APPROXIMATE ANALYTICAL APPROACH OF COUNTER CURRENT IMBIBITION PHENOMENON IN POROUS MEDIA

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Abstract: Approximate analytical approach is used to study counter-current imbibition phenomenon in porous medium. Counter-current imbibition phenomenon in porous media takes place during the process of secondary oil recovery. For the mathematical modelling, the permeability of the porous medium is considered as a function of variable. Mathematical formulation of the phenomenon is governed by one-dimensional non-linear partial differential equation. The solution of governing equation is obtained by modified variational iteration method (MVIM).

***Index Terms* -** Imbibition, Porous Media, Counter-Current, MVIM
Ams Subject Classification -76S05**,** 78M30, 80M30

I. Introduction

A spontaneous flow of the resident fluid from the medium occurs when a porous medium is filled with some fluid which preferentially wets the medium. The counter-current imbibition phenomenon is arising due to the difference in the wetting abilities of the fluid. Counter current imbibition is one of the most important recovery mechanisms during oil recovery.

Various authors have investigated this phenomenon from different viewpoints analytically as well as numerically with different assumptions and initial and boundary conditions. Mishra, Pradhan and Mehta used Homotopy perturbation transform method (HPTM) with linear relative permeability [1], M.F.El-Amina, Amgad S. and Shuyu S. have discussed numerical and dimensional investigation of two-phase counter current imbibition in porous media [2], M.A. Patel and N. B. Desai [3] have used Homotopy analysis method with variable porosity and permeability in heterogeneous porous medium, S.Pathak and T.Singh [4] have used optimal Homotopy analysis method .

II. BASICS OF MODIFIED VARIATIONAL ITERATION METHOD

Many problems in applied sciences involved non-linear partial differential equations with initial and boundary conditions. Such problems are solved by several techniques including decomposition, variational iteration, finite difference, polynomial spline and Homotopy perturbation [5-7]. The motivation to develop other methods for solving these problems is that these methods require huge computational work. The variational iteration method (VIM) is developed by He [8-10] for solving linear and nonlinear initial and boundary value problems. The solution obtained by this method is given in an infinite series usually converging to an accurate solution [11-14]. In this paper, the solution is obtained by the modified variational iteration method (MVIM). It is shown that the MVIM provides the solution in a rapid convergent series with easily computable components [15].

To illustrate the basic concept of the Modified variational decomposition method, consider the general differential equation [16],

$$L[u(x,t)]+N[u(x,t)]=g(x,t) \tag{A}$$

Where L is a linear operator, N a non-linear operator and $g(x,t)$ is the source inhomogeneous term.
Constructing a correct functional as follow;

$$u_{n+1}(x,t)=u_n(x,t)+p\int_0^t \lambda\{Lu_n(x,\xi)+N\tilde{u}_n(x,\xi)-g(x,\xi)\}d\xi\,,\quad n\geq 0 \tag{B}$$

Where λ is a Lagrange's multiplier which can be identified optimally via variational iteration method. The subscript n denote the nth approximation and $\tilde{u}_n$ is considered as a restricted variation i.e. $\delta\tilde{u}_n=0$.
Applying Homotopy perturbation method,

$$\sum_0^\infty p^{(n)}u_n(x,t)=u_0(x,t)+p\int_0^t \lambda\left\{\sum_0^\infty p^{(n)}\left(Lu_n(x,\xi)+N\tilde{u}_n(x,\xi)\right)-g(x,\xi)\right\}d\xi$$

Which is the modified variational iteration method (MVIM) and it is formulated by the coupling of variational iteration method and He's polynomials. The comparison of like powers of p gives solutions of various orders.

III. STATEMENT OF THE PROBLEM

A finite cylindrical piece of homogenous porous matrix of length L is fully saturated with a native liquid. It is completely surrounded by an impermeable surface except for one end is exposed to an adjacent formation of injected fluid. It is assumed that injected fluid is preferentially more wetting than that of native fluid and this arrangement give rise to the phenomenon of linear counter-current imbibition, that a spontaneous linear flow of injected fluid into the medium and a counter flow of the resident fluid from the medium [17].

IV. MATHEMATICAL FORMULATION

From Darcy's law, the equations of seepage velocity of flowing fluids are written as:

$$V_i = -\frac{k_i}{\mu_i} k \frac{\partial P_i}{\partial x} \tag{1}$$

$$V_n = -\frac{k_n}{\mu_n} k \frac{\partial P_n}{\partial x} \tag{2}$$

Where V_i and V_n are seepage velocity of injected fluid and native fluid respectively, k is the permeability of the homogeneous medium, k_i and k_n are relative permetabilities of injected fluid and native fluid respectively, P_i and P_n are the pressures and μ_i and μ_n are viscosities of injected fluid and native liquid respectively.
The equations of continuity for the flowing phase are:

$$\phi \frac{\partial S_i}{\partial t} + \frac{\partial V_i}{\partial x} = 0 \tag{3}$$

$$\phi \frac{\partial S_n}{\partial t} + \frac{\partial V_n}{\partial x} = 0 \tag{4}$$

Where ϕ is the porosity of the medium and S_i and S_n are injected fluid and native fluid saturation respectively.
An analytic condition of governing imbibition phenomenon is given by

$$V_i = -V_n \tag{5}$$

$$P_C = P_n - P_i \tag{6}$$

Where P_C is the capillary pressure.
The nonlinear partial differential equation describing the imbibition phenomena in homogeneous porous medium is given as

$$\phi \frac{\partial S_i}{\partial t} + \frac{\partial}{\partial x}\left\{ k \frac{k_n k_i}{k_n \mu_i + k_i \mu_n} \frac{dP_C}{dS_i} \frac{\partial S_i}{\partial x} \right\} = 0 \tag{7}$$

Assuming $\frac{k_n k_i}{k_n \mu_i + k_i \mu_n} \approx \frac{k_n}{\mu_n}$ [18], we may write equation (7) in the form

$$\phi \frac{\partial S_i}{\partial t} + \frac{\partial}{\partial x}\left\{ k \frac{k_n}{\mu_n} \frac{dP_C}{dS_i} \frac{\partial S_i}{\partial x} \right\} = 0 \tag{8}$$

Assuming linear relative permeability for injected fluid and native liquid [19] as

$$k_i = S_i, k_n = 1 - \alpha S_i, \alpha \text{ is any scalar} \tag{9}$$

Capillary pressure as a function of saturation of injected fluid [20] is given by

$$P_C = -\beta S_i \tag{10}$$

$$\phi \frac{\partial S_i}{\partial t} - \frac{k\beta}{\mu_n} \frac{\partial}{\partial x}\left\{ (1 - \alpha S_i) \frac{\partial S_i}{\partial x} \right\} = 0 \tag{11}$$

Replacing $(1-\alpha S_i)$ by S, from equation (11), we get

$$\phi\frac{\partial S}{\partial t}-\frac{k\beta}{\mu_n}\frac{\partial}{\partial x}\left\{S\frac{\partial S}{\partial x}\right\}=0 \tag{12}$$

Using dimensionless variable $X=\frac{x}{L}$, $T=\frac{k\beta}{\phi\mu_n L^2}t$ in (12), we get

$$\frac{\partial S}{\partial T}-\frac{\partial}{\partial X}\left\{S\frac{\partial S}{\partial X}\right\}=0$$

$$\frac{\partial S}{\partial T}-S\frac{\partial^2 S}{\partial X^2}-\left(\frac{\partial S}{\partial X}\right)^2=0 \tag{13}$$

Equation (13) is the governing equation for this phenomenon.
The suitable initial and boundary conditions are given by

$$S(X,0)=f(X),\ X\geq 0 \tag{14}$$

$$S(0,T)=1,\ \frac{\partial S}{\partial X}(1,T)=0,\ T\geq 0 \tag{15}$$

V. MATHEMATICAL SOLUTION

Applying MVIM for equation (13) with initial condition (14), a correct functional is as follow;

$$S_{n+1}(X,T)=S_n(X,T)+p\int_0^t\lambda\{LS_n(X,\xi)+N\tilde{S}_n(X,\xi)-g(X,\xi)\}d\xi,\ n\geq 0$$

Applying Homotopy perturbation method,

$$\sum_0^\infty p^{(n)}S_n(X,T)=S_0(X,T)+p\int_0^t\lambda\left\{\sum_0^\infty p^{(n)}\left(LS_n(X,\xi)+N\tilde{S}_n(X,\xi)\right)-g(X,\xi)\right\}d\xi \tag{16}$$

Here $\lambda=-1,\ g(x,t)=0,\ S_0(X,T)=S(X,0)=e^{-X}$

From equation (16), we get

$$\sum_0^\infty p^{(n)}S_n(X,T)=e^{-X}+p\int_0^t\left(S_0S_{0XX}+pS_1S_{1XX}+p^2S_2S_{2XX}+\ldots\right)d\xi+p\int_0^\infty\left(S^2{}_{0X}+pS^2{}_{1X}+p^2S^2{}_{2X}+\ldots\right)d\xi$$

$$p^{(0)}S_0(X,T)+p^{(1)}S_1(X,T)+p^{(3)}S_3(X,T)+\ldots\ldots=e^{-X}+p\int_0^t\left(S_0S_{0XX}+pS_1S_{1XX}+p^2S_2S_{2XX}+\ldots\right)d\xi+p\int_0^\infty\left(S^2{}_{0X}+pS^2{}_{1X}+p^2S^2{}_{2X}+\ldots\right)d\xi$$

Comparing co-efficient of like powers of p, we get

$$p^{(0)}:S_0(X,T)=e^{-X}$$

$$p^{(1)}:S_1(X,T)=2e^{-2X}T$$

$$p^{(2)}:S_2(X,T)=32e^{-4X}\frac{T^3}{3}$$

$$p^{(3)}:S_3(X,T)=\left(\frac{32}{3}\right)^2e^{-8X}\frac{T^7}{7}$$

$$p^{(4)}:S_4(X,T)=\left(\frac{32}{21}\right)^2128e^{-16X}\frac{T^{15}}{15}$$

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The series solution is given by

$$S(X,T)=e^{-X}+2e^{-2X}T+32e^{-4X}\frac{T^3}{3}+\left(\frac{32}{3}\right)^2e^{-8X}\frac{T^7}{7}+\left(\frac{32}{21}\right)^2128e^{-16X}\frac{T^{15}}{15}+\ldots$$

Numerical values of saturation of injected fluid for different distance and time is shown in the following table.

T → X ↓	T = 0	T = 0.01	T = 0.02	T = 0.03	T = 0.04	T = 0.05
	S_i					
0.1	0.095163	0.078781	0.062356	0.045846	0.029207	0.012396
0.2	0.181269	0.167858	0.154418	0.140921	0.127337	0.113638
0.3	0.259182	0.248202	0.237204	0.226166	0.202242	0.203899
0.4	0.32968	0.320691	0.31169	0.302662	0.293596	0.284478
0.5	0.393469	0.38611	0.378743	0.371358	0.363947	0.356501
0.6	0.451188	0.445164	0.439133	0.433091	0.427031	0.420948
0.7	0.503415	0.498482	0.493546	0.488601	0.483645	0.478674
0.8	0.550671	0.546633	0.542592	0.538546	0.534491	0.530427
0.9	0.59343	0.590124	0.586816	0.583505	0.580188	0.576864
1	0.632121	0.629414	0.626706	0.623995	0.621281	0.618563

VI. GRAPHICAL ANALYSIS

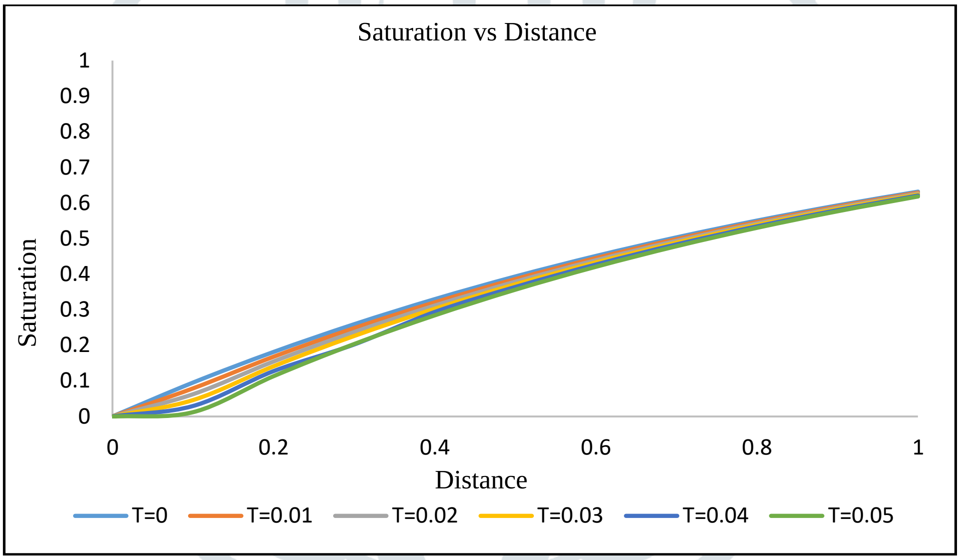

figure-1

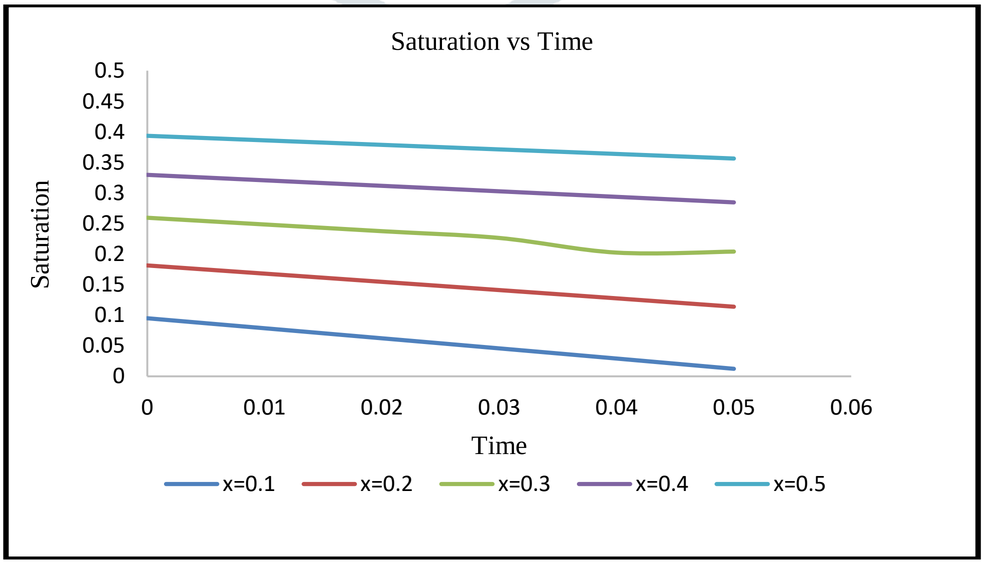

figure-2

VII. CONCLUSION

From figure-1, it is concluded that as distance increases, saturation of injected fluid increases and from figure-2, as time increases, saturation of injected fluid decreases. MVIM is used to obtain the solution. It may be concluded that the proposed method is very powerful and efficient in finding the analytical solutions for non-linear partial differential equation. The more series solution obtained by this method converge very rapidly in physical problems.

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