INTUITIONISTIC GENERALIZED j- REGULAR CLOSED SETS

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Abstract: The article comprises of new concept of generalized j – regular closed sets in topological spaces. The generalized closed set is properly placed between the generalized regular closed set and regular generalized closed set. Some of their properties have been discussed and studied.

Key words: j – regular open, gj – regular open.

I. INTRODUCTION

Regular open sets have been introduced and investigated by Stone [13] and Benchalli and Wali [3] respectively. Levine [6,7] introduced and investigated semi open sets, generalized closed sets. N. Levine introduced generalized the notion of closed sets in general topology as a generalization of closed sets. This notion was found to be useful and many results in general topology were improved. N. Palaniappan [11] studied the concept of regular generalized closed set in a topological space. D.Sasikala and I.Arockiarani initiated $\lambda\alpha$-J closed sets in generalized topological spaces. In 2012, D.Sasikala and I.Arockiarani [12], also initiated the notion of Decomposition of J-closed sets in Bigeneralized Topological spaces. In this paper, we introduce a new class of set called as generalized j – regular closed set. Also, we have acquired some of their properties and characterization.

II. PRELIMINARIES

Definition 2.1: A subset A of a space X is called

1) a preopen set if $A \subseteq int(cl(A))$ and a preclosed set if $cl(int(A)) \subseteq A$.
2) a semiopen set if $A \subseteq cl(int(A))$ and a semiclosed set if $int(cl(A)) \subseteq A$.
3) a regular open set if $A = int(cl(A))$ and a regular closed set if $A = cl(int(A))$.
4) regular semi open if there is a regular open U such $U \subseteq A \subseteq cl(U)$.

Definition 2.2: A subset A of a topological space (X,τ) is called

1) a generalized closed set (briefly g closed) [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
2) a generalized semi-closed set (briefly gs-closed) [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).

3) a semi generalized closed set (briefly sg-closed) [4] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X,τ).

4) an generalized α-closed set (briefly gα-closed) [9] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (x,τ).

5) an α-generalized closed set (briefly αg-closed) [8] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (x,τ).

6) a generalized semi-preclosed set (briefly gsp-closed) [5] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

7) a generalized preclosed set (briefly gp-closed) [10] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

8) a generalized closed set (briefly g*- closed) [14] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in (X,τ).

9) a regular generalized closed set (briefly rg-closed) [11] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X,τ).

10) a generalized regular closed set (briefly gr-closed) [4] if rcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X,τ).

Definition 2.3: A subset A of a topological space (X, τ) is called a generalized regular closed set (briefly g - r closed) if RCl(A) $\subseteq$U whenever A $\subseteq$ U and U is an open subset of X

Definition 2.4: Let (X, τ) be a generalized topological space and A $\subseteq$X, then A is said to be J- open if A $\subseteq$int (pcl(A)). The complement of τ - J-open is called as τ - J- closed.

Definition 2.5: i. A complement of gj- closed set is called gj-open.

ii. If (X, τ) is jg- closed if cl(A)$\subseteq$G whenever A$\subseteq$G and G is j-open in (X, τ). The complement of jg- closed set is called jg-open.

III. GENERALIZED j – REGULAR CLOSED SETS

Definition 3.1: A subset A of a space X is called j-regular open set if there is a regular open set U such that U$\subseteq$A$\subseteq$jcl(A). The family of all regular j-open sets of X is denoted by RjO(X).

Definition 3.2: A subset A of a topological space (X, τ) is called generalized j regular closed set (briefly gjr-closed) if jcl(A)$\subset$U whenever A$\subset$U and U is regular open in (X, τ). The family of all generalized j-regular closed sets of X is denoted by GjRC(X).

Theorem 3.3: Every regular generalized closed set (rg – closed) is gjr-closed but the converse is not true.

Proof: Consider a rg-closed set be A which is contained in X. Let $A\subset U$ and U be regular open. Then we have $cl(A)\subset U$. A is rg-closed. Since, every closed set is j- closed set, $jcl(A)\subset cl(A)$.Therefore, $jcl(A)\subset U$. Hence A is gjr-closed.

Example 3.4: Consider , X={a,b,c,d,e} τ = { ϕ ,{a,b},{c,d},{a,b,c,d},x}. Consider A={a,b}. Then U={a,b} where U is regular open. Then $A\subset U$. cl(A) = {a,b,e} and jcl(A) = {a,b}. Therefore A is gjr-closed but not rg- closed.

Theorem 3.5: Every gj –closed set is gjr-closed but the converse is not true.

Proof: Consider, a gj – closed set be A in (X,τ) and $A\subset U$ where U is regular open. Since every regular open set is open and A is gj – closed, $jcl(A)\subset U$. Hence A is gjr-closed.

Example 3.6: Consider X={a,b,c}, τ = { ϕ ,{b},{c},{b,c},x}. Consider A= {b, c}. Then U={x} where U is regular open. Then $A\subset U$. But jcl(A) = {x} which is not contained in A. Therefore A is not gj –closed.

Theorem 3.7: A subset A of a space (X,τ) is gjr-closed if and only if for each $A\subseteq U$ and U is regular open, there exists a j- closed set V such that $A\subseteq V\subseteq U$.

Proof: Consider A be a gjr-closed set, then $A\subseteq U$, where U is regular open set. Therefore, jcl (A) $\subseteq H$. If we put V = jcl (A), then $A\subseteq V\subseteq U$. Conversely assume that $A\subseteq U$ and U is regular open. Then by given, there exist a j-closed set V such that $A\subseteq V\subseteq U$. So, $A\subseteq jcl(A)\subseteq V$. Since $jcl(A)\subseteq H$. Therefore, A is gjr-closed.

Theorem 3.8: If A is closed and B is a gjr-closed subset of a space X, then $A\cup B$ is also gjr-closed.

Proof: If $A\cup B\subseteq U$ and U is regular open set. Then, we have $A\subseteq U$ and $B\subseteq U$. But B is gjr-closed, then $jcl(B)\subseteq U$ and hence $A\cup B\subseteq A\cup jcl(B)\subseteq U$. but we have, $A\cup jcl(B)$ is j-closed set, hence there exists a j-closed set $A\cup jcl(B)$ such that, $A\cup B\subseteq A\cup jcl(B)\subseteq U$. Therefore, by theorem 3.7, $A\cup B$ is gjr-closed.

Theorem 3.9: If a subset A of X is a gjr-closed set in X, then jcl (A)\A does not contain any non-empty j-open set in X.

Proof: Consider A be a gjr-closed set in X. Then, consider U be a j-open set such that, $U\subseteq jcl(A)\setminus A$ and $U\neq\phi$. Therefore, $U\subseteq jcl(A)\setminus A$. $\therefore U\subseteq X\setminus A$

$\Rightarrow \quad A\subseteq X\setminus U$.

Since U is j-open set, then X\U is also j-open in X. But A is gjr-closed set in X, then jcl (A)$\subseteq$X\U. So, U$\subseteq$ X\jcl (A). Also, U$\subseteq$jcl (U). Hence U$\subseteq$(jcl(A)$\cap$(X\jcl(A))) = ϕ.

$\Rightarrow$ U=φ which is a contradiction. Therefore, jcl (A)\A does not contain any non empty j- open set in X.

Theorem 3.10: If A is both open and g-closed subset in X, then A is gjr-closed set in X.

Proof: Assume that A is an open and g – closed subset in X and A$\subseteq$U, where U is regular open set in X. Then by given statement, jcl (A)$\subseteq$A, i.e., jcl(A)$\subseteq$U. Therefore A is a gjr – closed set in X.

Theorem 3.11: For a topological space (X,τ), if RjO(X,ϕ) = {X,ϕ}, then every subset of X is gjr-closed subset in X.

Proof: Consider a topological space (X,τ) and RjO(X,τ) = {X,ϕ}. Now, let A be a subset of X. If A = ϕ, then A is gjr-closed subset in X. If A $\neq \phi$, then X is the only regular open set in X containing A and so jcl(A)$\subseteq$X. Hence A is gjr – closed subset in X.

Theorem 3.12: Let A be a gjr-closed set in X. Then A is j-closed if and only if jcl (A)\A is regular open.

Proof: If A is j-closed in X, then jcl (A) = A then, jcl(A)\A = φ, which is regular open in X. Conversely, let jcl (A)\A is a regular open set in X. Since A is gjr-closed, then by theorem (3.9), jcl (A)\A does not contain any non empty regular open set in X. Then, jcl (A) =φ. Therefore, A is j – closed set in X.

Theorem 3.13: If A is regular open and gr – closed, then A is gjr- closed set in X.

Proof: Consider U be any regular open set in X such that A$\subseteq$U. Since A is regular open and gr- closed, then jcl (A)$\subseteq$A. Therefore, jcl (A)$\subseteq$A$\subseteq$U. Hence A is gjr-closed set in X.

Theorem 3.14: If A is gjr-closed set of X such that A$\subseteq$B$\subseteq$jcl (A), then B is gjr-closed set in X.

Proof: Let U be a j- open set of X such that B$\subseteq$U. Then A$\subseteq$U. But A is a gjr-closed set of X. Then, jcl (A)$\subseteq$U. Now, jcl(B)$\subseteq$jcl(jcl(A)) = jcl(A)$\subseteq$U. $\therefore$ B is gjr- closed set in X.

Theorem 3.15: If A is regular open and gjr-closed, then A is regular closed and hence j-clopen.

Proof: Consider A is regular open and gjr-closed. We know that, every regular open set is j-open and A$\subseteq$ A, therefore jcl (A)$\subseteq$A. But, A$\subseteq$jcl (A). $\therefore$ A = jcl (A), i.e., A is j-closed. Since A is regular open and j-open. Therefore A is regular closed and j-clopen.

Theorem 3.16: If a subset A is both regular open and gjr-closed in topological space (X, τ), then A is j – closed.

Proof: Let A is both regular open and gjr-closed in topological space (X, τ). Then jcl (A)$\subseteq$A. Hence A is j-closed.

Theorem 3.17: If A is both regular open and gjr –closed subset in X and V is a closed set in X, then A$\cap$V is a gjr – closed set in X.

Proof: Assume that A is regular open and gjr-closed subset in X and V is a closed set in X. Then by theorem (3.16), A is j- closed. So, A$\cap$V is j – closed. Therefore, A$\cap$V is a gjr- closed set in X.

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