# TRIPLE CONNECTED ROMAN DOMINATION NUMBER ON SOME CLASS OF GRAPHS

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**Abstract** : The triple connected graph with real life application was introduced by Paulraj Joseph J etal, considering the existence of a path containing any three vertices of G. We introduced the concept of triple connected Roman domination function. The Roman dominating function $f: V \rightarrow \{0,1,2\}$ is called triple connected Roman domination function (TRCDF) of G, if the induced subgraph $\langle V_1 \cup V_2 \rangle$ $or$ $\langle V_2 \rangle$ is triple connected, then the Roman dominating function is called triple connected Roman dominating function (TCRDF). The triple connected Roman domination number $\gamma_{TRC}(G)$ is the minimum weight of a triple connected Roman dominating function (TCRDF) on$G$. In this paper we have determined the $\gamma_{TRC}(G)$ for some standard class of graphs.

**Keywords:** Connected graphs, Triple connected graphs, Domination number, Roman domination number and Connected Roman domination number

**Subject Classification number:** 05C40, 05C69, 05C70

## 1. Introduction

A set D is a subset of $V(G)$ in G is a dominating set, if $N[D] = V(G)$. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set. If $D$ is a subset of $V(G)$, then we denote by$\langle D \rangle$, the subgraph induced by $D$. A subset $D$ of vertices is independent, if $\langle D \rangle$ has no edge. For notation and graph theory terminology in general we follow [2]. A Roman dominating function on a graph $G$ is a function $f: V \rightarrow \{0,1,2\}$ satisfying the condition that every vertex $u \in V$ for which $f(u) = 0$ is adjacent to at least one vertex $v \in V$ for which $f(v) = 2$. The weight of a Roman dominating function is the value$f(v) = \sum_{v \in V} f(v)$. The Roman domination number $\gamma_R(G)$ is the minimum weight of a Roman dominating function on$G$. A graph is said to be triple connected if any three vertices lie on a path in G [8] In this context we introduce triple connected Roman domination number. In a Roman dominating function if the induced subgraph $\langle V_1 \cup V_2 \rangle$ $or$ $\langle V_2 \rangle$ is triple connected, and then the Roman dominating function is called triple connected Roman dominating function (TCRDF). The triple connected Roman domination number $\gamma_{TRC}(G)$ is the minimum weight of a triple connected Roman dominating function (TCRDF) on$G$

## 2.Results

In this Section we found the triple connected Roman domination number of corona $C_n \odot H$,almost bipartite graph, book graphs.

**Theorem 1** For $\gamma_{TRC}(C_n \odot H) = 2n$

Proof:

Let us consider the corona graph $G = (C_n \odot H)$ where H is any connected graph

$V(G) = \{u_i, v_{ij}: 1 \le i \le n, 1 \le j \le m\}$,

$E(G) = \{u_i u_{i+1}: 1 \le i \le n-1\} \cup \{u_1 u_n\} \cup \{u_i v_{ij}: 1 \le i \le n, 1 \le j \le m\}$ (see the figure)

Let $f = (V_0, V_1, V_2)$ where $V_2 = \{u_i: 1 \le i \le n\}$, $V_1 = \emptyset$, $V_0 = \{v_{ij}: 1 \le i \le n, 1 \le j \le m\}$

Then, $V_2$ is a dominating set of $V_0$ and $\langle V_2 \rangle$ is triple connected.

Hence $f$ is a Roman dominating function and it is triple connected. Therefore $f$ is triple connected Roman dominating function $f(v) \ge 3$, here $V_0 \ne \emptyset$ and $V_2 \ne \emptyset$. Hence $f(v) \ge 2n$, since $f(v) = 2n$ is a minimum weight of triple connected Roman domination number. Hence $\gamma_{TRC}(C_n \odot H) = 2n$

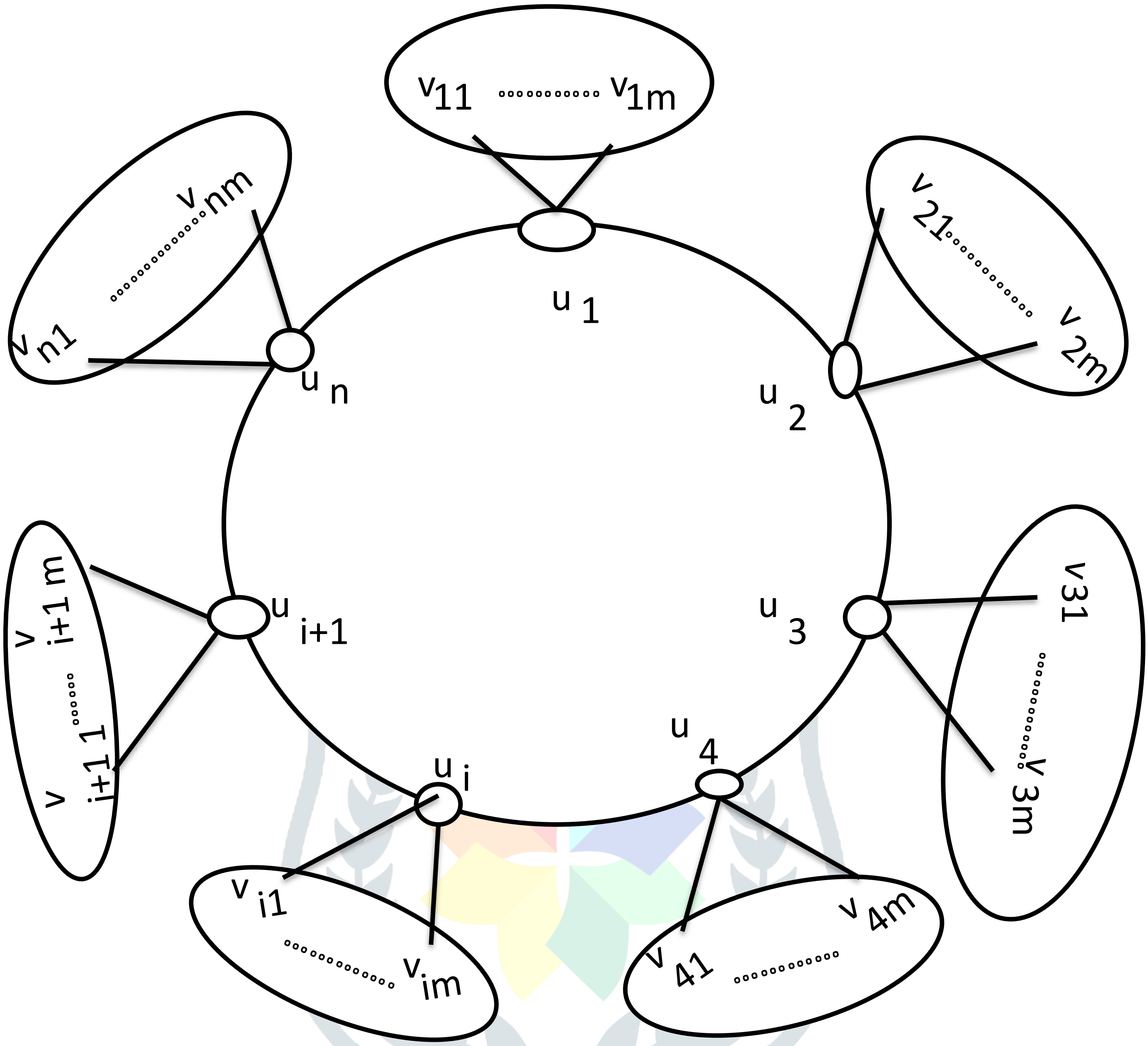


Figure1: Corona graph $C_n \odot H$,

**Observation 2**: From theorem 1 we can observe that the triple connected Roman domination number of Corona of $G \odot H$, is $2n$. Where G is a connected graph of $n$ vertices and H is connected graph of $m$ vertices.

**Theorem 3.** For $\gamma_{TRC}(K_{m,n} + e) = 5$

Proof: Let$G = K_{m,n} + e$, let $(X, Y)$be the bipartition of the graph G.

$V(G) = \{u_i, v_j : u_i \in X, v_j \in Y, 1 \le i \le m, 1 \le j \le n\}, E(G) = \{u_1u_2, u_iv_j : 1 \le i \le m, 1 \le j \le n\}$

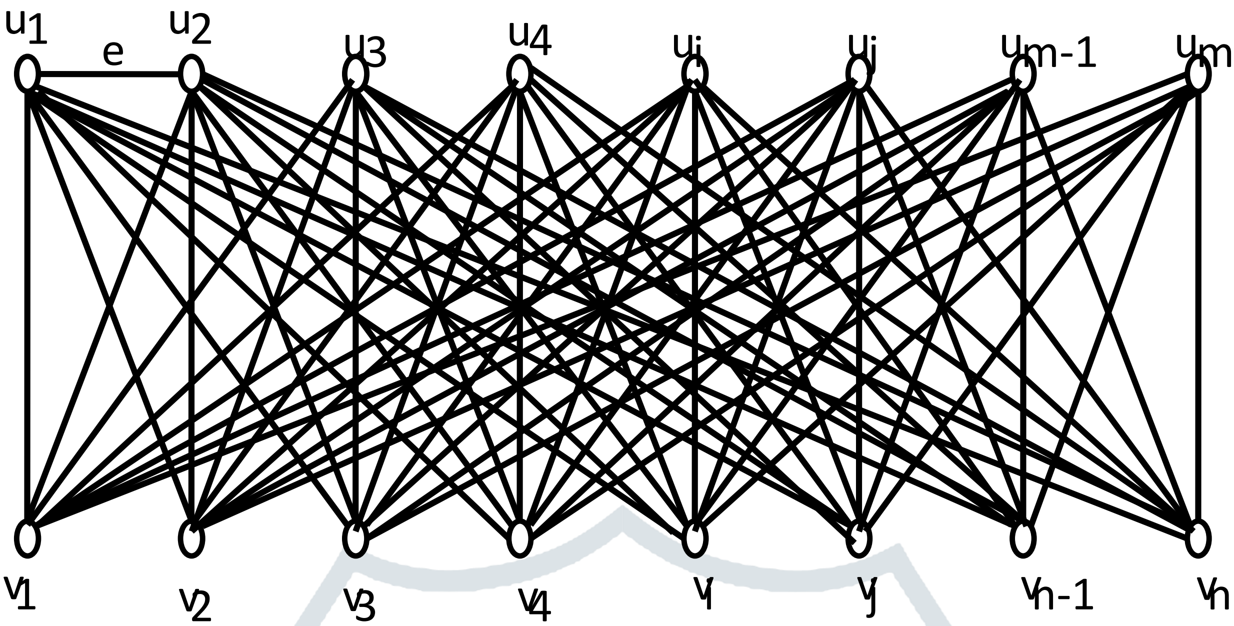


Figure 3: Almost Complete Bipartite Graph $K_{m,n} + e$

Let $x = u_i$ for some $i$, $1 \leq i \leq m$, $y = v_j$ for some $j$, $1 \leq j \leq n$. Define $f = (V_0, V_1, V_2)$ where $V_2 = \{x, y\}$ $V_1 = \{w\}$, where w ∈ V \ {x,y} and $V_0 = V \backslash (V_1 \cup V_2)$. Then $V_2$ is a dominating set of $V_0$ and $\langle V_1 \cup V_2 \rangle$ is triple connected. Therefore $\gamma_{TRC}(K_{m,n} + e) = 5$

**Theorem 4.** For $\gamma_{TRC}(B_{1,n}) = 5$

Proof: Let $G = B_{1,n}$, a triangle book graph, $V(G) = \{x, y, u_i: 1 \leq i \leq n\}$, $E(G) = \{xy, xu_i, yu_i: 1 \leq i \leq n\}$

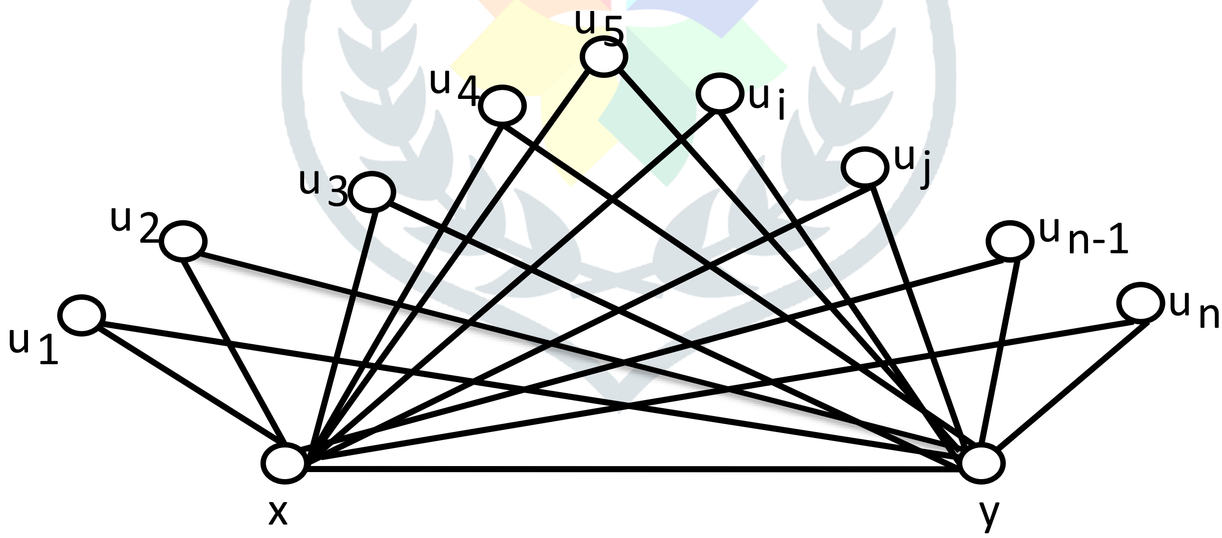


Figure 4: Triangle Book graph $B_{1,n}$

Let $z = u_i$ for some $i$, $1 \leq i \leq n$. Define $f = (V_0, V_1, V_2)$ where $V_2 = \{x, y\}$, $V_1 = \{z\}$ and $V_0 = V \backslash (V_1 \cup V_2)$. Then $V_2$ is a dominating set of $V_0$ and $\langle V_1 \cup V_2 \rangle$ is triple connected.

Hence $f$ is a Roman dominating function and it is triple connected. Therefore $\gamma_{TRC}(B_{1,n}) = 5$

**Theorem 5**. The triple connected Roman domination number of a square book graph$\gamma_{TRC}(B_{2,n})$ =6

Proof: Let G be a square book graph. $V(G) = \{x, y, u_i, v_i: 1 \leq i \leq n\}$, $E(G) = \{xy, xu_i, yv_i, u_iv_i: 1 \leq i \leq n\}$

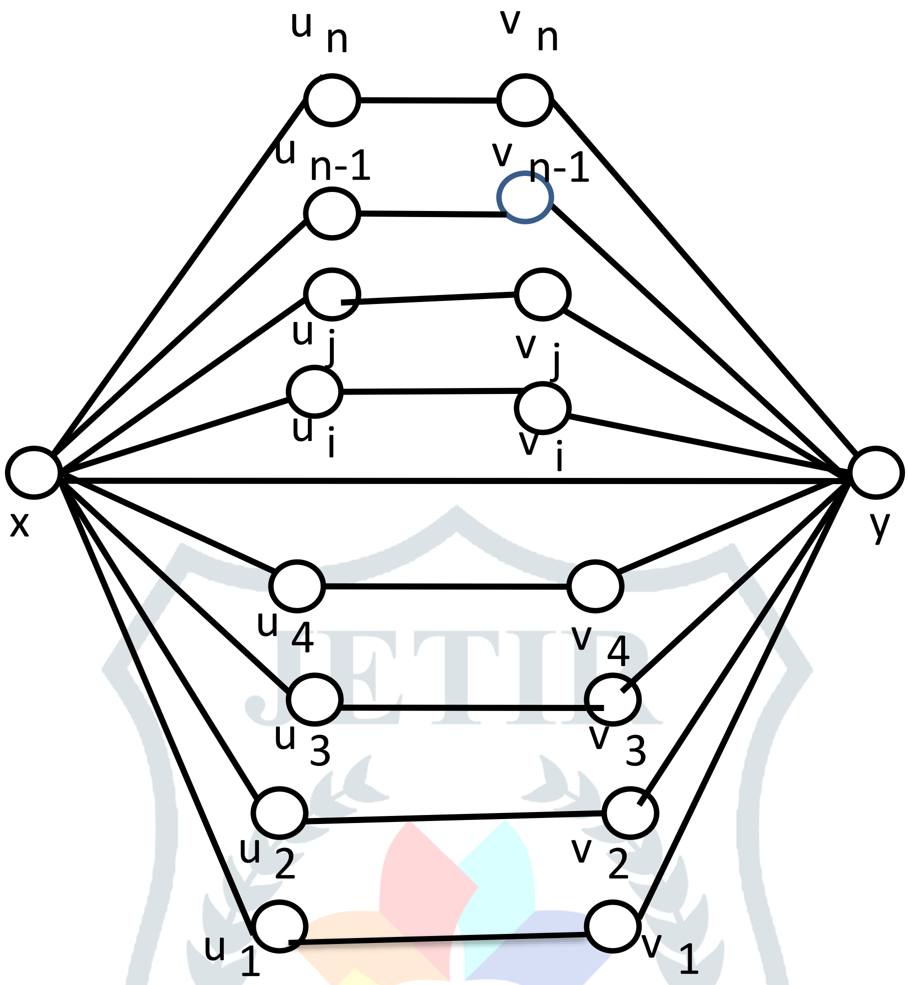


Figure 5: Square book graph

Let $u = u_i$ for some $i$, $1 \leq i \leq n$, $v = v_j$ for some $j$, $1 \leq j \leq n$.Define $f = (V_0, V_1, V_2)$ where $V_2 = \{x, y\}$, $V_1 = \{u, v\}$ for some $i$ and $V_0 = V \backslash (V_1 \cup V_2)$. Then, $V_2$ is a dominating set of $V_0$ and $\langle V_1 \cup V_2 \rangle$ is triple connected. Hence $f$ is a Roman dominating function and it is triple connected. Therefore $\gamma_{TRC}(B_{2,n})$ =6

**Conclusion:** In this paper we have found the triple connected Roman Domination number for Corona of graph, almost bipartite graph, triangular book, and square book. We can also find triple connected Roman Domination number for some other special graph and some general graph also.

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