SOME PROPERTIES OF SYMMETRIC BI-(σ,τ) SEMI DERIVATIONS IN NEAR RINGS

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Abstract: let N be a 3-prime near ring with centre Z. The main results in this paper are (1) If N is a 2-torsion free and $D(N,N) \subset Z$ then N is a commutative ring. (2) If $[d(t),t]_{(\sigma,\tau)} = 0$ then N is commutative with respect to addition.

Keywords: (σ,τ)-derivation, symmetric bi- derivation, symmetric bi-(σ,τ) semi derivations

1. Introduction

Maksa introduced Symmetric bi-derivation in [4]. Symmetric bi-derivation of Near-ring was introduced by Ozturk and Jun and also they studied some properties. In this paper we study some properties of symmetric bi-(σ,τ) semi derivations.

In this paper N is a zero-symmetric left near-ring with multiplicative center Z.A near-ring N is said to be 3-prime if $tNu = 0 \Rightarrow t = 0$ (or) $u = 0$.σ,τ are automorphisms of near-ring N. Here $[t,u] = tu - ut, [t,u]_{\sigma,\tau} = t\sigma(u) - \tau(u)t, (t,u) = t + u - t - u \ \forall\, x,y \in N$.A function D: N×N → N is called symmetric if $D(t,u) = D(u,t)\ \forall\, t,u \in N$. A function d: N → N is said to be the trace of D if $d(t) = D(t,t)$ where D: N×N → N is a symmetric function. The trace of D satisfies the relation $d(t+u) = d(t) + 2D(t,u) + d(u)\ \forall\, t,u \in N$. A symmetric bi-additive function D: N×N → N is said to be symmetric bi-(σ,τ) semi derivation if $D(tu,v) = D(t,v)g(\sigma(u)) + \tau(t)D(u,v)\ \forall\, t,u,v \in N$.

2. Primary results

Lemma 2.1: ([4, lemma 3]). Suppose N be a 3-prime near ring.
(i) If $v \in Z - \{0\}, \Rightarrow v$ is not a zero divisor.
(ii) If $Z - \{0\}$ has an element v for which $v + v \in Z, \Rightarrow$ N is abelian with respect to addition.

Lemma 2.2: Suppose N is a 2-torsion free 3-prime near ring, D be a symmetric bi- $(\sigma,\tau) -$ semi derivation of N and d be the trace of D. if $td(N) = 0\ \ \forall\, t \in N \Rightarrow t = 0\ (or) D = 0$.

Proof:
We know that $d(u+v) = d(u) + 2D(u,v) + d(v)\ \forall\, u,v \in N$
Multiply with t on both sides and also from the given hypothesis
$$td(u+v) = td(u) + 2tD(u,v) + td(v)$$
$$0 = 2tD(u,v)$$
Since N is 2-torsion free, we have
$$tD(u,v) = 0 \qquad \text{Equation(1)}$$
Substitute uw in the place of u in the above equation then
$$tD(uw,v) = 0$$
$t(D(u,v)g(\sigma(w)) + \tau(u)D(w,v)) = 0\ \forall\, t,u,v,w \in N$
$$tD(u,v)g(\sigma(w)) + t\,\tau(u)D(w,v) = 0$$
From equation (1) we have $t\,\tau(u)D(w,v) = 0$ for all for all $t,u,v,w \in N$
By the automorphism τ of N, we have $t\,ND(w,v) = 0$
By the 3-primeness of N we get $t = 0\ (or)\ D = 0$

Lemma2.3: Suppose N is a near ring, D be a symmetric bi- (σ,τ) semi derivation of N ⟺ $D(tu,v) = \tau(t)D(u,v) + D(t,v)g(\sigma(u))\ \forall\, t,u,v \in N$

Proof:

Suppose D is a symmetric bi- (σ,τ) semi derivation of N, $\forall\, t,u,v \in N, \sigma$ is an automorphism we have

$D(t(u+u),v) = D(t,v)g\big(\sigma(u+u)\big) + \tau(t)D(u+u,v)$

$D(t(u+u),v) = D(t,v)g\big(\sigma(u)\big) + D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v) + \tau(t)D(u,v)$ Equation(2)

And also $D(t(u+u),v) = D(tu,v) + D(tu,v)$

$D(t(u+u),v) = D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v) + D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v)$ Equation(3)

From equations (2) and (3) we will get

$$D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v) = \tau(t)D(u,v) + D(t,v)g\big(\sigma(u)\big)$$

$D(tu,v) = \tau(t)D(u,v) + D(t,v)g\big(\sigma(u)\big)\ \forall\, t,u,v \in N$

Similarly we can prove converse.

Lemma 2.4: Suppose N is a near ring, D be a symmetric bi- (σ,τ) semi derivation of N and d be the trace of D, then $\forall\, t,u,v,w \in N$.

(i) $[D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v)]w = D(t,v)g\big(\sigma(u)\big)w + \tau(t)D(u,v)w.$

(ii) $[\tau(t)D(u,v) + D(t,v)g\big(\sigma(u)\big)]w = \tau(t)D(u,v)w + D(t,v)g\big(\sigma(u)\big)w.$

Proof:

(i) Suppose we have

$D(tu,v) = D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v)\ \forall\, t,u,v \in N$

$D((tu)w,v) = D(tu,v)g\big(\sigma(w)\big) + \tau(tu)D(w,v)$

$D((tu)w,v) = \Big(D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v)\Big)\, g\big(\sigma(w)\big) + \tau(t)\tau(u)\, D(w,v)$ Equation(4)

And also

$D(t(uw),v) = D(t,v)g\big(\sigma(uw)\big) + \tau(t)D(uw,v)$

$= D(t,v)g\big(\sigma(u)\sigma(w)\big) + \tau(t)\big(D(u,v)g\big(\sigma(w)\big) + \tau(u)\, D(w,v)\big)$

$D(t(uw),v) = D(t,v)g\big(\sigma(u)\big)g\big(\sigma(w)\big) + \tau(t)D(u,v)g\big(\sigma(w)\big) + \tau(t)\,\tau(u)\, D(w,v)$ Equation(5)

From equations (4) and (5) we have

$\Big(D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v)\Big)\, g\big(\sigma(w)\big) = D(t,v)g\big(\sigma(u)\big)g\big(\sigma(w)\big) + \tau(t)D(u,v)g\big(\sigma(w)\big)$

Since g is onto and σ,τ are automorphisms

$\Big(D(t,v)g\big(\sigma(u)\big) + \tau(t)D(u,v)\Big)w = D(t,v)g\big(\sigma(u)\big)w + \tau(t)D(u,v)\, w.$

(ii) Suppose $D(tu,v) = \tau(t)D(u,v) + D(t,v)g\big(\sigma(u)\big)\ \forall\, t,u,v \in N$

From the associative law, we have

$D((tu)w,v) = \tau(tu)D(w,v) + D(tu,v)g\big(\sigma(w)\big)$

$D((tu)w,v) = \tau(t)\,\tau(u)\, D(w,v) + \big(\tau(t)D(u,v) + D(t,v)g\big(\sigma(u)\big)\big)\, g\big(\sigma(w)\big)$

Since g is onto and σ is an automorphism then

$D((tu)w,v) = \tau(t)\,\tau(u)\, D(w,v) + \big(\tau(t)D(u,v) + D(t,v)g\big(\sigma(u)\big)\big)w$ Equation(6)

And also

$D(t(uw),v) = \tau(t)D(uw,v) + D(t,v)g\big(\sigma(uw)\big)$

$D(t(uw),v) = \tau(t)\big(\tau(u)\, D(w,v) + D(u,v)g\big(\sigma(w)\big)\big) + D(t,v)g\big(\sigma(u)\big)g\big(\sigma(w)\big)$

$D(t(uw),v) = \tau(t)\tau(u)\, D(w,v) + \tau(t)D(u,v)g\big(\sigma(w)\big) + D(t,v)g\big(\sigma(u)\big)g\big(\sigma(w)\big)$

Since g is onto and σ is an automorphism then

$D(t(uw),v) = \tau(t)\tau(u)\, D(w,v) + \tau(t)D(u,v)w + D(t,v)g\big(\sigma(u)\big)w$ Equation(7)

From equations (6) and (7) we have

$\Big(\tau(t)D(u,v) + D(t,v)g\big(\sigma(u)\big)\Big)w = \tau(t)D(u,v)w + D(t,v)g\big(\sigma(u)\big)w$

Lemma 2.5: Suppose N is a 3-prime near ring, D be a non-zero symmetric bi- $(\sigma,\tau) -$ semi derivation of N. if $D(N,N)t = 0\ \forall\, t \in N \Rightarrow t = 0$ and if $tD(N,N) = 0\ \forall\, t \in N \Rightarrow t = 0$.

Proof:

Let us consider $D(u,v)t = 0 \ \forall \ t,u,v \in N$.

Now replace u with uw and from lemma 2.4 (i),

$$D(uw,v)t = 0$$
$$\big(D(u,v)g(\sigma(w)) + \tau(u)D(w,v)\big)t = 0$$
$$D(u,v)g(\sigma(w))t + \tau(u)D(w,v)t = 0$$
$$D(u,v)g(\sigma(w))t = 0.$$

Since g is onto and by the automorphism σ of N we have

$$D(u,v)Nt = 0 \ \forall \ t,u,v \in N.$$

By the 3-primeness of N and D is non-zero,then we have $t = 0$.

If $tD(N,N) = 0$,

Then$\forall \ t,u,v \in N$, $tD(u,v) = 0$

Now replace u with uw in the above and from lemma 2.4 (i)

$$tD(uw,v) = 0 \ \forall \ t,u,v,w \in N$$
$$t(D(u,v)g(\sigma(w)) + \tau(u)D(w,v)) = 0$$
$$tD(u,v)g(\sigma(w)) + t\,\tau(u)D(w,v) = 0$$
$$t\,\tau(u)D(w,v) = 0$$

Since τ is an automorphism then we have

$$t\,ND(w,v) = 0 \ \forall \ t,u,v,w \in N$$

Since N is a 3-prime near ring and D is non-zero, we get $t = 0$

3. Main Results

Theorem 3.1: Let N be a 3-prime near ring, D be a non-zero symmetric bi- $(\sigma,\tau) -$ semi derivation of N. if N is a 2-torsion free and $D(N,N) \subset Z$ then N is a commutative ring.

Proof:

Let $D(N,N) \subset Z$ and D is a non trivial, then there exists non zero elements $t,u \in N$ such that $D(t,u) \in Z - \{0\}$, then

$$D(t,u+u) = D(t,u) + D(t,u) \in Z$$

And by lemma 2.1, N is abelian with respect to addition.

$D(t,u+u) \in Z$ for all $t,u \in N$ implies that

$$vD(t,u) = D(t,u)v \text{ for all } v \in N$$

Substitute vw in the place of v then we have

$$vD(tw,u) = D(tw,u)v$$
$$v\big(D(t,u)g(\sigma(w)) + \tau(t)D(w,u)\big) = \big(D(t,u)g(\sigma(w)) + \tau(t)D(w,u)\big)v$$

From lemma 2.4(i) and $D(N,N) \subset Z$ we have

$$D(t,u)v\,g(\sigma(w)) + D(w,u)v\,\tau(t) = D(t,u)g(\sigma(w))v + \tau(t)D(w,u)v$$

(or) N is abelian with respect to addition for all $t,u,v,w \in N$

$$D(t,u)\big[v,g(\sigma(w))\big] = D(w,u)[v,\tau(t)].$$

Since g is onto

$$D(t,u)[v,\sigma(w)] = D(w,u)[v,\tau(t)].$$

Substitute $D(x,y)$ in the place of $\sigma(w)$ for all $x,y \in N$ and since $D(x,y) \in Z$ we have

$D(D(x,y),u)[v,\tau(t)] = 0$ for all $x,y,t,u,v \in N$

Since D is non zero we have $[v,\tau(t)] = 0$ for all $t,v \in N$

Since τ is an automorphism

Therefore N is a commutative ring.

Theorem 3.2: Let N be a 3-prime near ring, D be a non-zero symmetric bi- $(\sigma,\tau)-$ semi derivation of N and d be the trace of D. If $[d(t),t]_{(\sigma,\tau)}=0$ then N is commutative with respect to addition.

Proof:
Let D is a non-zero symmetric bi- $(\sigma,\tau)-$ semi derivation then
$D(t(t+u),t)=D(t,t)g\big(\sigma(t+u)\big)+\tau(t)D(t+u,t)$
$D(t(t+u),t)=d(t)g(\sigma(t))+d(t)g\big(\sigma(u)\big)+\tau(t)D(t,t)+\tau(t)D(u,t)$
$D(t(t+u),t)=d(t)g\big(\sigma(t)\big)+d(t)g\big(\sigma(u)\big)+\tau(t)d(t)+\tau(t)D(u,t)$ Equation(8)
And also $D(t(t+u),t)=D(t^2+tu,t)=D(t^2,t)+D(tu,t)$
$D(t(t+u),t)=D(tt,t)+D(tu,t)$
$D(t(t+u),t)=D(t,t)g\big(\sigma(t)\big)+\tau(t)D(t,t)+D(t,t)g\big(\sigma(u)\big)+\tau(t)D(u,t)$
$D(t(t+u),t)=d(t)g\big(\sigma(t)\big)+\tau(t)d(t)+d(t)g\big(\sigma(u)\big)+\tau(t)D(u,t)$ Equation(9)
From equations (8) and (9) we have

$$d(t)g\big(\sigma(u)\big)+\tau(t)d(t)=\tau(t)d(t)+d(t)g\big(\sigma(u)\big)$$

Since g is onto and $[d(t),t]_{(\sigma,\tau)}=0$

$$d(t)\sigma(u)+\tau(t)d(t)=\tau(t)d(t)+d(t)\sigma(u)$$
$$d(t)\sigma(u)+d(t)\sigma(t)-d(t)\sigma(u)-d(t)\sigma(t)=0$$
$$d(t)\big(\sigma(u)+\sigma(t)-\sigma(u)-\sigma(t)\big)=0$$

$d(t)\big(\sigma(t),\sigma(u)\big)=0$ for all $t,u\in N$
Since σ is an automorphism and from lemma 2.2 we have N is abelian with respect to addition.

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