RADIO MEAN Dd-DISTANCE NUMBER OF DEGREE SPLITTING GRAPHS

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Abstract

A Radio Mean Dd-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to the ℕ such that for two distinct vertices u and v of G, D^{Dd}(u, v) + $\left\lceil\frac{f(u)+f(v)}{2}\right\rceil$ ≥ 1 + $diam^{Dd}$(G), where D^{Dd}(u, v) denote the Dd-distance between u and v and $diam^{Dd}$(G) denotes the Dd-diameter of G. The radio mean Dd-distance number of f, rmn^{Dd}(f) is the maximum label assigned to any vertex of G. The radio mean Dd-distance number of G, rmn^{Dd}(G) is the minimum value of f of G. In this paper we find the radio mean Dd-distance number of degree splitting graphs.

Keywords: Dd-distance, Radio mean Dd-distance, Radio Dd-distance number.

Subject Classification (2010): 05C78, 05C15

1. Introduction

By a graph G = (V(G), E(G)) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

The Dd-distance was introduced by A. Anto Kinsley and P. Siva Ananthi [1]. For a connected graph G, the Dd-length of a connected u – v path is defined as D^{Dd}(u, v) = D(u, v) + deg(u) + deg(v). The Dd-radius, denoted by r^{Dd}(G) is the minimum Dd-eccentricity among all vertices of G. That is r^{Dd}(G) = min{e^{Dd}(G) : v ∈ V(G)}. Similarly the Dd-diameter, D^{Dd}(G) is the maximum Dd-eccentricity among all vertices of G. We observe that for any two vertices u, v of G, we have d(u, v) ≤ D^{Dd}(u, v). The equality holds if and only if u, v are identical. If G is any connected graph then the Dd-distance is a metric on the set of vertices of G. We can check easily r^{Dd}(G) ≤ D^{Dd}(G) ≤ 2r^{Dd}(G). The lower bound is clear from the definition and the upper bound follows from the triangular inequality.

We introduced the concept of radio mean Dd-distance colouring of a graph G. Radio mean Dd-distance coloring is a function f : V(G) → ℕ such that D^{Dd}(u, v) + $\left\lceil\frac{f(u)+f(v)}{2}\right\rceil$ ≥ $diam^{Dd}$(G) + 1, where $diam^{Dd}$(G) is the Dd-distance diameter of G. A Dd-distance radio coloring number of G is the maximum color assigned to any vertex of G. It is denoted by rmn^{Dd}(G).

Radio labeling can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by W. K. Hale [6]. G. Chartrand et al.[2] introduced the concept of radio labeling of graph. Also G. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [10]. However G. Chartrand et al.[2] obtained different values for them. They found the lower and upper bound for the radio number of cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. M. M. Rivera et al.[21] gave the radio number of $C_n \times C_n$, the Cartesian product of C_n. In [4] C. Fernandez et al. found the radio number for Complete graph, Star graph, Complete Bipartite graph, Wheel graph and Gear graph. M. T. Rahim and I. Tomescu [17] investigated the radio number of Helm graph. The radio number for the generalized prism graphs were presented by Paul Martinez et al. in [11]. In this paper, we fined the radio mean Dd-distance coloring of degree splitting graphs.

2. Main Results

Theorem 2.1

The radio mean Dd-distance number of degree splitting graph of path DS(P_n), rmn^{Dd}(DS(P_n)) ≤ 2n – 5, n ≥ 7.

.Proof.

Let V(DS(P_n)) = {w_1, w_2, v_i : 1≤ i ≤ n }and E(DS(P_n)) = $v_i v_{i+1}$: 1 ≤ i ≤ n – 1, $v_1 w_2, v_n w_2, w_1 v_i$: 2≤ i ≤ n – 1}.Then D^{Dd}(v_1,w_2) = n + 5, D^{Dd}(v_2,v_3) = n + 7, D^{Dd}(v_2,w_1) = 2n + 2. So diam^{Dd}(DS(P_n)) = 2n + 2.

Without loss of generality, Let, f(w_1) < f(v_2) < f(v_3) < . . . < f(v_n) < f(v_1) < f(w_2).

The radio mean Dd-distance condition is D^{Dd}(u, v) + $\left\lceil\frac{f(u)+f(v)}{2}\right\rceil$ ≥ diam^{Dd}(G) + 1.

Now, D^{Dd}(w_1, v_2) + $\left\lceil\frac{f(w_1)+f(v_2)}{2}\right\rceil$ ≥ diam^{Dd}(DS(P_n)) + 1

f(w_1) + f(v_2) ≥ 1, which implies f(w_1) = n – 6 and f(v_2) = n – 5.

D^{Dd}(v_2, v_3) + $\left\lceil\frac{f(v_2)+f(v_3)}{2}\right\rceil$ ≥ diam^{Dd}($DS(P_n)$) + 1, f(v_2) + f(v_3) ≥ 2n – 9. Therefore, f(v_3) = n – 4.

f(v_i) = n + i – 7, 2 ≤ i ≤ n – 1.

D^{Dd}(v_{n-1}, v_n) + $\left\lceil\frac{f(v_{n-1})+f(v_n)}{2}\right\rceil$ ≥ diam^{Dd}($DS(P_n)$) + 1, f(v_{n-1}) + f(v_n) ≥ 2n – 7, which implies f(v_n) = 2n – 7.

D^{Dd}(v_n, v_1) + $\left\lceil\frac{f(v_n)+f(v_1)}{2}\right\rceil$ ≥ diam^{Dd}($DS(P_n)$) + 1, f(v_n) + f(v_1) ≥ 2n – 3, which implies f(v_1) = 2n – 6.

D^{Dd}(v_1, w_2) + $\left\lceil\frac{f(v_1)+f(w_2)}{2}\right\rceil$ ≥ diam^{Dd}($DS(P_n)$) + 1, f(v_1) + f(w_2) ≥ 2n – 5, which implies f(w_2) = 2n – 5.

Therefore, rmn^{Dd}($DS(P_n)$) ≤ 2n – 5, n ≥ 7. ■

Note: mn^{Dd}($DS(P_n)$) ≤ n + 2, 4 ≤ n ≥ 6.

Theorem.2.2

The radio mean Dd-distance number of degree splitting of a star graph DS($K_{1,n}$) is

rmn^{Dd} (DS($K_{1,n}$)) ≤ 3n – 7, n ≥ 5.

Proof.

Let V(DS($K_{1,n}$)) = {v_0, v_1, v_2, . . . , v_n}and E(DS($K_{1,n}$)) ={$v_0 v_i$, $w v_i$: 1 ≤ i ≤ n }

Then, D^{Dd}(v_0, v_i) = n + 5, D^{Dd}(v_i, w) = n + 5 and D^{Dd}(v_0, w) = 2n + 2. So,diam^{Dd}(DS($K_{1,n}$)) = 2n + 2. Without loss of generality, Let, f(v_0) < f(w) < f(v_1) < f(v_2) < . . . < f(v_n).

Now, D^{Dd}(v_0, w) + $\left\lceil\frac{f(v_0)+f(w)}{2}\right\rceil$ ≥ 2n + 3, f(v_0) + f(w) ≥ 1, f(v_0) = 2n – 8 and f(w) = 2n – 7.

D^{Dd}(w, v_1) + $\left\lceil\frac{f(w)+f(v_1)}{2}\right\rceil$ ≥ 2n + 3, f(w) + f(v_1) ≥ 2n – 5, which implies that f(v_1) = 2n – 6.

D^{Dd}(v_1, v_2) + $\left\lceil\frac{f(v_1)+f(v_2)}{2}\right\rceil$ ≥ 2n + 3, f(v_1) + f(v_2) ≥ 4n – 11, which implies that f(v_2) = 2n – 5.

D^{Dd}(v_2, v_3) + $\left\lceil\frac{f(v_2)+f(v_3)}{2}\right\rceil$ ≥ 2n + 3, f(v_2) + f(v_3) ≥ 4n – 11, which implies that f(v_3) = 2n – 4.

f(v_i) = 2n + i – 7, 1 ≤ i ≤ n. Hence rmn^{Dd} (DS($K_{1,n}$)) ≤ 3n – 7, n ≥ 5. ■

Note: rmn^{Dd}($DS(K_{1,n})$) ≤ n + 2, 2 ≤ n ≤ 4.

Theorem 2.3

The radio mean Dd-distance number of degree splitting of a bistar graph DS($B_{n,n}$), rmn^{Dd}(DS($B_{n,n}$)) ≤ 5n – 3, n ≥ 3.

.**Proof.**

Let V (DS ($B_{n,n}$)) = {u, v, w_1, w_2, u_i, v_i : 1≤ i ≤ n } and E (DS ($B_{n,n}$)) = {uu_i, vv_i, w_1u_i, w_1v_i, uv, uw_2, w_2v : 1≤ i ≤ n}.

Then D^{Dd}(u, v) = 2n + 8, D^{Dd}(u, w_1) = 3n + 6, D^{Dd}(w_1, w_2) = 2n + 6

D^{Dd}(u,w_2) = D^{Dd}(w_2,v) = D^{Dd}(u,u_i) = D^{Dd}(v,v_i) = n + 9, 1≤ i ≤ n, D^{Dd}(v_i,v_j) = D^{Dd}(u_i,u_j) = 10, i ≠ j

D^{Dd}(w_1,u_i) = D^{Dd}(w_1,v_i) = 2n + 7, 1≤ i ≤ n. So diam^{Dd}(DS($B_{n,n}$)) = 3n + 6

Without loss of generality, Let, f(w_1) < f(u) < f(v) < f(v_1) < . . . < f(v_n) < f(u_1) < . . . < f(u_n) < f(w_2).

Now, D^{Dd}(w_1, u) + $\left\lceil \frac{f(w_1)+f(u)}{2} \right\rceil$ ≥ diam^{Dd}(DS($B_{n,n}$)) + 1, f(w_1) + f(u) ≥ 1, which implies f(w_1) = 3n – 6 and f(u) = 3n – 5.

Now, D^{Dd}(u, v) + $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ ≥ diam^{Dd}(DS($B_{n,n}$)) + 1, f(u) + f(v) ≥ 1, which implies f(v) = 3n – 4.

D^{Dd}(v, v_1) + $\left\lceil \frac{f(v)+f(v_1)}{2} \right\rceil$ ≥ diam^{Dd}($DS(B_{n,n})$) + 1, f(v) + f(v_1) ≥ 4n – 5, which implies f(v_1) = 3n – 3.

D^{Dd}(v_1, v_2) + $\left\lceil \frac{f(v_1)+f(v_2)}{2} \right\rceil$ ≥ diam^{Dd}($DS(B_{n,n})$) + 1, f(v_1) + f(v_2) ≥ 6n – 7, which implies f(v_2) = 3n – 2.

D^{Dd}(v_2, v_3) + $\left\lceil \frac{f(v_2)+f(v_3)}{2} \right\rceil$ ≥ diam^{Dd}($DS(B_{n,n})$) + 1, f(v_2) + f(v_3) ≥ 6n – 7 which implies that f(v_3) = 3n – 1.

so f(v_i) = 3n + i – 4, 1 ≤ i ≤ n.

D^{Dd}(v_n, u_1) + $\left\lceil \frac{f(v_n)+f(u_1)}{2} \right\rceil$ ≥ 3n + 7, f(v_n) + f(u_1) ≥ 6n – 7, which implies that f(u_1) = 4n – 3

D^{Dd}(u_1, u_2) + $\left\lceil \frac{f(u_1)+f(u_2)}{2} \right\rceil$ ≥ 3n + 7, f(u_1) + f(u_2) ≥ 6n – 7, which implies that f(u_2) = 4n – 2.

so f(u_i) = 4n + i – 4, 1 ≤ i ≤ n.

D^{Dd}(u_n, w_2) + $\left\lceil \frac{f(u_n)+f(w_2)}{2} \right\rceil$ ≥ 3n + 7, f(u_n) + f(w_2) ≥ 6n – 3, which implies that f(w_2) = 5n – 3.

Hence, rmn^{Dd}($DS(B_{n,n})$) ≤ 5n – 3, n ≥ 3. ∎

Note. If n = 2, then, rmn^{Dd}($DS(B_{n,n})$) = 8.

Theorem.2.4

The radio mean Dd-distance number of degree splitting of wheel graph DS(W_n) is

rmn^{Dd} (DS(W_n)) ≤ 3n – 7, n ≥ 6.

Proof.

Let V(DS(W_n)) = {u, v, v_1,v_2, . . . , v_n}and E(DS(W_n)) = {uv_i, vv_i, v_iv_{i+1}: 1 ≤ i ≤ n }

Then D^{Dd}(u, v) = 3n + 1, D^{Dd}(v_1, v_2) = n + 8, D^{Dd}(u, v_i) = D^{Dd}(v, v_i) = 2n + 5, 1 ≤ i ≤ n

So, diam^{Dd}(DS(W_n)) = 3n + 1. Without loss of generality, Let, f(u) < f(v) < f(v_1) < . . . < f(v_n).

Now, D^{Dd}(u, v) + $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ ≥ 3n + 2, f(u) + f(v) ≥ 1, which implies that f(u) = 2n – 8 and f(v) = 2n – 7

D^{Dd}(v, v_1) + $\left\lceil \frac{f(v)+f(v_1)}{2} \right\rceil$ ≥ 3n + 2, f(v) + f(v_1) ≥ 2n – 7, which implies that f(v_1) = 2n – 6.

D^{Dd}(v_1, v_2) + $\left\lceil \frac{f(v_1)+f(v_2)}{2} \right\rceil$ ≥ 3n + 2, f(v_1) + f(v_2) ≥ 4n – 13, which implies that f(v_2) = 2n – 5.

D^{Dd}(v_2, v_3) + $\left\lceil \frac{f(v_2)+f(v_3)}{2} \right\rceil$ ≥ 3n + 2, f(v_2) + f(v_3) ≥ 4n – 13, which implies that f(v_3) = 2n – 4 so $f(v_i) = 2n + i - 7, 1 \leq i \leq n$. Hence, $rmn^{Dd}(DS(W_n)) \leq 3n - 7, n \geq 6$. ■

Theorem.2.5

The radio mean Dd-distance number of degree splitting of $K_2 + mK_1$ is $rmn^{Dd}(DS(K_2 + mK_1)) \leq 3m - 3, m \geq 4$

Proof.

Let $V(DS(K_2 + mK_1)) = \{u, v, w_1, w_2, v_1, v_2, . . . , v_m\}$,and $E(DS(K_2 + mK_1)) = \{uv, uw_2, vw_2, uv_i, vv_i, w_1v_i : 1 \leq i \leq m\}$.

Then $D^{Dd}(u, v) = 2m + 8$, $D^{Dd}(v, v_1) = m + 10$, $D^{Dd}(v_1, v_2) = 12$, $D^{Dd}(v_m, w_1) = m + 8$, $D^{Dd}(w_1, w_2) = m + 7$

So, $diam^{Dd}(DS(K_2 + mK_1)) = 2m + 8$. Without loss of generality, Let, $f(u) < f(v) < f(v_1) < . . . < f(v_n) < f(w_1) < f(w_2)$.

Now, $D^{Dd}(u, v) + \left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 2m + 9$, $f(u) + f(v) \geq 1$, which implies $f(u) = 2m - 6$, $f(v) = 2m - 5$.

$D^{Dd}(v, v_1) + \left\lceil\frac{f(v)+f(v_1)}{2}\right\rceil \geq 2m + 9$, $f(v) + f(v_1) \geq 2m - 3$,which implies that $f(v_1) = 2m - 4$.

$D^{Dd}(v_1, v_2) + \left\lceil\frac{f(v_1)+f(v_2)}{2}\right\rceil \geq 2m + 9$, $f(v_1) + f(v_2) \geq 4m - 7$, which implies that $f(v_2) = 2m - 3$.

$D^{Dd}(v_2, v_3) + \left\lceil\frac{f(v_2)+f(v_3)}{2}\right\rceil \geq 2m + 9$, $f(v_2) + f(v_3) \geq 4m - 7$, which implies that $f(v_3) = 2m - 2$.

so $f(v_i) = 2m + i - 5, 1 \leq i \leq m$.

$D^{Dd}(v_m, w_1) + \left\lceil\frac{f(v_m)+f(w_1)}{2}\right\rceil \geq 2m + 9$, $f(v_m) + f(w_1) \geq 2m + 1$,which implies that $f(w_1) = 3m - 4$.

$D^{Dd}(w_1, w_2) + \left\lceil\frac{f(w_1)+f(w_2)}{2}\right\rceil \geq 2m + 9$, $f(w_1) + f(w_2) \geq 2m + 3$,which implies that $f(w_2) = 3m - 3$.

Hence, $rmn^{Dd}(DS(K_2 + mK_1) \leq 3m - 3, m \geq 4$. ■

Note. $rmn^{Dd}(DS(K_2 + mK_1)) \leq m + 4, m = 2, 3$.

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