# CONVERGENCE ANALYSIS OF MULTISTEP PREDICTORS OF UNIFORM SPACES

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Abstract. In the uniform spaces one-step ahead predictor is identi ed for the solution of any topological spaces subject to some feasibility constraints, whose initialization depends on the initial state. The purpose of this paper is to optimize the results of uncertainty, by using a new formulation based on the convergence analysis of the uniform spaces. In this way, the feasibility conditions are expressed as simple constraints on the solution of the topological space, also preserving strict analogy with the results on one-step prediction and multi predicted measures. The transiant and asymptotic behaviour of the multistep predictor is analyzed. More precisely, su cient conditions on the initial state uncertainity are worked out, which guarantee the existence of the pridictor over an arbitrarily long time interval and its convergence to steady-state even in the uncertainity.

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## I. Introduction

The uniform space one step ahead predictor is one of the generalizations of a topological metric space. The uniform structure's concept lies between the topological metric space and its structure, in the sense that every metric space is a uniform space and every uniform space is a topological space [1,8]. The importance of uniform structures lies on the fact that uniform spaces preserve the main features of topological spaces [2,4]. With this we can deal here with the topological concepts of classical analysis such as completeness, total boundedness, Cauchy filters, uniform continuity etc [3,5].

## II. Design of predictors

As for notation, $\|.\|$ denots the Euclidean vector norm. The symbols $[X_+]$, $[X_-]$ represent the positive and negative part of a symmetric matrix $X$ respectively. Namely, $X = [X_+]+[X_-]$ where $[X_+]\geq 0,\ [X_-]\leq 0$ and the positive eigenvalues of $X$ coincide with the non-null eigenvalues of $[X_+]$.

Consider the finite time horizon $[0, N]$ and the function [10],

$$Y_N = \sum_{k=0}^{N} \| z_k - \hat{z}_{k|k-j} \|^2 - \gamma^2 \left( \sum_{k=0}^{N} \| w_k \|^2 + x_{0'}\Pi_0 x_0 \right) \quad (1)$$

where $\hat{z}_{k|k-j}$ is an estimate of $z_k$ based on the output observations $\{y_0, y_1,, y_{k-l}\}$, namely $\hat{z}_{k|k-j}$ represents an $l$-step ahead prediction, with $j \geq 1$. In $(1)$, the scalar $\gamma > 0$ is the prescribed level of disturbances attenuation, and $\Pi_0 = \Pi_{0'} > 0$ is a given weighting matrix reflecting the uncertainty on the initial state $x_0$. When $Y_N < 0$, the ratio of the prediction error energy to the energy of the disturbances (including the initial state $x_0$) is less than $\gamma^2$. It will be said that a predictor guarantees a prescribed attenuation level $\gamma$ if it ensures that $Y_N < 0$ for each finite nonzero $(x_0, \{w_0, w_1,, w_N\})$.

The main results are summarized in the following thoerem [12].

**Theorem 2.1** *An $l$-step predictor guaranteeing a prescribed attenuation level $\gamma$ exists if and only if there exist two sequences of matrices $\{S_k\}_{k=0}^{N-l}$ and $\{Q_m\}_{m=0}^{l-1}$ satisfying the recursions*

$$\begin{aligned} S_{k+1} &= \left(AS_k^{-1}A' + BB'\right)^{-1} + C'C - L'L_\gamma^{-2} \\ S_0 &= \Pi_0^{-1} + C'C - L'L_\gamma^{-2} \end{aligned} \tag{2}$$

$$Q_{m-1} = A'\left[\left(L'L_\gamma^{-2} + Q_m\right) + \left(L'L_\gamma^{-2} + Q_m\right)B\left(I - B'\left(L'L_\gamma^{-2} + Q_m\right)B\right)^{-1}\right] \tag{3}$$

$$Q_{l-1} = 0 \tag{4}$$

and the conditions

$$S_k = Q_0 + C'C, \qquad K = 0,1,,N-l \tag{5}$$

$$0 < I - B'\left(L'L_\gamma^{-2} + Q_m\right)B, \qquad m = 0,1,,l-1 \tag{6}$$

In that case, an admissible predictor is given by

$$\hat{x}_{k+1} = A\hat{x}_k + K_k \begin{bmatrix} y_k - C\hat{x}_k \\ \hat{z}_{k|k-l} - L\hat{x}_k \end{bmatrix}, \hat{x}_0 = 0 \tag{7}$$

$$\hat{x}_{k+m+1}^k = A\hat{x}_{k+m}^k + K_{k+m}^k\left(\hat{z}_{k+m|k+m-l} - L\hat{x}_{k+m}^k\right), \hat{x}_k^k = \hat{x}_k \tag{8}$$

$$\begin{aligned} \hat{z}_{k|k-l} &= L\hat{x}_{k+l-1}^k, k \geq l \\ &= 0, k < l \end{aligned} \tag{9}$$

where

$$K_k = AP_k\left[C'L'\right]\left(\begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C \\ L \end{bmatrix} P_k\left[C'L'\right]\right)^{-1} \tag{10}$$

$$P_k = \left(S_k - C'C + L'L_{\gamma^{-2}}\right)^{-1} \tag{11}$$

$$K_{k+m}^k = A\left(S_{k+m}^k + L'L_{\gamma^{-2}}\right)^{-1} L'\left(-\gamma^2 I + L\left(S_{k+m}^k + L'L_{\gamma^{-2}}\right)^{-1} L'\right)^{-1} \tag{12}$$

$$S_{k+m+l}^k = \left(A\left(S_{k+m}^k\right)^{-1} A' + BB'\right)^{-1} - L'L_{\gamma^{-2}}, S_k^k = S_k - C'C \tag{13}$$

where $m = 0,1,,l-1$

Note that the gains $K_k$ and $K_{k+m}^k$ used in the predictor depend on the solutions of any compact spaces and unifrom spaces. $(13)$ has the form of a Bounded Real Lemma equation and must be solved at any time instant for $l-1$ step with an initial condition that is dictated by the solution $S_k$ of the main equation $(2)$.

**Lemma 2.2** *Let ( $z^1$,..., $z^n$ ) be any basis of $\mathrm{K}^n$. Then the following statements are equivalent [7].*

$(i)$ The system $(1)$ is stable at time $t_0$ ( uniformly stable )

$(ii)$ There exists a constant $M$ which may depend on $t_0$ ( independent of $t_0$ ) such that $\|\phi(t,t_0)\| \leq M$ for all $t \in T_{t_0}$.

$(iii)$ There exists a constant $M$ which may depend on $t_0$ ( independent of $t_0$ ) such that $\left\|\phi(t,t_0)Z^i\right\| \le M$ for all $t \in T_{t_0}.,\ i \in n$

*Proof.* $(i) \Rightarrow (ii)$. Suppose that $(1)$ is stable at time $t_0$ ( uniformly stable ), then for $\varepsilon = 1$, there exists $\delta > 0$ depending on $t_0$ such that

$$\left\|x^0\right\| \le \delta \Rightarrow \left\|\phi(t,t_0)x^0\right\| \le 1, \quad t \in T_{t_0}$$

hence $\left\|\phi(t,t_0)\right\| \le \delta^{-1}$ for all $t \in T_{t_0}$

As $(ii) \Rightarrow (iii)$ is trivial it only remains to prove $(iii) \Rightarrow (i)$ . Suppose $(iii)$ holds. Since there exist $a, b > 0$ such that

$$a\max_{i\in n}\left|\xi_i\right| \le \left\|\sum_{i=1}^{n}\xi_i Z^i\right\| \le b\max_{i\in n}\left|\xi_i\right|, \xi \in k^n. \qquad (14)$$

we have for all $x^0 = \sum_{i=1}^{n}\xi_i Z^i \in \mathrm{K}^n$,

$$\begin{aligned}\left\|\phi(t,t_0)x^0\right\| &= \left\|\Phi(t,t_0)\sum_{i=1}^{n}\xi_i Z^i\right\| \\ &\le \max_{i\in n}\left|\xi_i\right|\sum_{i=1}^{n}\left\|\phi(t,t_0)Z^i\right\| \le a^{-1}nM\left\|x^0\right\|.\end{aligned}$$

This proves (i)

**Lemma 2.3** *Let ( $z^1$,..., $z^n$ ) be any basis of $\mathrm{K}^n$ . Then the following statements are equivalent.*

$(i)$ The system $(1)$ is asymptotically stable at time $t_0$ (uniformly asymptotically stable).

$(ii)$ The system $(1)$ is globally asymptotically stable at time $t_0$ (globally uniformly asymptotically stable ).

$(iii)$ $\left\|\phi(t,t_0)\right\| \to 0 \quad as \quad t \to \infty$ (uniformly in $t_0$ ).

$(iv)$ For $i \in n$, $\left\|\phi(t,t_0)Z^i\right\| \to 0 \quad as \quad t \to \infty$ (uniformly in $t_0$ ).

*Proof.* $(i) \Rightarrow (ii)$ follows directly from linearity and $(ii) \Rightarrow (iv)$ and $(iii) \Rightarrow (i)$ are trivial. $(iv) \Rightarrow (iii)$ . Suppose $(iv)$ holds, then for every $\varepsilon > 0$ there exists a time $\tau(\varepsilon)$ depending on $t_0$ such that $\left\|\phi(t,t_0)Z^i\right\| < \varepsilon$ for all $t \in T_{t_0+\tau(\varepsilon)}$, $i \in n$ . But then, for every

$x^0 = \sum_{i=1}^{n}\xi_i Z^i$, $\left\|x^0\right\| = 1$ we have $\max_{i\in n}\left|\xi_i\right| \le a^{-1}$ where $a > 0$ satisfies $(14)$ and thus

$$\left\|\phi(t,t_0)x^0\right\| = \left\|\sum_{i=1}^{n}\xi_i\phi(t,t_0)Z^i\right\| \le a^{-1}n\varepsilon, t \in T_{t_0+\tau(\varepsilon)}.$$

hence $(iii)$ holds.

## III. Sufficient conditions for convergence

Before formulating the main result, consider the equation associated with $(2)$, i.e.

$$S = \left(AS^{-1}A' + BB'\right)^{-1} + C'C - L'L_{\gamma^{-2}} \qquad (15)$$

and assume that it admits a solution $S_S$ which is stabilizing [9,11]. Consider the eigenvalues of $\hat{A} = \left(A^{-1}\right)\left(I + S_S A^{-1}BB'\left(A^{-1}\right)\right)^{-1}$ which lies inside the unit circle. Moreover, supppose that $S_S > Q_0 + C'C$, where $Q_0$ has been defined in section $2$ through $(3)$, $(4)$. In this case, $S_S$ hold its feasibility [6]. Furthermore, we introduce the following notation:

$$\Psi = A^{-1}B\left(I + B'\left(A^{-1}\right)S_S A^{-1}B\right)^{-1} B'\left(A^{-1}\right)$$
$$\Theta = \left(\hat{A}^{-1}\right)\left(\left(S_S - Q_0 - C'C\right)^{-1} - \Psi\right)\hat{A}^{-1} - \left(S_S - Q_0 - c'c\right)^{-1}$$

Finally, consider the Lyapunov equation

$$X = \hat{A}'Z\hat{A} + [\Theta]_-$$

and let

$$\overline{S_0} = S_S - \left(\left(S_S - Q_0 - C'C\right)^{-1} - \hat{A}'X\hat{A}\right)^{-1} \quad (16)$$

Then, a sufficient condition for the existence and convergence of the $l$-step ahead predictor is illustrated in the following theorem.

**Theorem 3.1** *Suppose that the pair* $(A, B)$ *is reachable, and the stabilizing solution of* $(15)$ *is feasible. Then, the condition* $S_0 > \overline{S_0}$ *implies that*

$(i)$ a predictor guaranteeing a prescribed attenuation level $\gamma$ exists over an interval of arbitrary length $N$, and one possible predictor is that given by $(7)$ - $(8)$;

$(ii)$ as $N \to \infty$, the time-varying predictor $(7)$ - $(8)$ tends to the stationary one described by

$$\hat{x}_{k+1} = A\hat{x}_k + \overline{K}\begin{bmatrix} y_k - C\hat{x}_k \\ \hat{z}_{k|k-l} - L\hat{x}_k \end{bmatrix}, \hat{x}_0 = 0$$

$$\hat{x}^k_{k+m+1} = A\hat{x}^k_{k+m} + \overline{K}_m\left(\hat{z}_{k+m|k+m-l} - L\hat{x}^k_{k+m}\right), \hat{x}^k_k = \hat{x}_k$$
$$\hat{z}_{k|k-l} = L\hat{x}^k_{k+l-1}$$

where

$$\overline{K} = AP_S\left[C'L'\right]\left(\begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C \\ L \end{bmatrix} P_S\left[C'L'\right]\right)^{-1}$$
$$P_S = \left(S_S - C'C + L'L_{\gamma^{-2}}\right)^{-1}$$
$$\overline{K}_m = A\left(\overline{S}_m + L'L_{\gamma^{-2}}\right)^{-1} L'\left(-\gamma^2 I + L\left(\overline{S}_m + L'L_{\gamma^{-2}}\right)^{-1} L'\right)^{-1}$$
$$\overline{S}_{m+1} = \left(A\left(\overline{S}_m\right)^{-1} A' + BB'\right)^{-1} - L'L_{\gamma^{-2}}, \overline{S}_0 = S_S - C'C$$
$$m = 0,1,,l-1$$

*Proof.* The proof of this theorem is easily follows from the application of Lemma 2.2. The assumption on the reachability of $(A, B)$ is of positive definite matrices and $(15)$ admits also an antistabilizing solution.

**Remark 3.2** *In the discrete time case*

$$\phi(t, t_0) = A(t-1)A(t-2)...A(t_0), t \in T_{t_0} \quad (17)$$

So if $(15)$ is asymptotically stable at time $t_0 \in T$ it will also be asymptotically stable at time $\tau \in T$ for all $\tau < t_0$. A similar statement also holds for $\tau \in T$, $\tau > t_0$ provided that det $A(k) \neq 0$ for $k = t_0, \ldots, \tau - 1$. Furthermore, taking norms in $(17)$ we obtain $(\forall t \in T : \|A(t)\| \leq \gamma) \Rightarrow \|\phi(t,t_0)\| \leq \gamma^{t-t_0}$, $t_0 \in T$, $t \in T_{t_0}$.

Hence the zero state of $(15)$ will be uniformly stable if $\|A(t)\| \leq 1,\ t \in T$ and it will be uniformly asymptotically stable if $\|A(t)\| \leq \gamma < 1$ for all $t \in T$. These conditions, however, are far from being necessary.

The following Lemma shows that for periodic systems and the stability properties of time-invariant nonlinear system.

**Lemma 3.3** *Suppose the generators* $A(\cdot)$ *of* $(17)$ *are periodic with period* $\tau \in T,\ T = \mathrm{R}$ *or* $\mathrm{Z}$ *,* $\tau > 0$ *:* $A(t+\tau) = A(t),\ t \in T$. *Then* $(17)$ *is uniformly stable (uniformly asymptotically stable) if and only if the time-invariant discrete time system*

$$\hat{x}(k+1) = \phi(\tau,0)\hat{x}(k), k \in \mathrm{N} \tag{18}$$

is asymptotically stable where $\phi$ is the evolution operator generated by $(17)$.

*Proof.* By periodicity $\phi(t,t_0) = \phi(t+\tau, t_0+\tau),\ t \in T_{t_0}$, $t_0 \in T$. Hence if $t \in T_{t_0}$ and

$$t_0 = k_0\tau + t_o^{'}, \quad t = k\tau + t^{'}, \quad 0 \leq t_o^{'}, t^{'} < \tau, \quad k, k_0 \in \mathrm{N}, \tag{19}$$

then

$$\begin{aligned} \phi(t,t_0) &= \phi(t,k\tau)\phi(k\tau,(k-1)\tau)\cdots\phi((k_0+1)\tau,t_0) \\ &= \phi(t^{'},0)\phi(\tau,0)^{k-k_0-1}\phi(\tau,t_0^{'}). \end{aligned} \tag{20}$$

But there exists $c > 0$ such that $\|\phi(t^{'},0)\|$ , $\|\phi(\tau,t_0^{'})\| \leq c$ for all $t_0^{'}, t^{'} \in [0,\tau] \cap T$. Therefore $(20)$ implies $\|\phi(t,t_0)\| \leq c^2 \left\|\phi(\tau,0)^{k-k_0-1}\right\|,\ t \in T_{t_0}$. Applying Lemma 2.2 and Lemma 2.3 we see that $(17)$ is uniformly asymptotically stable if $(18)$ holds this property. The converse implication is obvious since $\phi(\tau,0)^k = \phi(k\tau,0)$.

The following example shows that a system $(??)$ may be unstable even though every time-invariant stable $\dot{x}(t) = A(\tau)x(t)$ $(x(t+1) = A(\tau)x(t))$ at time $\tau \in T$ is asymptotically stable. It is also possible that every system is unstable yet $(17)$ is stable.

**Example 3.4** *Consider the two dimensional periodic system of period* $2\pi$ *, where*

$$A(t) = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix} \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$$

Then $\sigma(A(\tau)) = -1$, $\tau \in \mathrm{R}+$ . The evolution operator generated by $A(\cdot)$ is such that

$$\phi(t,0) = \begin{bmatrix} e^t(\cos t + 1/2\sin t) & e^{-3t}(\cos t - 1/2\sin t) \\ e^t(\sin t - 1/2\cos t) & e^{-3t}(\sin t + 1/2\cos t) \end{bmatrix}$$

which is clearly unbounded.

Let us now turn to exponential stability. The non linear system $(17)$ is $(uniformly)$ exponentially stable if there exist for every $t_0 \in T$ a constant $M > 0$, and a decay rate $\omega < 0$ which may be depend upon $t_0$

$$\|\phi(t,t_0)\| \leq Me^{\omega(t-t_0)}, t \in T_{t_0}. \tag{21}$$

**Theorem 3.5** *The system* $(2.2)$ *is uniformly exponentially stable if and only if it is uniformly asymptotically stable.*

*Proof.* The only if part follows immediately from $(21)$ and Lemma 2.3 . Conversely suppose that $(??)$ is uniformly asymptotically stable. By Lemma 2.3 there exists $\tau \in T$ such that $\|\phi(t+\tau,t)\| \le 1/2$ for all $t \in T$. Hence using the concatenation property of $\phi$.

$$\|\phi(t_0+k\tau,t_0)\| \le \|\phi(t_0+k\tau,t_0+(k-1)\tau)\|\ldots\|\phi(t_0+\tau,t_0)\| \le 2^{-k}.$$

Now suppose $t_0+k\tau \le t < t_0+(k+1)\tau,\ t\in T_{t_0}, K\in \mathrm{N}$ then

$$\|\phi(t,t_0)\| \le \|\phi(t,t_0+k\tau)\|\|\phi(t_0+k\tau,t_0)\| \le \|\phi(t,t_0+k\tau)\|2^{-k}.$$

By Lemma 2.2 there exists $M^{'} > 0$ such that $\|\phi(t,t_0+k\tau)\| \le M^{'}$ for all $t \ge t_0+k\tau,\ k\in \mathrm{N},$ and hence

$$\|\phi(t,t_0)\| \le M^{'}2^{-\left[t-t_0/\tau-1\right]},\quad t\in T_{t_0}, t_0 \in T$$

Setting $M = 2M^{'}$, $\omega = -(ln2)/\tau$ we obtain $(21)$.

**Definition 3.6** ***(Liapunov exponents)*** *If $\phi(\cdot,\cdot)$ is the evolution operator and $t_0 \in T$, the upper and lower Liapunov exponents $\overline{\alpha}(\phi)$ , $\underline{\alpha}(\phi)$ are defined by*

$$\overline{\alpha}(\phi) = \inf\left\{\omega\in R; \exists M_\omega > 0; t\in T_{t_0} : \|\phi(t,t_0)\| \le M_\omega e^{\omega(t-t_0)}\right\}$$

$$\underline{\alpha}(\phi) = \sup\left\{\omega\in R; \exists M_\omega > 0; t\in T_{t_0}\ \forall x\in K^n : \|\phi(t,t_0)x\| \ge M_\omega e^{\omega(t-t_0)}\|x\|\right\}.$$

It is easily seen that the two Liapunov exponents do not depend upon $t_0$ in the continuous time case. In the discrete time case this is also true if $detA(t) \ne 0$ for all $t\in T.$ But, if $detA(t_1)=0$ for some $t_1 \in T$ then $det\phi(t,t_0)=0$ for all $(t,t_0)$ with $t_0 \le t_1 \le t.$ So by $(17)$ we get $\overline{\alpha}(\phi) = -\infty$ if we choose $t_0 \le t_1$. Therefore we need not indicate the dependency on $t_0$ in our notation of the Liapunov exponents.

**Lemma 3.7** $\overline{\beta}(\phi) < \infty$ *if and only if*

$$\sup_{t_0,t\in T, 0\le t-t_0 \le 1} \|\phi(t,t_0)\| < \infty \qquad (22)$$

Note that in the discrete time case $(22)$ holds if and only if $\sup_{t\in T}\|A(t)\| =: \gamma < \infty$ in which case $\overline{\alpha}(\phi) \le \overline{\beta}(\phi) \le ln\gamma$ .

**Definition 3.8** ***(Liapunov transformation).*** *[?] A time-varying transformation*

$S(\cdot)\in PC^1(T;Gln(C))\ \ (S(t)\in Gl_n(C), t\in T)$ is called a Liapunov transformation. if

$$\inf\left\{\varepsilon\in R; \exists M_\varepsilon > 0 \forall t,s\in T : \|S(t)^{-1}\|\|S(s)\| \le M_\varepsilon e^{\varepsilon|t-s|}\right\} = 0$$

It is easily seen that the Liapunov transformations on $T$ form a group with respect to pointwise multiplication, and this group of transformations preserves the properties of stability, instability and asymptotic stability.

**Lemma 3.9** *Suppose the generator $A(\cdot)$ of $(??)$ is periodic with period $\tau > 0$, $\tau\in T$: $A(t+\tau) = A(t)$, $t\in T$, and $detA(t)\ne 0$, $t\in T$ in the discrete time case. Then there exists a Liapunov transformation such that the transformed system*

$$\dot{\tilde{x}}(t) = \tilde{A}(t)\tilde{x}(t), t\in T(\tilde{x}(t+1) = \tilde{A}(t)\tilde{x}(t)), t\in T \qquad (23)$$

is time-invariant.

*Proof.* Suppose $\phi(\cdot,\cdot)$ is generated by $A(\cdot)$. Since $A(\cdot)$ is periodic, we have

$$\dot{\phi}(t+\tau,0) = A(t)\phi(t+\tau,0), t\in T(\phi(t+\tau+1,0) = A(t)\phi(t+\tau,0), t\in T).$$

So there must exist a consant nonsingular matrix $V$ such that $\phi(t+\tau,0) = \phi(t,0)V$ .

Choose $L \in C^{n\times n}$ such that $e^{L} = V$ and $S(t) = \phi(t,0)e^{-tL/\tau}$ , $t \in T$. Then

$$S(t+\tau) = \phi(t+\tau,0)e^{-(tL/\tau)-L} = \phi(t,0)e^{L}e^{-(tL/\tau)-L} = S(t).$$

Hence $S(\cdot)$ is periodic with period $\tau$ . In the continuous time case $\phi$ is automatically invertible and in the discrete time case this is a consequence of the assumption that $\det A(t) \neq 0$, $t \in T$. It follows that $S(t)$, $t \in T$ is invertible. Moreover

$$\dot{s}(t) = A(t)\phi(t,0)e^{-tL/\tau} - \phi(t,0)e^{-tL/\tau}\tau^{-1}L = A(t)S(t) - S(t)\tau^{-1}L, t \geq 0.$$

And in the discrete case

$$S(t+1) = \phi(t+1,0)e^{-(t+1)L/\tau} = A(t)S(t)e^{-L/\tau}, t \in T.$$

So the transformed system $(23)$ is given by $\hat{A}(t) = \tau^{-1}L$, $t \in T$. Hence $S$ is a Liapunov transformation and this completes the proof.

## IV. Conclusion

One-step ahead predictor is identified for the solution of topological spaces subject to some feasibility constraints are obtained in this paper. New results of uncertainity, based on the convergence analysis of the uniform spaces is arrived.

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