THE INVERSE BONDAGE NUMBER OF A GRAPH

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Abstract: The inverse bondage number $b^{-1}(G)$ of a graph G to be the cardinality of a smallest set $E^{'} \subseteq E$ of edges for which $\gamma^{-1}(G-E^{'}) > \gamma^{-1}(G)$. Thus, the inverse bondage number of G is the smallest number of edges whose removal will render every minimum inverse dominating set in G a "non inverse dominating set" set in the resultant spanning sub graph.

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I. INTRODUCTION:

The graphs considered here are finite, nontrivial, connected, undirected without loops or multiple edges. The study of domination in graphs was begun by Ore and Berge [1]. A set D of vertices in a graph G is a dominating set if every vertex not in D is adjacent to at least one vertex in D. The domination number $\gamma(G)$ of G is the order of a smallest dominating set in G. In a communication network, let D denote the set of transmitting stations so that every station not belonging to D have a link with at least one station in D. If this set of stations fails, then one has to find another disjoint such set of stations. This leads to define the domatic number. The domatic number d (G) of G is the maximum number of dominating sets in G [2]. Kulli and Sigarkanti consider the problem of selecting such set of transmitting stations so that if it fails and the existence of a disjoint set of transmitting stations should contains minimum number of stations, or if it is required to work on two disjoint set of transmitting stations D_1 (D_2) so that each station not belonging to D_1 (D_2) have a link with at least one station in D_1 (D_2). This leads us to define the inverse domination number. Let D be a minimum dominating set of G. If V-D contains a dominating set say D^1 of G, then D^1 is called an inverse dominating set with respect to D. The inverse domination number $\gamma^{-1}(G)$ of G is the order of a smallest inverse dominating set in G.

This concept was first defined by Kulli and Sigarkanti [2]. Application concerning the vulnerability of the communications network under link failure, was given in [3]. In particular, suppose that some one does not know which sites in the network act as transmitters, but does know that the set of such site corresponds to a minimum dominating set in the related graph. What is the fewest number of communication links that he must sever so that at least one additional transmitter would be received in order that communication with all sites be possible ? With this idea we now introduce inverse bondage number of a graph.

II. SOME EXACT VALUES.

We begin our investigation of the inverse bondage number by computing the value for several well known classes of graphs.

Proposition 1: The inverse bondage number of the complete graph K_p ($p \geq 3$) is $b^{-1}(K_p) = \left\lfloor \frac{p}{2} \right\rfloor$

Next we find the inverse bondage number of a cycle C_p with p vertices as follows.

Proposition 2: For any cycle C_p other than C_4

$$b^{-1}(C_p) = \begin{cases} 1 & \text{if } p \equiv 0 \pmod 3 \\ 2 & \text{otherwise} \end{cases}$$

In the next result we find inverse bondage number of a path P_p with p vertices.

Proposition 3: For any path P_p with order $p \geq 5$

$$b^{-1}(P_p) = 1$$

Proposition 4: Let $K_{m,n}$ be a complete bipartite graph other than C_4 with $2 \leq m \leq n$. Then $b^{-1}(K_{m,n}) = m-1$.

Proposition 5: For any wheel W_p, $b^{-1}(W_p) = 2$

III. GENERAL BOUNDS

In the following proposition we give on upper bound on inverse bondage number in terms degree of G.

Proposition 6: If G is a non empty graph then,
$b^{-1}(G) \leq \min\{\deg u + \deg v - 2 : u \text{ and } v \text{ are adjacent vertices}\}$.

Proposition 7: If $\Delta(G)$ and $\delta(G)$ are the maximum and minimum degrees of a non empty connected graph G, then $b^{-1}(G) \leq \Delta(G) + \delta(G) - 2$.

Proposition 8: for any graph G, $\gamma(G) + \gamma^{-1}(G) = p$ if any only if $b^{-1}(G)=0$

In the following theorem, we give Nordhaus - Gaddum type results.

Proposition 9: Let G be a graph for which both $b^{-1}(G)$ and $b^{-1}(\overline{G})$ exits. Then

(i) $b^{-1}(G) + b^{-1}(\overline{G}) \leq 2(p-3)$

(ii) $b^{-1}(G) \,.\, b^{-1}(\overline{G}) \leq (2p - \Delta - 2\delta - 2)\,\Delta + (2p - \delta - 2\Delta - 2)\delta + 8$

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