SQUARE SUM LABELING OF MOSER SPINDLE GRAPH,GOLOMB GRAPH, SOIFER GRAPH ANDNIGH COMPLETE BIPARTITE GRAPH

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Abstract
A (p,q)-graph G is said to be square sum, if there exists a bijection $f{:}V(G)\rightarrow\{0,1,2,\ldots\ldots, p-1\}$ such that the induced function $f^*: E(G)\rightarrow N$ defined by $f^*(uv)=[f(u)]^2+[f(v)]^2$, for every $uv\in E(G)$, is injective. In this paper, we show that Moser spindle graph, Golomb graph, Soifer graph and Nigh complete bipartite graph $K_{n,n}+ e_1 + e_2$, $3 \le n \le 10$,admits square sum labeling.

Keywords
Square sum labeling, Moser spindle graph, Almost bipartite graph, Golomb graph, Soifer graph
Nigh complete bipartite graph.

AMS Subject Classification (2010):05C78.

1 Introduction
Labeledgraphs[8] are utilized in various correlated fields of engineering, technology, etc.An exceptional method of labeling looks attractive if there emerges a number of problems that lights the interest of the researchers. one among the types of labeling is square sum labeling[1][3][4][5][6]. In this paper we deal only finite, simple, connected and undirected graphs obtained through graph operations.

Definition 1.1
An almost-bipartite graph[2] is a non-bipartite graph with the property that the removal of a particular single edge renders the graph bipartite.

Definition 1.2
Let G be a (p, q)-graph. G is said to be a square sum graph if there exist a bijection $f{:}V(G)\rightarrow\{0,1,2, \ldots\ldots, p-1\}$such that the induced function $f^*: E(G)\rightarrow N$ given by $f^*(uv)=[f(u)]^2+[f(v)]^2$, for every $uv\in E(G)$ are all distinct.

Definition 1.3
A nigh complete bipartite graph[7] or a nigh graph is a graph $K_{m,n} + e_1 + e_2(m =n)$ where e_i , i = 1,2has both ends in V_i, i = 1,2 respectively such that the removal of that two edges $e_1$and $e_2$renders complete bipartite graph.

Definition 1.4
Moser spindle graph is an undirected graph with seven vertices and eleven edges. It is a unit distance graph some times called as hajos graph.

Definition 1.5
Golomb graph is a polyhedral graph with 10 vertices and 18 edges. It is a unit distance graph

Definition 1.6
The soifer graph is a planar graph with 9-nodes and 20 edges.It is an undirected graph

2. Results

Theorem 2.1 The Moser spindle graph is square sum.

Proof:
Let G denote the moser spindle graph which contains 7 vertices and 11 edges having the vertex set $V = \{u_0,u_1,u_2,u_3,u_4,u_5,u_6\}$ where u_2,u_3,u_4,u_5,u_6,u_2forms the cycle C_5 and the vertex u_0 is adjacent to u_2,u_3 and u_4 and u_1 is adjacent to u_2,u_5 and u_6.
Define $f: V\rightarrow\{0, 1, 2,\ldots\ldots,6\}$ by
$f(u_i) = i$, $0 \le i \le 6$
The function f induces a square sum labeling on G and the edge labels of G, f^* is given in table-1

Table-1: Egde labels of Moser spindle graph

S.NO	EDGE	EDGE LABEL (f*)
1	u_0u_2	4
2	u_0u_3	9
3	u_0u_4	16
4	u_1u_2	5
5	u_1u_6	37
6	u_1u_5	26
7	u_2u_3	13
8	u_3u_4	25
9	u_4u_5	41
10	u_5u_6	61
11	$u_6\, u_2$	40

Hence for any two edges of G the edge labels are distinct .
$\therefore f^*$ is injective. Hence G is square sum.

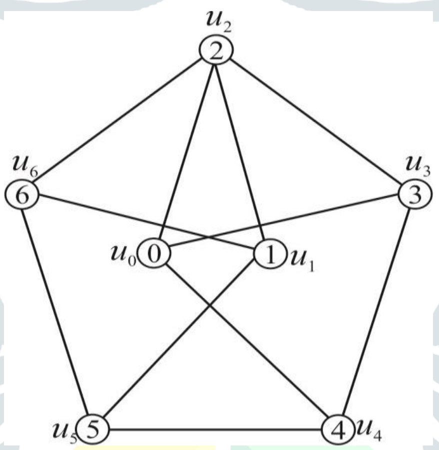

Figure 1: Square sum labeling of Moser spindle graph

Theorem 2.2 The Golomb graph is square sum.
Proof:
Let G denote the golomb graph which contains 10 vertices and 18 edges having the vertex set $V = \{ v_1, v_2, v_3, u_1, u_2, u_3, u_4, u_5, u_6, w\}$ where v_1, v_2 and v_3 are the vertices of K_3 and u_1, u_2, u_3, u_4, u_5 and u_6 are the vertices of the wheel and w is the central vertex of the wheel.
Define $f : V \rightarrow \{0, 1, 2, \ldots, 9\}$ and the function f defined is shown in table-2

Table-2: Vertex labels (f) of Golomb graph

S.NO	VERTEX	VERTEX LABEL (f)
1	w	0
2	v_1	2
3	v_2	3
4	v_3	5
5	u_1	1
6	u_2	4
7	u_3	6
8	u_4	7
9	u_5	8
10	u_6	9

Let $E = \{ wu_1, wu_2, wu_3, wu_4, wu_5, wu_6, u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_1, u_1v_1, u_3v_2, u_5v_3\, v_1v_2, v_2v_3, v_3v_1 \}$

The function f induces a square sum labeling on G and the edge labels of G, f^* is given in table-3

Table-3: Egde labels of Golomb graph

S.NO	EDGE	EDGE LABEL (f *)
1	wu_1	1
2	wu_2	16
3	wu_3	36
4	wu_4	49
5	wu_5	64
6	wu_6	81
7	u_1u_2	17
8	u_2u_3	52
9	u_3u_4	85
10	u_4u_5	113
11	u_5u_6	145
12	u_6u_1	82
13	u_1v_1	5
14	u_3v_2	45
15	u_5v_3	89
16	v_1v_2	13
17	v_2v_3	34
18	v_3v_1	29

Hence for any two edges of G the edge labels are distinct .
$\therefore f$ * is injective
Hence G is square sum.

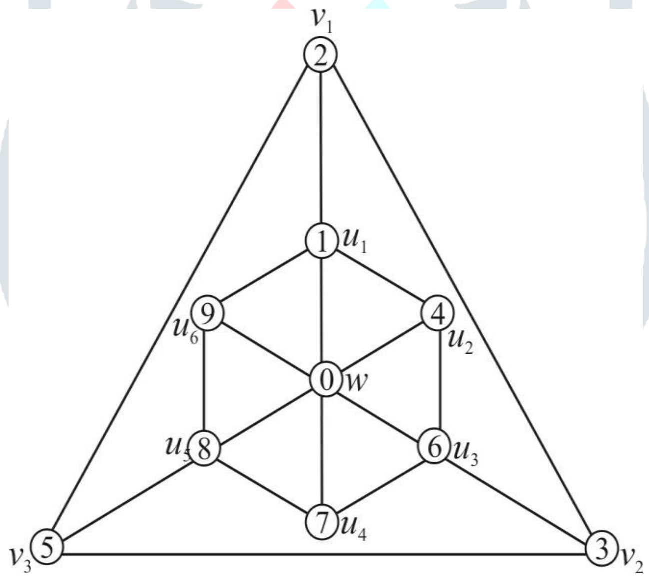

Figure 2: Square sum labeling of Golomb graph

Theorem 2.3 The Soifer graph is square sum.
Proof:
Let G denote the soifer graph which contains 9 vertices and 20 edges having the vertex set $V = \{ v_1, v_2, v_3, v_4, u_1, u_2, u_3, u_4, u_5 \}$ where v_1, v_2, v_3 and v_4 are the vertices of the outer cycle C_4 and u_1, u_2, u_3, u_4 and u_5 are the vertices of the inner cycle C_5 of G.
Define $f: V \rightarrow \{0, 1, 2, \ldots\ldots, 8\}$ and the function f defined is shown in table- 4

Table-4: Vertex labels (f) of Soifer graph

S.NO	VERTEX	VERTEX LABEL (f)
1	v_1	0
2	v_2	1

3	v_3	2
4	v_4	3
5	u_1	4
6	u_2	5
7	u_3	6
8	u_4	7
9	u_5	8

Let$E$ = {v_1v_2, v_2v_3, v_3v_4, v_4v_1, u_1u_2, u_2u_3, u_3u_4 ,u_4u_5, u_5u_1, v_1u_2, v_1u_1, v_1u_5, v_2u_2, v_2u_3, v_3u_3, v_3u_4, v_4u_4, v_4u_5 ,u_1u_3, u_1u_4}

The function f induces a square sum labeling on G and the edge labels of G, f* is given in

table-5

Table-5: Egde labels of Soifer graph

S.NO	EDGE	EDGE LABEL (f*)
1	v_1v_2	1
2	v_2v_3	5
3	v_3v_4	13
4	v_4v_1	9
5	u_1u_2	41
6	u_2u_3	61
7	u_3u_4	85
8	u_4u_5	113
9	u_5u_1	80
10	v_1u_1	16
11	v_1u_2	25
12	v_1u_5	64
13	v_2u_2	26
14	v_2u_3	37
15	v_3u_3	40
16	v_3u_4	53
17	v_4u_4	58
18	v_4u_5	73
19	u_1u_3	52
20	u_1u_4	65

Hence for any two edges of G the edge labels are distinct .
$\therefore f$* is injective. Hence G is square sum.

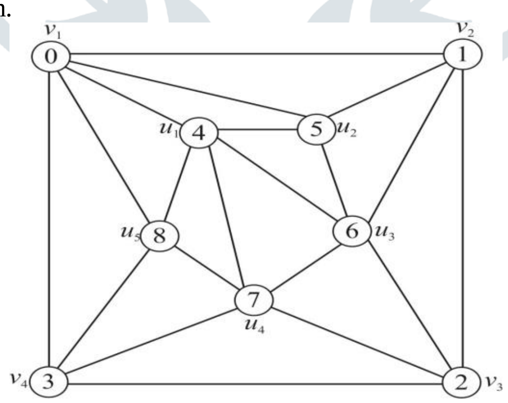

Figure 3: Square sum labeling of Soifer graph

Theorem 2.4 : The Nigh complete bipartite graph $K_{n,n}+\ e_1 + e_2$, $3 \le n \le 10$ is square sum.
Proof:
Case-1: $K_{n,n}+\ e_1 + e_2$, $3 \le n \le 7$
Consider the graph $G= K_{n,n}+\ e_1 + e_2, 3 \le n \le 7$. Let V_1 and V_2 be the bipartition of $V(G)$, where $V_1 = \{v_1, v_2, ..., v_n\}$ and $V_2= \{ u_1, u_2, ..., u_n\}$. Suppose $e_1 = v_1v_2$ and $e_2 = u_1u_2$
we note that $|E(G)|= n^2+2$.
Define $f : V(G) \to \{0,1, 2, ..., 2n- 1\}$ by
$f(v_1) = 0$, $f(u_1) = 1$, $f(v_2) = 2$, $f(u_2) =3$, $f(u_3) = 4$, $f(v_3) = 5$, $f(v_4) = 6$, $f(u_4) = 7$,
$f(u_5) = 8$,……………………….., $f(v_n) =2n–1$,*if n is odd.*

$f(v_1) = 0$, $f(u_1) = 1$, $f(v_2) = 2$, $f(u_2) = 3$, $f(u_3) = 4$, $f(v_3) = 5$, $f(v_4) = 6$, $f(u_4) = 7$, $f(u_5) = 8, \ldots\ldots\ldots, f(u_n) = 2n-1$, *if n is even.*

The function f induces a square sum labeling on G.

For, if $e_1 = u_1v_1$ and $e_2 = u_2v_2$ then

(i)$u_1 = u_2 \Rightarrow f(v_1) < f(v_2)\ or\ f(v_1) > f(v_2)$,

then $f^*(e_1) = [f(u_1)]^2 + [f(v_1)]^2 \neq [f(u_2)]^2 + [f(v_2)]^2 = f^*(e_2)$.

(ii)If $u_1 \neq u_2$, then $f(u_1) < f(u_2) \Rightarrow f(v_1) < f(v_2)$.

Hence $f^*(e_1) = [f(u_1)]^2 + [f(v_1)]^2 \neq [f(u_2)]^2 + [f(v_2)]^2 = f^*(e_2)$.

so that f^* admits square sum labeling of $K_{n,n} + e_1 + e_2$, $3 \le n \le 7$

Hence G is square sum.

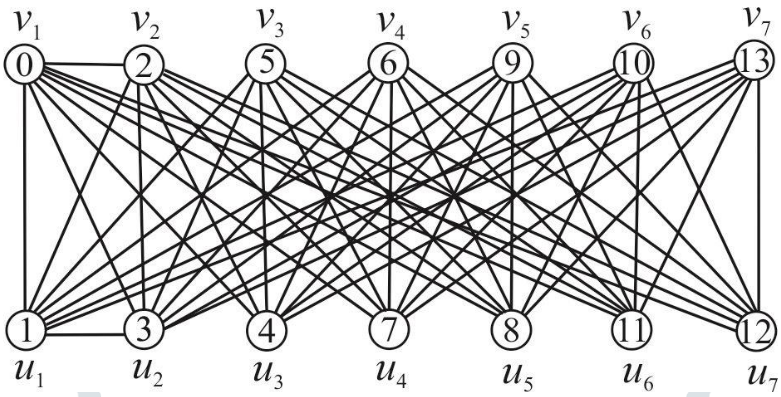

Figure 4: Square sum labeling of nigh complete bipartite graph $K_{7,7} + e_1 + e_2$.

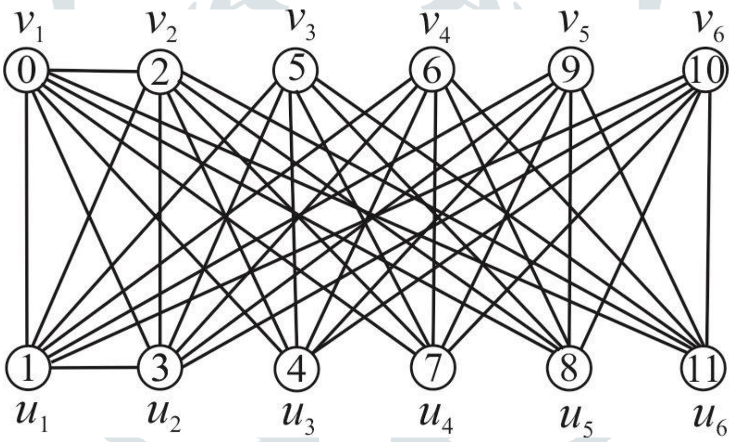

Figure 5: Square sum labeling of nigh complete bipartite graph $K_{6,6} + e_1 + e_2$.

Case-2 :$K_{8,8} + e_1 + e_2$

Consider the graph G = $K_{8,8} + e_1 + e_2$. Let V_1 and V_2 be the bipartition of $V(G)$, where $V_1 = \{v_1, v_2, ..., v_n\}$ and $V_2 = \{u_1, u_2, ..., u_n\}$.

Suppose Suppose $e_1 = v_1v_2$ and $e_2 = u_1u_2$

we note that $|E(G)| = n^2+2$.

Define $f: V(G) \to \{0, 1, 2, ..., 2n-1\}$ by

$$f(v_1) = 0,\ f(v_2) = 1,\ f(u_1) = 2,\ f(u_2) = 3,\ f(u_3) = 4,\ f(v_3) = 5,\ f(u_4) = 6,\ f(u_5) = 7,$$

$$f(v_4) = 8,\ f(u_6) = 9,\ f(v_5) = 10,\ f(u_7) = 11,\ f(v_6) = 12,\ f(u_8) = 13,\ f(v_7) = 14,\ f(v_8) = 15.$$

The function f induces a square sum labeling on G.

clearly for any two edges of G the edge labels are distinct .Hence f^* is injective

So f^* admits square sum labeling for $K_{8,8} + e_1 + e_2$

Hence G is square sum.

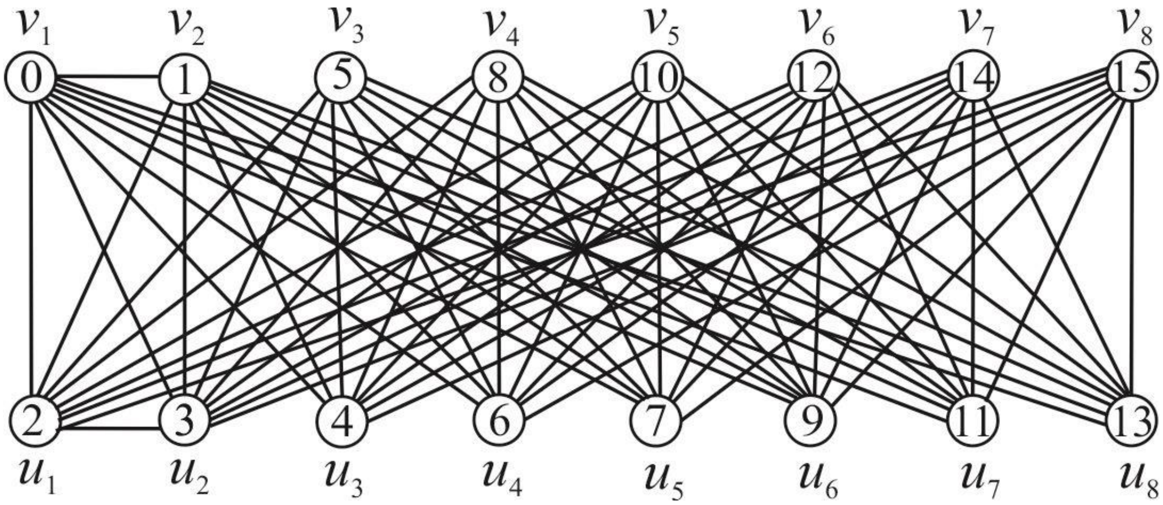

Figure 6: Square sum labeling of nigh complete bipartite graph $K_{8,8} + e_1 + e_2$

Case-3 : $K_{9,9} + e_1 + e_2$

Consider the graph G = $K_{9,9} + e_1 + e_2$

Let V_1 and V_2 be the bipartition of $V(G)$, where $V_1 = \{v_1, v_2, ..., v_n\}$ and $V_2 = \{u_1, u_2, ..., u_n\}$. Suppose $e_1 = v_1v_2$ and $e_2 = u_1u_2$ we note that $|E(G)| = n^2+2$.

Define $f : V(G) \rightarrow \{0,1, 2, ..., 2n-1\}$ by

$$f(v_1) = 0,\ f(v_2) = 1,\ f(u_1) = 2,\ f(u_2) = 3,\ f(u_3) = 4,\ f(v_3) = 5,\ f(u_4) = 6,\ f(u_5) = 7,$$

$$f(v_4) = 8,\ f(u_6) = 9,\ f(v_5) = 10,\ f(u_7) = 11,\ f(v_6) = 12,\ f(u_8) = 13,\ f(v_7) = 14,$$

$$f(v_8) = 15,\ f(u_9) = 16,\ f(v_9) = 17.$$

The function f induces a square sum labeling on G.

clearly for any two edges of G the edge labels are distinct .Hence f^* is injective

So f^* admits square sum labeling for $K_{9,9} + e_1 + e_2$

Hence G is square sum.

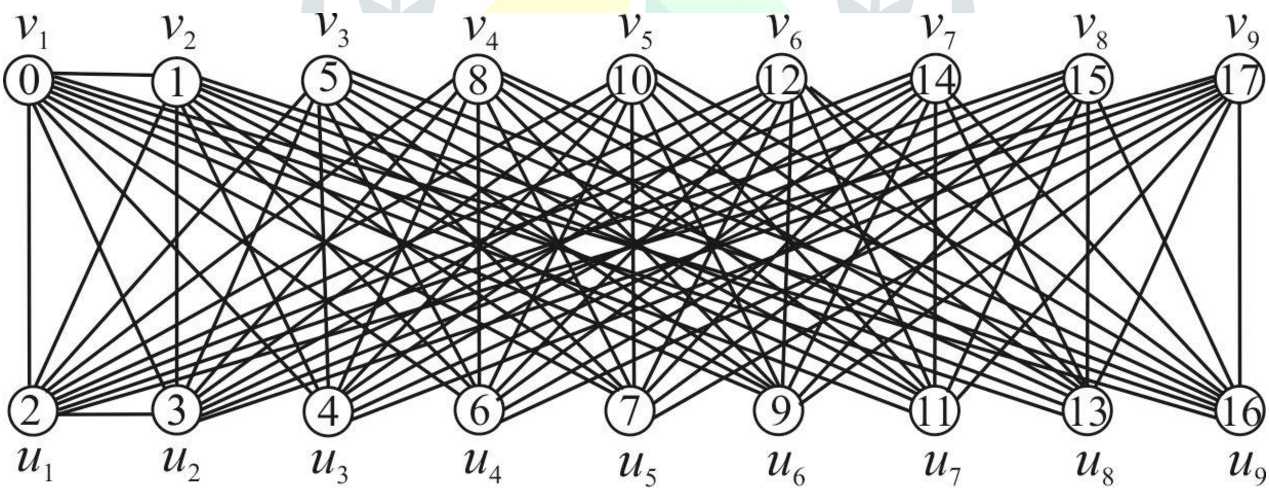

Figure 7: Square sum labeling of nigh complete bipartite graph $K_{9,9} + e_1 + e_2$

Case-4 : $K_{10,10} + e_1 + e_2$

Consider the graph G = $K_{10,10} + e_1 + e_2$

Let V_1 and V_2 be the bipartition of $V(G)$, where $V_1 = \{v_1, v_2, ..., v_n\}$ and $V_2 = \{u_1, u_2, ..., u_n\}$. Suppose $e_1 = v_1v_2$ and $e_2 = u_1u_2$, we note that $|E(G)| = n^2+2$.

Define $f : V(G) \rightarrow \{0,1, 2, ..., 2n-1\}$ by

$$f(v_1) = 0,\ f(v_2) = 1,\ f(u_1) = 2,\ f(u_2) = 3,\ f(u_3) = 4,\ f(v_3) = 5,\ f(u_4) = 6,\ f(u_5) = 7,$$

$$f(v_4) = 8,\ f(u_6) = 9,\ f(v_5) = 10,\ f(u_7) = 11,\ f(v_6) = 12,\ f(u_8) = 13,\ f(v_7) = 14,$$

$$f(v_8) = 15,\ f(u_9) = 16,\ f(v_9) = 17,\ f(v_{10}) = 18,\ f(u_{10}) = 19.$$

The function f induces a square sum labeling on G.

clearly for any two edges of G the edge labels are distinct .Hence f^* is injective

So f^* is a square sum labeling of $K_{10,10} + e_1 + e_2$

Hence G is square sum.

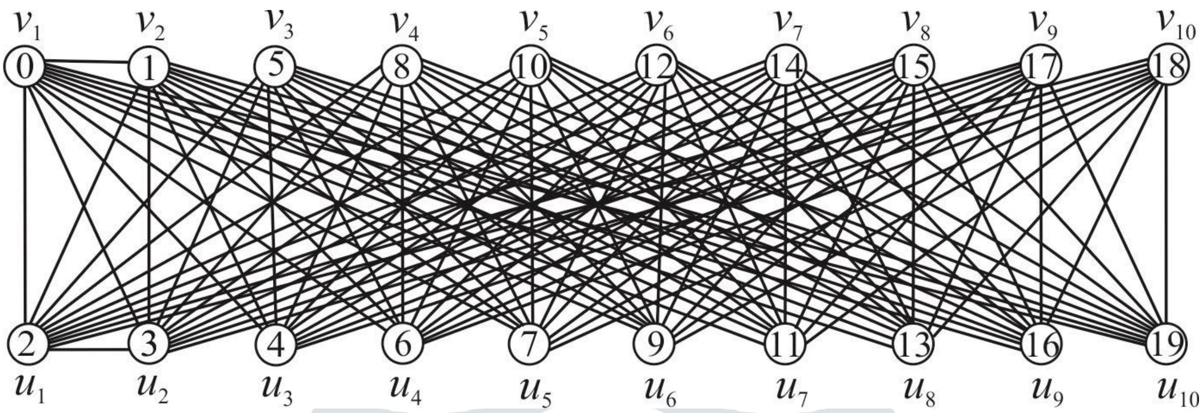

Figure 8: Square sum labeling of nigh complete bipartite graph $K_{10,10} + e_1 + e_2$

3Conclusion

It is very interesting to study graphs which admit square sum labeling. Here we have examined and verified that moser spindle graph, golomb graph, soifer graph is square sum, also we examined another special case of almost bipartite graph,the nigh complete bipartite graph $K_{n,n} + e_1 + e_2$and we proved it to be square sum for certain cases. To examine equivalent results for different types of graphs is an open area of research.

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