"A STUDY ON HISTORY OF MATHEMATICS"

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ABSTRACT

The history of mathematics is nearly as old as humanity itself. Since antiquity, mathematics has been fundamental to advances in science, engineering, and philosophy. It has evolved from simple counting, measurement and calculation, and the systematic study of the shapes and motions of physical objects, through the application of abstraction, imagination and logic, to the broad, complex and often abstract discipline we know today. From the notched bones of early man to the mathematical advances brought about by settled agriculture in Mesopotamia and Egypt and the revolutionary developments of ancient Greece and its Hellenistic empire, the story of mathematics is a long and impressive one. The East carried on the baton, particularly China, India and the medieval Islamic empire, before the focus of mathematical innovation moved back to Europe in the late Middle Ages and Renaissance. Then, a whole new series of revolutionary developments occurred in 17th Century and 18th Century Europe, setting the stage for the increasing complexity and abstraction of 19th Century mathematics, and finally the audacious and sometimes devastating discoveries of the 20th Century. Follow the story as it unfolds in this series of linked sections, like the chapters of a book. Read the human stories behind the innovations, and how they made - and sometimes destroyed

Mathematics may be defined as "the study of relationships among quantities, magnitudes and properties, and also of the logical operations by which unknown quantities, magnitudes, and properties may be deduced" (*Microsoft Encarta Encyclopedia*) or "the study of quantity, structure, space and change" (*Wikipedia*). Historically, it was regarded as the science of quantity, whether of magnitudes (as in geometry) or of numbers (as in arithmetic) or of the generalization of these two fields (as in algebra). Some have seen it in terms as simple as a search for patterns. During the 19th Century, however, mathematics broadened to encompass mathematical or symbolic logic, and thus came to be regarded increasingly as the science of relations or of drawing necessary conclusions (although some see even this as too restrictive). The discipline of mathematics now covers - in addition to the more or less standard fields of number theory, algebra, geometry, analysis (calculus), mathematical logic and set theory, and more applied mathematics such as probability theory and statistics - a bewildering array of specialized areas and fields of study, including group theory, order theory, knot theory, sheaf theory, topology, differential geometry, fractal geometry, graph theory, functional analysis, complex analysis, singularity theory, catastrophe theory, chaos theory, measure theory, model theory, category theory, control theory, game theory, complexity theory and many more.

THE STORY OF MATHEMATICS

Follow the story as it unfolds in this series of linked sections, like the chapters of a book. Read the human stories behind the innovations, and how they made - and sometimes destroyed - the men and women who devoted their lives to the Story of Mathematics.

PREHISTORICATHEMATICS
THE ISHANGO BONE, A TALLY STICK FROM CENTRAL AFRICA, DOES FROM ABOUT 20,000 YEARS AGO

Our prehistoric ancestors would have had a general sensibility about amounts, and would have instinctively known the difference between, say, one and two antelopes. But the intellectual leap from the concrete idea of two things to the invention of a symbol or word for the abstract idea of "two" took many ages to come about.

Even today, there are isolated hunter-gatherer tribes in Amazonia which only have words for "one", "two" and "many", and others which only have words for numbers up to five. In the absence of settled agriculture and trade, there is little need for a formal system of numbers.

Early man kept track of regular occurrences such as the phases of the moon and the seasons. Some of the very earliest evidence of mankind thinking about numbers is from notched bones in Africa dating back to 35,000 to 20,000 years ago. But this is really mere counting and tallying rather than mathematics as such.

According to some authorities, there is evidence of basic arithmetic and geometric notations on the petroglyphs at Knowth and Newgrange burial mounds in Ireland (dating from about 3500 BCE and 3200 BCE respectively). These utilize a repeated zig-zag glyph for counting, a system which continued to be used in Britain and Ireland into the 1st millennium BCE. Stonehenge, a Neolithic ceremonial and astronomical monument in England, which dates from around 2300 BCE, also arguably exhibits examples of the use of 60 and 360 in the circle measurements, a practice which presumably developed quite independently of the sexagesimal counting system of the ancient Sumerian and Babylonians.

SUMERIAN/BABYLONIAN MATHEMATICS

Sumer (a region of Mesopotamia, modern-day Iraq) was the birthplace of writing, the wheel, agriculture, the arch, the plow, irrigation and many other innovations, and is often referred to as the Cradle of Civilization. The Sumerians developed the earliest known writing system - a pictographic writing system known as cuneiform script, using wedge-shaped characters inscribed on baked clay tablets - and this has meant that we actually have more knowledge of ancient Sumerian and Babylonian mathematics than of early Egyptian mathematics. Indeed, we even have what appear to school exercises in arithmetic and geometric problems.

They were perhaps the first people to assign symbols to groups of objects in an attempt to make the description of larger numbers easier. They moved from using separate tokens or symbols to represent sheaves of wheat, jars of oil, etc, to the more abstract use of a symbol for specific numbers of anything. Starting as early as the 4th millennium BCE, they began using a small clay cone to represent one, a clay ball for ten, and a large cone for sixty. Over the course of the third millennium, these objects were replaced by cuneiform equivalents so that numbers could be written with the same stylus that was being used for the words in the text.

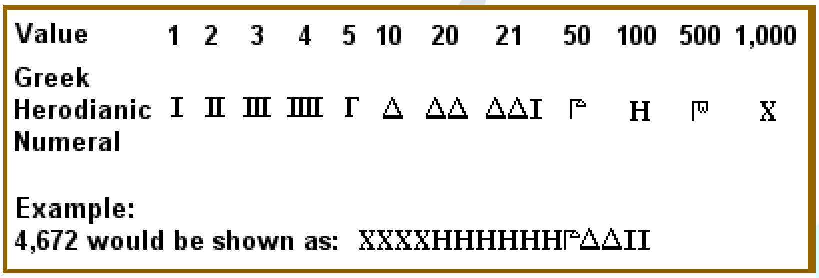
EGYPTIAN MATHEMATICS

The early Egyptians settled along the fertile Nile valley as early as about 6000 BCE, and they began to record the patterns of lunar phases and the seasons, both for agricultural and religious reasons. The Pharaoh’s surveyors used measurements based on body parts (a palm was the width of the hand, a cubit the measurement from elbow to fingertips) to measure land and buildings very early in Egyptian history, and a decimal numeric system was developed based on our ten fingers. The oldest mathematical text from ancient Egypt discovered so far, though, is the Moscow Papyrus, which dates from the Egyptian Middle Kingdom around 2000 - 1800 BCE.

The Rhind Papyrus, dating from around 1650 BCE, is a kind of instruction manual in arithmetic and geometry, and it gives us explicit demonstrations of how multiplication and division was carried out at that time. It also contains evidence of other mathematical knowledge, including unit fractions, composite and prime numbers, arithmetic, geometric and harmonic means, and how to solve first order linear equations as well as arithmetic and geometric series. The Berlin Papyrus, which dates from around 1300 BCE, shows that ancient Egyptians could solve second-order algebraic (quadratic) equations.

Unit fractions could also be used for simple division sums. For example, if they needed to divide 3 loaves among 5 people, they would first divide two of the loaves into thirds and the third loaf into fifths, then they would divide the left over third from the second loaf into five pieces. Thus, each person would receive one-third plus one-fifth plus one-fifteenth (which totals three-fifths, as we would expect).

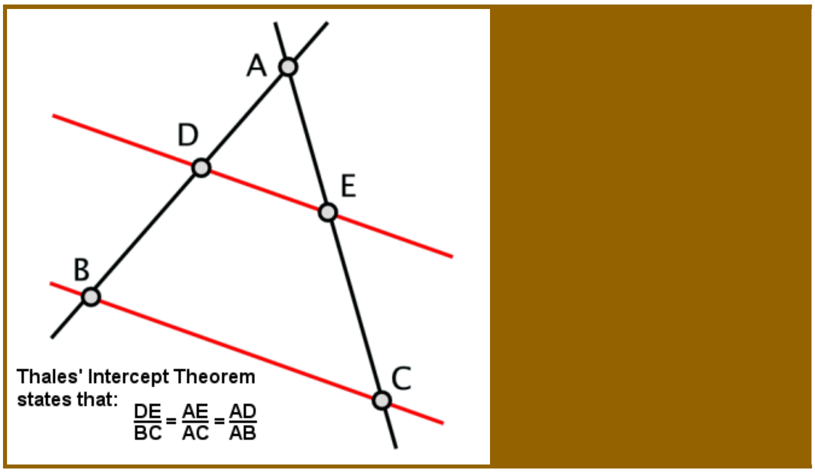
GREEK MATHEMATICS



Ancient Greek Herodianic numerals

As the Greek empire began to spread its sphere of influence into Asia Minor, Mesopotamia and beyond, the Greeks were smart enough to adopt and adapt useful elements from the societies they conquered. This was as true of their mathematics as anything else, and they adopted elements of mathematics from both the Babylonians and the Egyptians. But they soon started to make important contributions in their own right and, for the first time, we can acknowledge contributions by individuals. By the Hellenistic period, the Greeks had presided over one of the most dramatic and important revolutions in mathematical thought of all time.

The ancient Greek numeral system, known as Attic or Herodianic numerals, was fully developed by about 450 BCE, and in regular use possibly as early as the 7th Century BCE. It was a base 10 system similar to the earlier Egyptian one (and even more similar to the later Roman system), with symbols for 1, 5, 10, 50, 100, 500 and 1,000 repeated as many times needed to represent the desired number. Addition was done by totalling separately the symbols (1s, 10s, 100s, etc) in the numbers to be added, and multiplication was a laborious process based on successive doublings (division was based on the inverse of this process).



Thales' Intercept Theorem

But most of Greek mathematics was based on geometry. Thales, one of the Seven Sages of Ancient Greece, who lived on the Ionian coast of Asian Minor in the first half of the 6th Century BCE, is usually considered to have been the first to lay down guidelines for the abstract development of geometry, although what we know of his work (such as on similar and right triangles) now seems quite elementary.

Thales established what has become known as Thales' Theorem, whereby if a triangle is drawn within a circle with the long side as a diameter of the circle, then the opposite angle will always be a right angle (as well as some other related properties derived from this). He is also credited with another theorem, also known as Thales' Theorem or the Intercept Theorem, about the ratios of the line segments that are created if two intersecting lines are intercepted by a pair of parallels (and, by extension, the ratios of the sides of similar triangles).

To some extent, however, the legend of the 6th Century BCE mathematician Pythagoras of Samos has become synonymous with the birth of Greek mathematics. Indeed, he is believed to have coined both the words "philosophy" ("love of wisdom") and "mathematics" ("that which is learned"). Pythagoras was perhaps the first to realize that a complete system of mathematics could be constructed, where geometric elements corresponded with numbers. Pythagoras' Theorem (or the Pythagorean Theorem) is one of the best known of all mathematical theorems. But he remains a controversial figure, as we will see, and Greek mathematics was by no means limited to one man.

Three geometrical problems in particular, often referred to as the Three Classical Problems, and all to be solved by purely geometric means using only a straight edge and a compass, date back to the early days of Greek geometry: “the squaring (or quadrature) of the circle”, “the doubling (or duplicating) of the cube” and “the trisection of an angle”. These intransigent problems were profoundly influential on future geometry and led to many fruitful discoveries, although their actual solutions (or, as it turned out, the proofs of their impossibility) had to wait until the 19th Century.

Hippocrates of Chios (not to be confused with the great Greek physician Hippocrates of Kos) was one such Greek mathematician who applied himself to these problems during the 5th Century BCE (his contribution to the “squaring the circle” problem is known as the Lune of Hippocrates). His influential book “The Elements”, dating to around 440 BCE, was the first compilation of the elements of geometry, and his work was an important source for Euclid's later work.

It was the Greeks who first grappled with the idea of infinity, such as described in the well-known paradoxes attributed to the philosopher Zeno of Elea in the 5th Century BCE. The most famous of his paradoxes is that of Achilles and the Tortoise, which describes a theoretical race between Achilles and a tortoise. Achilles gives the much slower tortoise a head start, but by the time Achilles reaches the tortoise's starting point, the tortoise has already moved ahead. By the time Achilles reaches that point, the tortoise has moved on again, etc, etc, so that in principle the swift Achilles can never catch up with the slow tortoise.

Paradoxes such as this one and Zeno's so-called Dichotomy Paradox are based on the infinite divisibility of space and time, and rest on the idea that a half plus a quarter plus an eighth plus a sixteenth, etc, etc, to infinity will never quite equal a whole. The paradox stems, however, from the false assumption that it is impossible to complete an infinite number of discrete dashes in a finite time, although it is extremely difficult to definitively prove the fallacy. The ancient Greek Aristotle was the first of many to try to disprove the paradoxes, particularly as he was a firm believer that infinity could only ever be potential and not real.

Democritus, most famous for his prescient ideas about all matter being composed of tiny atoms, was also a pioneer of mathematics and geometry in the 5th - 4th Century BCE, and he produced works with titles like "On Numbers", "On Geometrics", "On Tangencies", "On Mapping" and "On Irrationals", although these works have not survived. We do know that he was among the first to observe that a cone (or pyramid) has one-third the volume of a cylinder (or prism) with the same base and height, and he is perhaps the first to have seriously considered the division of objects into an infinite number of cross-sections.

However, it is certainly true that Pythagoras in particular greatly influenced those who came after him, including Plato, who established his famous Academy in Athens in 387 BCE, and his protégé Aristotle, whose work on logic was regarded as definitive for over two thousand years. Plato the mathematician is best known for his description of the five Platonic solids, but the value of his work as a teacher and popularizer of mathematics can not be overstated.

Plato’s student Eudoxus of Cnidus is usually credited with the first implementation of the “method of exhaustion” (later developed by Archimedes), an early method of integration by successive approximations which he used for the calculation of the volume of the pyramid and cone. He also developed a general theory of proportion, which was applicable to incommensurable (irrational) magnitudes that cannot be expressed as a ratio of two whole numbers, as well as to commensurable (rational) magnitudes, thus extending Pythagoras’ incomplete ideas.

Perhaps the most important single contribution of the Greeks, though - and Pythagoras, Plato and Aristotle were all influential in this respect - was the idea of proof, and the deductive method of using logical steps to prove or disprove theorems from initial assumed axioms. Older cultures, like the Egyptians and the Babylonians, had relied on inductive reasoning, that is using repeated observations to establish rules of thumb. It is this concept of proof that give mathematics its power and ensures that proven theories are as true today as they were two thousand years ago, and which laid the foundations for the systematic approach to mathematics of Euclid and those who came after him.

HELLENISTIC MATHEMATICS

By the 3rd Century BCE, in the wake of the conquests of Alexander the Great, mathematical breakthroughs were also beginning to be made on the edges of the Greek Hellenistic empire.

In particular, Alexandria in Egypt became a great centre of learning under the beneficent rule of the Ptolemies, and its famous Library soon gained a reputation to rival that of the Athenian Academy. The patrons of the Library were arguably the first professional scientists, paid for their devotion to research. Among the best known and most influential mathematicians who studied and taught at Alexandria were Euclid, Archimedes, Eratosthenes, Heron, Menelaus and Diophantus.

During the late 4th and early 3rd Century BCE, Euclid was the great chronicler of the mathematics of the time, and one of the most influential teachers in history. He virtually invented classical (Euclidean) geometry as we know it. Archimedes spent most of his life in Syracuse, Sicily, but also studied for a while in Alexandria. He is perhaps best known as an engineer and inventor but, in the light of recent discoveries, he is now considered of one of the greatest pure mathematicians of all time. Eratosthenes of Alexandria was a near contemporary of Archimedes in the 3rd Century BCE. A mathematician, astronomer and geographer, he devised the first system of latitude and longitude, and calculated the circumference of the earth to a remarkable degree of accuracy. As a mathematician, his greatest legacy is the "Sieve of Eratosthenes" algorithm for identifying prime numbers.

It is not known exactly when the great Library of Alexandria burned down, but Alexandria remained an important intellectual centre for some centuries. In the 1st century BCE, Heron (or Hero) was another great Alexandrian inventor, best known in mathematical circles for Heronian triangles (triangles with integer sides and integer area), Heron's Formula for finding the area of a triangle from its side lengths, and Heron's Method for iteratively computing a square root. He was also the first mathematician to confront at least the idea of $\sqrt{-1}$ (although he had no idea how to treat it, something which had to wait for Tartaglia and Cardano in the 16th Century).

ROMAN MATHEMATICS

I	V	X	L	C	D	M
1	5	10	50	100	500	1,000

Example:
1,944 = MDCCCCXXXXIIII = MCMXLIV

Romannumerals

By the middle of the 1st Century BCE, the Roman had tightened their grip on the old Greek and Hellenistic empires, and the mathematical revolution of the Greeks ground to halt. Despite all their advances in other respects, no mathematical innovations occurred under the Roman Empire and Republic, and there were no mathematicians of note. The Romans had no use for pure mathematics, only for its practical applications, and the Christian regime that followed it (after Christianity became the official religion of the Roman empire) even less so.

Roman numerals are well known today, and were the dominant number system for trade and administration in most of Europe for the best part of a millennium. It was decimal (base 10) system but not directly positional, and did not include a zero, so that, for arithmetic and mathematical purposes, it was a clumsy and inefficient system. It was based on letters of the Roman alphabet - I, V, X, L, C, D and M - combines to signify the sum of their values (e.g. VII = V + I + I = 7).

Later, a subtractive notation was also adopted, where VIIII, for example, was replaced by IX (10 - 1 = 9), which simplified the writing of numbers a little, but made calculation even more difficult, requiring conversion of the subtractive notation at the beginning of a sum and then its re-application at the end (see image at right). Due to the difficulty of written arithmetic using Roman numeral notation, calculations were usually performed with an abacus, based on earlier Babylonian and Greek abaci.

MAYAN MATHEMATICS

The Mayan civilisation had settled in the region of Central America from about 2000 BCE, although the so-called Classic Period stretches from about 250 CE to 900 CE. At its peak, it was one of the most densely populated and culturally dynamic societies in the world.

The importance of astronomy and calendar calculations in Mayan society required mathematics, and the Maya constructed quite early a very sophisticated number system, possibly more advanced than any other in the world at the time (although the dating of developments is quite difficult).

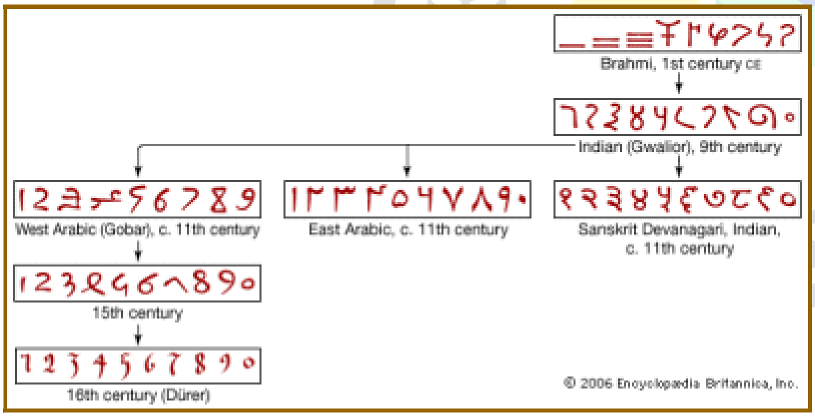
CHINESE MATHEMATICS

Even as mathematical developments in the ancient Greek world were beginning to falter during the final centuries BCE, the burgeoning trade empire of China was leading Chinese mathematics to ever greater heights.

The simple but efficient ancient Chinese numbering system, which dates back to at least the 2nd millennium BCE, used small bamboo rods arranged to represent the numbers 1 to 9, which were then places in columns representing units, tens, hundreds, thousands, etc. It was therefore a decimal place value system, very similar to the one we use today - indeed it was the first such number system, adopted by the Chinese over a thousand years before it was adopted in the West - and it made even quite complex calculations very quick and easy.

Written numbers, however, employed the slightly less efficient system of using a different symbol for tens, hundreds, thousands, etc. This was largely because there was no concept or symbol of zero, and it had the effect of limiting the usefulness of the written number in Chinese.

INDIAN MATHEMATICS



The evolution of Hindu-Arabic numerals

Despite developing quite independently of Chinese (and probably also of Babylonian mathematics), some very advanced mathematical discoveries were made at a very early time in India.

Mantras from the early Vedic period (before 1000 BCE) invoke powers of ten from a hundred all the way up to a trillion, and provide evidence of the use of arithmetic operations such as addition, subtraction, multiplication, fractions, squares, cubes and roots. A 4th Century CE Sanskrit text reports Buddha enumerating numbers up to 10^{53}, as well as describing six more numbering systems over and above these, leading to a number equivalent to 10^{421}. Given that there are an estimated 10^{80} atoms in the whole universe, this is as close to infinity as any in the ancient world came. It also describes a series of iterations in decreasing size, in order to demonstrate the size of an atom, which comes remarkably close to the actual size of a carbon atom (about 70 trillionths of a metre).

As early as the 8th Century BCE, long before Pythagoras, a text known as the “Sulba Sutras” (or "Sulva Sutras") listed several simple Pythagorean triples, as well as a statement of the simplified Pythagorean theorem for the sides of a square and for a rectangle (indeed, it seems quite likely that Pythagoras learned his basic geometry from the "Sulba Sutras"). The Sutras also contain geometric solutions of linear and quadratic equations in a single unknown, and give a remarkably accurate figure for the square root of 2, obtained by adding $1 + \frac{1}{3} + \frac{1}{(3 \times 4)} - \frac{1}{(3 \times 4 \times 34)}$, which yields a value of 1.4142156, correct to 5 decimal places.

As early as the 3rd or 2nd Century BCE, Jain mathematicians recognized five different types of infinities: infinite in one direction, in two directions, in area, infinite everywhere and perpetually infinite. Ancient Buddhist literature also demonstrates a prescient awareness of indeterminate and infinite numbers, with numbers deemed to be of three types: countable, uncountable and infinite.

Like the Chinese, the Indians early discovered the benefits of a decimal place value number system, and were certainly using it before about the 3rd Century CE. They refined and perfected the system, particularly the written representation of the numerals, creating the ancestors of the nine numerals that (thanks to its dissemination by medieval Arabicmathematicans) we use across the world today, sometimes considered one of the greatest intellectual innovations of all time.

The Indians were also responsible for another hugely important development in mathematics. The earliest recorded usage of a circle character for the number zero is usually attributed to a 9th Century engraving in a temple in Gwalior in central India. But the brilliant conceptual leap to include zero as a number in its own right (rather than merely as a placeholder, a blank or empty space within a number, as it had been treated until that time) is usually credited to the 7th Century Indian mathematicians Brahmagupta - or possibly another Indian, Bhaskara I - even though it may well have been in practical use for centuries before that. The use of zero as a number which could be used in calculations and mathematical investigations, would revolutionize mathematics.

ISLAMIC MATHEMATICS - AL-KHWARIZMI

One of the first Directors of the House of Wisdom in Bagdad in the early 9th Century was an outstanding Persian mathematician called Muhammad Al-Khwarizmi. He oversaw the translation of the major Greek and Indian mathematical and astronomy works (including those of Brahmagupta) into Arabic, and produced original work which had a lasting influence on the advance of Muslim and (after his works spread to Europe through Latin translations in the 12th Century) later European mathematics.

The word “algorithm” is derived from the Latinization of his name, and the word "algebra" is derived from the Latinization of "al-jabr", part of the title of his most famous book, in which he introduced the fundamental algebraic methods and techniques for solving equations.

Perhaps his most important contribution to mathematics was his strong advocacy of the Hindu numerical system, which Al-Khwarizmi recognized as having the power and efficiency needed to revolutionize Islamic and Western mathematics. The Hindu numerals 1 - 9 and 0 - which have since become known as Hindu-Arabic numerals - were soon adopted by the entire Islamic world. Later, with translations of Al-Khwarizmi’s work into Latin by Adelard of Bath and others in the 12th Century, and with the influence of Fibonacci’s “Liber Abaci” they would be adopted throughout Europe as well.

MEDIEVAL MATHEMATICS

During the centuries in which the Chinese, Indian and Islamic mathematicians had been in the ascendancy, Europe had fallen into the Dark Ages, in which science, mathematics and almost all intellectual endeavour stagnated. Scholastic scholars only valued studies in the humanities, such as philosophy and literature, and spent much of their energies quarrelling over subtle subjects in metaphysics and theology, such as "How many angels can stand on the point of a needle?"

From the 4th to 12th Centuries, European knowledge and study of arithmetic, geometry, astronomy and music was limited mainly to Boethius' translations of some of the works of ancient Greek masters such as Nicomachus and Euclid. All trade and calculation was made using the clumsy and inefficient Roman numeral system, and with an abacus based on Greek and Roman models.

16TH CENTURY MATHEMATICS - TARTAGLIA, CARDANO & FERRARI

In the Renaissance Italy of the early 16th Century, Bologna University in particular was famed for its intense public mathematics competitions. It was in just such a competition, in 1535, that the unlikely figure of the young Venetian Tartaglia first revealed a mathematical finding hitherto considered impossible, and which had stumped the best mathematicians of China, India and the Islamic world.

Niccolò Fontana became known as Tartaglia (meaning "the stammerer") for a speech defect he suffered due to an injury he received in a battle against the invading French army. He was a poor engineer known for designing fortifications, a surveyor of topography (seeking the best means of defence or offence in battles) and a bookkeeper in the Republic of Venice.

17TH CENTURY MATHEMATICS

In the wake of the Renaissance, the 17th Century saw an unprecedented explosion of mathematical and scientific ideas across Europe, a period sometimes called the Age of Reason. Hard on the heels of the "Copernican Revolution" of Nicolaus Copernicus in the 16th Century, scientists like Galileo Galilei, Tycho Brahe and Johannes Kepler were making equally revolutionary discoveries in the exploration of the Solar system, leading to Kepler's formulation of mathematical laws of planetary motion.

The invention of the logarithm in the early 17th Century by John Napier (and later improved by Napier and Henry Briggs) contributed to the advance of science, astronomy and mathematics by making some difficult calculations relatively easy. It was one of the most significant mathematical developments of the age, and 17th Century physicists like Kepler and Newton could never have performed the complex calculatons needed for their innovations without it. The French astronomer and mathematician Pierre Simon Laplace remarked, almost two centuries later, that Napier, by halving the labours of astronomers, had doubled their lifetimes.

18TH CENTURY MATHEMATICS

Most of the late 17th Century and a good part of the early 18th were taken up by the work of disciples of Newtonand Leibniz, who applied their ideas on calculus to solving a variety of problems in physics, astronomy and engineering.

The period was dominated, though, by one family, the Bernoulli's of Basel in Switzerland, which boasted two or three generations of exceptional mathematicians, particularly the brothers, Jacob and Johann. They were largely responsible for further developing Leibniz's infinitesimal calculus - paricularly through the generalization and extension of calculus known as the "calculus of variations" - as well as Pascal and Fermat's probability and number theory.

Basel was also the home town of the greatest of the 18th Century mathematicians, Leonhard Euler, although, partly due to the difficulties in getting on in a city dominated by the Bernoulli family, Euler spent most of his time abroad, in Germany and St. Petersburg, Russia. He excelled in all aspects of mathematics, from geometry to calculus to trigonometry to algebra to number theory, and was able to find unexpected links between the different fields. He proved numerous theorems, pioneered new methods, standardized mathematical notation and wrote many influential textbooks throughout his long academic life.

19TH CENTURY MATHEMATICS

The 19th Century saw an unprecedented increase in the breadth and complexity of mathematical concepts. Both France and Germany were caught up in the age of revolution which swept Europe in the late 18th Century, but the two countries treated mathematics quite differently.

After the French Revolution, Napoleon emphasized the practical usefulness of mathematics and his reforms and military ambitions gave French mathematics a big boost, as exemplified by “the three L’s”, Lagrange, Laplace and Legendre (see the section on 18th Century Mathematics), Fourier and Galois.

Joseph Fourier's study, at the beginning of the 19th Century, of infinite sums in which the terms are trigonometric functions were another important advance in mathematical analysis. Periodic functions that can be expressed as the sum of an infinite series of sines and cosines are known today as Fourier Series, and they are still powerful tools in pure and applied mathematics. Fourier (following Leibniz, Euler, Lagrange and others) also contributed towards defining exactly what is meant by a function, although the definition that is found in texts today - defining it in terms of a correspondence between elements of the domain and the range - is usually attributed to the 19th Century German mathematician Peter Dirichlet.

In 1806, Jean-Robert Argand published his paper on how complex numbers (of the form a + b*i*, where *i* is √-1) could be represented on geometric diagrams and manipulated using trigonometry and vectors. Even though the Dane Caspar Wessel had produced a very similar paper at the end of the 18th Century, and even though it was Gauss who popularized the practice, they are still known today as Argand Diagrams.

20TH CENTURY MATHEMATICS

The 20th Century continued the trend of the 19th towards increasing generalization and abstraction in mathematics, in which the notion of axioms as “self-evident truths” was largely discarded in favour of an emphasis on such logical concepts as consistency and completeness.

It also saw mathematics become a major profession, involving thousands of new Ph.D.s each year and jobs in both teaching and industry, and the development of hundreds of specialized areas and fields of study, such as group theory, knot theory, sheaf theory, topology, graph theory, functional analysis, singularity theory, catastrophe theory, chaos theory, model theory, category theory, game theory, complexity theory and many more.

The eccentric British mathematician G.H. Hardy and his young Indian protégé Srinivasa Ramanujan, were just two of the great mathematicians of the early 20th Century who applied themselves in earnest to solving problems of the previous century, such as the Riemann hypothesis. Although they came close, they too were defeated by that most intractable of problems, but Hardy is credited with reforming British mathematics, which had sunk to something of a low ebb at that time, and Ramanujan proved himself to be one of the most brilliant (if somewhat undisciplined and unstable) minds of the century.

Others followed techniques dating back millennia but taken to a 20th Century level of complexity. In 1904, Johann Gustav Hermes completed his construction of a regular polygon with 65,537 sides ($2^{16} + 1$), using just a compass and straight edge as Euclid would have done, a feat that took him over ten years.

The early 20th Century also saw the beginnings of the rise of the field of mathematical logic, building on the earlier advances of Gottlob Frege, which came to fruition in the hands of Giuseppe Peano, L.E.J.

Brouwer, David Hilbert and, particularly, Bertrand Russell and A.N. Whitehead, whose monumental joint work the “Principia Mathematica” was so influential in mathematical and philosophical logicism.

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