements. 0 denotes the identity element of the group $(N,+)$.

Definition 2.3:

A fuzzy subgroup B of N is said to be fuzzy bi-ideal if $(BNB) \cap (BN * B) \subseteq B$. In zero-symmertric if $BNB \subseteq B$.

Definition 2.4:

Let N be a near-ring. A fuzzy set μ of N is called an anti fuzzy bi-ideal of N if for all$x,y,z \in N$

(i) $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$
(ii) $\mu(xyz) \leq \max\{\mu(x), \mu(y)\}$.

Definition2.5:

A mapping $\mu: X \times Q \longrightarrow [0,1]$, where X is an arbitrary non-empty set and is called a Q-fuzzy set in X.

Definition 2.6:

A family of a Q-fuzzy set $\{\mu_i / i \in \Omega\}$ is a near-ring N, the union of $\bigcup_{i\in\Omega} \mu_i$ of $\{\mu_i / i \in \Omega\}$ is defined by $\bigcup_{i\in\Omega} \mu_i(x,q) = sup\{\mu_i(x,q)/i \in \Omega\}\, \forall x \in N, q \in Q$.

Definition 2.7:

A family of Q-fuzzy set $\{\mu_i / i \in \Omega\}$ is near-ring N, the intersection of $\bigcap_{i\in\Omega} \mu_i$ of $\{\mu_i / i \in \Omega\}$ is defined by $\bigcap_{i\in\Omega} \mu_i(x,q) = \inf\{\mu_i(x,q)/i \in \Omega\}\, \forall\, x \in N$ and $q \in Q$.

Definition 2.8:

Let f be a mapping from a set N to a set N'. Let μ and λ be a Q-fuzzy set of N and N' respectively. Then $f(\mu)$, the image of μ under f, is a subset of N' defined by

$$f(\mu)(y,q) = \begin{cases} \inf\limits_{x\in f^{-1}(y,q)} \mu(x,q) & if\ f^{-1}(y,q) \neq \phi \\ 0 & Otherwise \end{cases}$$

And pre-image of λ under f is a Q-fuzzy subset of N defined by $f^{-1}(\lambda(x,q)) = \lambda(f(x,q))$, for all $x \in N, q \in Q$ and $f^{-1}(y,q) = \{(x,q)/x \in N, q \in Q, f(x,q) = (y,q)\}$

Definition 2.9:

Let μ and λ be any two Q-fuzzy subsets of N defined by
$(\mu \cap \lambda)(x,q) = \min\{\mu(x,q), \lambda(y,q)\}$
$(\mu \cup \lambda)(x,q) = \max\{\mu(x,q), \lambda(x,q)\}$

$$(\mu - \lambda)(x,q) = \begin{cases} \inf\limits_{x=y-z} \max\{\mu(y,q), \lambda(z,q)\} & if\ x\ can\ be\ expressed\ as\ x = y - z \\ 0 & Otherwise \end{cases}$$

$$(\mu\lambda)(x,q) = \begin{cases} \inf\limits_{x=yz} max\{\mu(y,q), \lambda(z,q)\} & if\ x\ can\ be\ expressed\ as\ x = yz \\ 0 & otherwise \end{cases}$$

$$(\mu * \lambda)(x,q) = \begin{cases} \inf\limits_{x=ac-a(b-c)} \min\{\mu(a,q), \lambda(c,q)\} & if\ x\ can\ be\ expressed\ as\ x = ac - a(b-c) \\ 0 & otherwise \end{cases}$$

Throughout this paper, f_I is the characteristic function of the subset I of N and the characteristic function of $N \times Q$ denoted by χ, that means $\chi: N \times Q \longrightarrow [0,1]$ mapping every elements of $N \times Q$ to 1.

Definition 2.10:

For any Q-fuzzy set μ in X and $t \in [0,1]$. We define two sets $U(\mu;t) = \{(x,q)/x \in N, q \in Q, \mu(x,q) \geq t\}$ and $L(\mu;t) = \{(x,q)/x \in N, q \in Q, \mu(x,q) \leq t\}$, which are called an upper and lower t-level cut of μ respectively.

Definition 2.11:

A mapping $f: N \longrightarrow N'$ is called a near-ring homomorphism if $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y), \forall\, x, y \in N$.

Definition 2.12:

A mapping $f: N \longrightarrow N'$ is called a near-ring anti homomorphism if $f(x+y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)\ \forall x, y \in N$

Definition 2.13:

A Q-fuzzy set μ of a group G is called Q-fuzzy subgroup if
$\mu(x-y,q) \leq \max\{\mu(x,q), \mu(y,q)\}, \forall\, x, y \in \text{G}, q \in Q$

Definition 2.14:

A Q-fuzzy set μ in N is a Q-fuzzy bi-ideal of N if
(i) $\mu(x-y,q) \geq \min\{\mu(x), \mu(y)\}\ \forall x, y \in N$
(ii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}\ \forall x, y, z \in N$

Definition 2.15:

A Q-fuzzy set μ of a group G is called an anti Q-fuzzy subgroup if
$\mu(x-y,q) \leq \max\{\mu(x,q), \mu(y,q)\}\ \forall x, y \in G, q \in Q$

3.Anti Q-fuzzy bi-ideal in near-ring:

Definition 3.1:

A Q-fuzzy set μ in N is an anti Q-fuzzy bi-ideal if for all $x, y, z \in N$ and $q \in Q$
(i) $\mu(x-y,q) \leq \max\{\mu(x,q), \mu(y,q)\}$
(ii) $\mu(xyz,q) \leq \max\{\mu(x,q), \mu(z,q)\}$

Example 3.2:

Let $N = \{0, a, b, c\}$ be the Klein's four group. Define multiplication in N as follows:

+	0	A	b	c
0	0	A	b	c
a	a	0	c	b
b	b	C	0	a
c	c	B	a	0

.	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Then $(N, +\cdot)$ is a near-ring. Define an anti Q-fuzzy set $\mu: N \times Q \longrightarrow [0,1]$ by $\mu(0,q) = 0.6$, $\mu(a,q) = 0.7, \mu(b,q) = \mu(c,q) = 0.8$. It is easy verify that μ is an anti Q-fuzzy bi-ideal of N.

Theorem 3.3:

Let $f: N \longrightarrow N'$ be an onto homomorphism of a near ring N.
1) If λ is an anti Q-fuzzy bi-ideal in N', then $f^{-1}(\lambda)$ is an anti Q-fuzzy bi-ideal in N.
2) If μ is an anti Q-fuzzy bi-ideal in N, then $f(\mu)$ is an anti Q-fuzzy bi-ideal in N'.

Proof:

1) Let λ be an anti Q-fuzzy bi-ideal of N'.
For $, y, z \in N$, q$\in Q$
(i) $f^{-1}(\lambda)(x-y,q) = \lambda(f(x-y,q))$
$= \lambda(f(x,q) - f(y,q))$
$\leq max\ \{\lambda f(x,q), \lambda f(y,q)\}$
$= max\ \{f^{-1}(\lambda)\ (x,q), f^{-1}(\lambda)\ (y,q)\}$
Therefore $f^{-1}(\lambda)\ (x-y,q) \leq max\ \{f^{-1}(\lambda)(x,q), f^{-1}(\lambda)\ (y,q)\}$

Thus $f^{-1}(\lambda)$ is an anti Q-fuzzy subgroup.

(ii) $f^{-1}(\lambda)\,(xyz,q) = \lambda\,(f(xyz,q))$

$= \lambda((f(x,q)f(y,q)f(z,q))$

$\leq max\,\{\lambda(f(x,q)), \lambda(f(z,q)\}$

$= max\,\{f^{-1}(\lambda)\,(x,q), f^{-1}(\lambda)\,(z,q)\}$

Therefore, $f^{-1}(\lambda)\,(xyz,q) \leq max\,\{f^{-1}(\lambda)\,(x,q), f^{-1}(\lambda)\,(z,q)\}$

Hence $f^{-1}(\lambda)$ is an anti Q-fuzzy bi-ideal in N.

2) Let μ be an anti Q-fuzzy bi-ideal in N.

Let $y_1, y_2, y_3 \in N'$ and $q \in Q$

Then we have ,

$\{(x,q)/(x,q) \in f^{-1}(y_1\text{-}y_2,q)\} \supseteq \{(x_1 - x_2,q)/(x_1,q) \in f^{-1}(y_1,q)$ and $(x_2,q) \in f^{-1}(y_2,q)\}$

And hence

(i) $f(\mu)\,(y_1 - y_2,q) = inf\,\{\mu(x,q)/(x,q) \in f^{-1}(y_1 - y_2,q)\}$

$\leq inf\,\{\mu(x_1 - x_2,q)/(x_1,q) \in f^{-1}(y_1,q)$ and $(x_2,q) \in f^{-1}(y_2,q)\}$

$\leq \inf\{max\{\mu(x_1,q),\mu(x_2,q)\}/(x_1,q) \in f^{-1}(y_1,q)$ and $(x_2,q) \in f^{-1}(y_2,q)\}$

$= max\,\{\inf\{\mu(x_1,q)/(x_1,q) \in f^{-1}(y_1,q)\}$ and $\inf\,\{\mu(x_2,q)/(x_2,q) \in f^{-1}(y_2,q)\}\}$

$= max\,\{f(\mu)(y_1,q), f(\mu)(y_2,q)\}$

Therefore, $f(\mu)(y_1 - y_2,q) \leq \max\{f(\mu)(y_1,q), f(\mu)(y_2,q)\}$

Thus $f(\mu)$ is an anti Q-fuzzy subgroup in N'.

(ii) Let $y_1, y_2, y_3 \in N'$ and q$\in Q$

Then we have,

$f(\mu)(y_1y_2y_3,q) = \inf\{\mu(x,q)/(x,q) \in f^{-1}(y_1y_2y_3,q)\}$

$\leq inf\{\mu(x_1x_2x_3,q)/x_1 \in f^{-1}(y_1,q)$ and $x_3 \in f^{-1}(y_3,q)\}$

$\leq inf\{\max\{\mu(x_1,q),\mu(x_3,q)/(x_1,q) \in f^{-1}(y_1,q)\ and\ (x_3,q) \in f^{-1}(y_3,q)\}$

$= \max\{\inf\{\mu(x_1,q)/(x_1 \in f^{-1}(y_1,q)\}$ and $\inf\{\mu(x_3,q)/(x_3,q) \in f^{-1}(y_3,q)\}$

$= max\{f(\mu)(y_1,q), f(\mu)(y_3,q)\}$

Therefore, $f(\mu)(y_1y_2y_3,q) \leq max\{f(\mu)(y_1,q), f(\mu)(y_3,q)\}$

Hence $f(\mu)$ is an anti Q-fuzzy bi-ideal of N'.

Theorem 3.4:

Let $\{\mu_i/i \in \Omega\}$ be a family of an anti Q-fuzzy bi-ideal of a near-ring N, then $\bigcup_{i\in\Omega}\mu_i$ is also an anti Q-fuzzy bi-ideal of N, where Ω is any index set.

Proof:

Let $\{\mu_i\}_{i\in\Omega}$ be a family of an anti Q-fuzzy bi-ideals of N.

Let $x, y, z \in N$ and q$\in Q$ and $\mu = \bigcup_{i\in\Omega}\mu_i$

Then, $\mu(x,q) = \bigcup_{i\in\Omega}\mu_i\,(x,q)$

$= \sup_{i\in\Omega}\mu_i(x,q)$

Now, $\mu(x-y,q) = \sup_{i\in\Omega}\mu_i\,(x-y,q)$

$\leq \sup_{i\in\Omega} max\{\mu_i(x,q), \mu_i(y,q)\}$

$= \max\{\sup_{i\in\Omega}\mu_i(x,q), \sup_{i\in\Omega}\mu_i(y,q)\}$

$= \max\{\bigcup_{i\in\Omega}\mu_i\,(x,q), \bigcup_{i\in\Omega}\mu_i(y,q)\}$

$= \max\{\mu(x,q), \mu(y,q)\}$

Therefore,$\mu(x-y) \leq \max\{\mu(x,q), \mu(y,q)\}$

Thus μ is an anti Q-fuzzy subgroup of N.

And $\mu(xyz,q) = \sup_{i\in\Omega}\mu_i(xyz,q)$

$\leq \sup_{i\in\Omega}\max\{\mu_i(x,q), \mu_i(z,q)\}$

$= \max\{\sup_{i\in\Omega}\mu_i(x,q), \sup_{i\in\Omega}\mu_i(z,q)\}$

$= \max\{\bigcup_{i\in\Omega}\mu_i\,(x,q), \bigcup_{i\in\Omega}\mu_i(z,q)\}$

$= \max\{\mu(x,q), \mu(z,q)\}$

Therefore,$\mu(xyz,q) \leq \max\{\mu(x,q), \mu(z,q)\}$

Hence, $\mu = \bigcup_{i\in\Omega}\mu_i$ is an anti Q-fuzzy bi-ideal of N, where Ω is any index set.

Theorem 3.5:

Let μ be an anti Q-fuzzy subgroup on N, then μ is an anti Q-fuzzy bi-ideal of N iff $\mu N\mu \supseteq \mu$.

Proof:

Assume that μ is an anti Q-fuzzy bi-ideal of N.
Let $x', x, y, x_1, x_2 \in N$ and q$\in Q$ such that $x' = xy, x = x_1x_2$
Now,$(\mu N\mu)(x', q) = \inf_{x'=xy} max\{(\mu N)(x, q), \mu(y, q)\}$
$= \inf_{x'=xy} max\{\inf_{x=x_1x_2} max\{\mu(x_1, q), N(x_2, q)\}, \mu(y, q)\}$
$= \inf_{x'=xy} max\{\inf_{x=x_1x_2} max\{\mu(x_1, q), 1\}, \mu(y, q)\}$
$= \inf_{x'=x_1x_2y} \max\{\mu(x, q), \mu(y, q)\}$
$\geq \inf_{x'=x_1x_2y} \mu(x_1x_2y, q)$
$= \mu(x, q)$
Thus $(\mu N\mu) \supseteq \mu$
Conversely, Assume that $(\mu N\mu) \supseteq \mu$
$\mu(xyz, q) \geq (\mu N\mu)(xyz, q)$
$= \inf_{xyz=ab} \max\{(\mu N)(a, q), \mu(b, q)\}$
$\leq \max\{(\mu N)(xy, q), \mu(z, q)\}$
$\leq \max\{\mu(x, q), N(x, q), \mu(z, q)\}$
$= \max\{\mu(x, q), 1, \mu(z, q)\}$
$= \max\{\mu(x, q), \mu(z, q)\}$
Thus $\mu(xyz, q) \leq \max\{\mu(x, q), \mu(z, q)\}$
Therefore, μ is an anti Q-fuzzy bi-ideal of N.

Theorem 3.6:

Let N and N' be two near rings. A mapping $f: N \longrightarrow N'$ be a homomorphism if μ is an anti Q-fuzzy bi-ideal of N' and $L(\mu; t)$ is a bi-ideal of N' then $L(f^{-1}(\mu); t)$ is a bi-ideal of N.

Proof:

A mapping $f: N \longrightarrow N'$ be a homomorphism if μ is an anti Q-fuzzy bi-ideal of N and $L(\mu; t)$ is a bi-ideal of N'.
Let $x, y \in L(f^{-1}(\mu); t)$
$\Longrightarrow f^{-1}(\mu)(x, q) \leq t$ and $f^{-1}(\mu)(y, q) \leq t$
$\Longrightarrow \mu(f(x, q)) \leq t$ and $\mu(f(y, q)) \leq t$
Now,$f^{-1}(\mu)(x - y, q) = \mu(f(x - y, q))$
$= \mu(f(x, q) - f(y, q))$
$\leq \max\{\mu(f(x, q)), \mu(f(y, q))\}$
$= t$
Therefore,$f^{-1}(\mu)(x - y, q) \leq t$
Hence $L(f^{-1}(\mu), t)$ is an anti Q-fuzzy subgroup of N'.
Let $x, z \in L(f^{-1}(\mu), t)$ and $y \in N$
Then $f^{-1}(\mu)(x, q) \leq t$ and $f^{-1}(\mu)(z, q) \leq t$
$\Longrightarrow \mu(f(x, q)) \leq t$ and $\mu(f(z, q)) \leq t$
Now, $f^{-1}(\mu)(xyz, q) = \mu(f(xyz, q))$
$= \mu(f(x, q)f(y, q)f(z, q))$
$\leq \max\{\mu(f(x, q)), \mu(f(z, q))\}$
$= t$
Therefore, $f^{-1}(\mu)(xyz, q) \leq t$
We get $xyz \in L(f^{-1}(\mu); t)$
Hence $L(f^{-1}(\mu); t)$ is a bi-ideal of N.

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