Investigation of Polynomial Functions by Galerkin Method for Flexure of Thin Rectangular Isotropic Plate.

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Abstract: The viability of coordinate multi-term polynomial functions for flexural analysis of thin rectangular isotropic plate subjected to a uniformly distributed load is herein dealt with. The rectangular plate has two opposite edges fixed and the other two opposite edges simply supported. Polynomial deflection functions with unknown coefficients, which satisfy the geometric and natural boundary conditions, corresponding to the first, second, truncated third and third approximations were derived. These deflection functions were substituted into the fourth order governing differential equation of the plate and using the Galerkin method, the unknown deflection coefficients were evaluated for the different approximations. With the help of these determined coefficients, the maximum deflection coefficient values and their corresponding maximum span moment coefficient values were determined for aspect ratios ranging from 1.0 to 2.0 for each approximation. The results of each approximation were compared with the results of the classical solution and their accuracy and convergence observed. The deflection coefficient values for the four different approximations showed close agreement with the classical solution; only the first approximation showed a good response pattern with respect to the maximum span moment coefficient values.

Keywords: Uniform load, Coordinate polynomial, Deflection function, Rectangular plate, Boundary conditions, Opposite edges, Galerkin method.

Notation

A	surface area of plate
a	primary axis of plate
b	secondary axis of plate
D	flexural rigidity
E	young modulus
h	plate thickness
M	maximum bending moment
p	aspect ratio (p = b/a)
q	external load
w (.) $\overline{w}$ (.)	deflection function, trial function
α	maximum deflection coefficient
β	maximum moment coefficient
ν	poisson's ratio

1. Introduction

The nature of engineering structures means that the end supports of rectangular plates could be subjected to different conditions namely, clamped, simple support, free etc. or a combination of these. These edge conditions make one rectangular plate unique from the other. A rectangular plate with two opposite edges simply supported and the other two clamped has found various applications in construction and engineering and some researchers have done different researches on them (Timoshenko and Woinowsky-Krieger, 1959, Aginam et al., 2012).

However, the theory of plate bending has enjoyed currency since man started using scientifically proven methods to build. Advances in this area date back to early nineteenth century when Navier (1823) used the double trigonometric series to obtain the first solution to the problem of the bending of simply supported rectangular plates. Levy (1899) suggested an alternative solution for the bending of rectangular plates that have two opposite edges simply supported and the other two edges arbitrary. Later, Nadai (1925) simplified the work done by Levy (1899) for uniformly loaded and simply supported rectangular plates. A more straight forward way to satisfy some particular boundary conditions was formulated by Papkovitch (1941). But these earlier solutions could not easily handle complex plate problems; hence, approximate methods were established. Approximate methods could be divided into two, namely numerical and analytical methods. The analytical methods consist most of the powerful tools of mathematical physics for the engineer and the Galerkin method is prominent amongst these (Szilard, 2004). With increasing application of rectangular plates in engineering and construction, many scholars are seeking for ways to better understand the behaviour of rectangular plates and make the mechanical properties converge faster. Polynomial, trigonometric, or hyperbolic deflection functions have been used in different approximate methods to solve for the bending of rectangular plates (Wojtaszak, 1937, Taylor and Govindjee, 2004, Wang and El-Sheikh, 2005, Imark and Gerdemeli, 2007). Mikhlin (1964) used the Ritz method to derive the corresponding one-term solution for a rectangular plate and the three-term solution for a square plate and got fairly good results. Some scholars have argued that increasing the number of terms of the deflection function increases the accuracy and convergence of the solutions (Mbakogu and Pavlovic, 2000, Osadebe and Aginam, 2011, Aginam et al., 2018).

To this end, this research applies the Galerkin method to the problem of a uniformly-loaded isotropic rectangular plate with two opposite edges clamped and the other two opposite edges simply supported. Different approximations of the deflection function of the plate corresponding to the first, second, truncated third and third approximations are examined in detail. The results of maximum deflections and span moments for each approximation are compared with the results of the classical solution (Timoshenko and Woinowsky-krieger, 1970) and their behaviours discussed in terms of how convergent or otherwise they are.

2. Theoretical background

The classical plate theory assumes that the material is elastic and that the stress normal to the middle plane σ_z is small and may be neglected. Hence, Hooke's law is obeyed two-dimensionally. The stress and displacement relations can be stated as (Birman, 2011):

$$\sigma_x = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \quad (1a)$$

$$\sigma_y = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \quad (1b)$$

$$\tau_{xy} = -\frac{Ez}{1+\nu}\frac{\partial^2 w}{\partial x \partial y} \quad (1c)$$

The moment-stress relations are calculated thus,

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{+h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} zdz \tag{2}$$

Integrating equation (2) over the thickness of the plate gives:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \tag{3a}$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \tag{3b}$$

$$M_{xy} = M_{yx} = -D\,(1-\nu)\,\frac{\partial^2 w}{\partial x\,\partial y} \tag{3c}$$

Where $D = Eh^3/12(1-\nu^2)$ is the flexural rigidity of the plate, E is the Young modulus, G is the shear modulus and ν is the Poisson's ratio.

But the general equation of plate is given as (Ventsel and Krauthammer, 2001):

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_y}{\partial y^2} = -P(x,y) \tag{4}$$

Substituting equations 3 (a-c) into the general equation of plate element yields the governing differential equation of isotropic plate as:

$$\frac{\partial^2 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{-P(x,y)}{D} \tag{5}$$

Where P is the applied lateral load.

3. Methodology

The approximation method adopted for this research is Galerkin. However, the Galerkin formulation of plate bending problem for an isotropic rectangular plate is given in Cartesian coordinate as follows:

$$\iint_A \left(D\frac{\partial^4 w}{\partial x^4} + 2D\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D\frac{\partial^4 w}{\partial y^4} - q\right)\bar{w}_1(x,y)dxdy = 0$$

$$\iint_A \left(D\frac{\partial^4 w}{\partial x^4} + 2D\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D\frac{\partial^4 w}{\partial y^4} - q\right)\bar{w}_2(x,y)dxdy = 0$$

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$$\iint_A \left(D\frac{\partial^4 w}{\partial x^4} + 2D\frac{\partial^4 w}{\partial x^2\,\partial y^2} + D\frac{\partial^4 w}{\partial y^4} - q\right)\bar{w}_N(x,y)dxdy = 0 \tag{6}$$

The integrals are evaluated over the entire surface area A of the plate and $\bar{w}_{1\ldots N}(x,y)$ are the linearly independent displacement functions that satisfy all the prescribed boundary conditions but not necessarily equation (5). w(x,y) is the plate deflection function which is being approximated in this study as an n-term polynomial, viz:

$w(x,y) = C_1X_1(x)Y_1(y) + C_2X_2(x)Y_2(y) + C_3X_3(x)Y_3(y) \ldots + C_nX_n(x)Y_n(y)$ (7)

Where $X_1, X_2, X_3 \ldots X_n$ and $Y_1, Y_2, Y_3, \ldots Y_n$ are derived coordinate functions in x and y axes respectively.

Equation (7) could be simplified further by putting

$\bar{w}_1 = X_1(x)Y_1(y)$

$$\bar{w}_2 = X_2(x)Y_2(y)$$
$$\bar{w}_3 = X_3(x)Y_3(y)$$
$$\vdots$$
$$\bar{w}_n = X_n(x)Y_n(y) \qquad (8)$$

Substituting equation (8) into equation (7), we obtain

$$w(x,y) = C_1\bar{w}_1 + C_2\bar{w}_2 + C_3\bar{w}_3 \ldots + C_n\bar{w}_n \qquad (9a)$$
$$w(x,y) = \bar{w}C \qquad (9b)$$

where $\bar{w} = [\bar{w}_1 \ \bar{w}_2 \ \bar{w}_3 \ \bar{w}_4 \ \bar{w}_5 \ \bar{w}_6]$ and $C = [C_1 \ C_2 \ C_3 \ C_4 \ C_5 \ C_6]$

Substituting equations 9 (a-b) into equation (6) and differentiating accordingly gives:

$$a_{11} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4\bar{w}_1}{\partial x^4} + 2\frac{\partial^4\bar{w}_1}{\partial x^2\,\partial y^2} + \frac{\partial^4\bar{w}_1}{\partial y^4}\right]\bar{w}_1(x,y)dxdy$$
$$a_{12} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4\bar{w}_1}{\partial x^4} + 2\frac{\partial^4\bar{w}_1}{\partial x^2\,\partial y^2} + \frac{\partial^4\bar{w}_1}{\partial y^4}\right]\bar{w}_2(x,y)dxdy$$
$$a_{13} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4\bar{w}_1}{\partial x^4} + 2\frac{\partial^4\bar{w}_1}{\partial x^2\,\partial y^2} + \frac{\partial^4\bar{w}_1}{\partial y^4}\right]\bar{w}_3(x,y)dxdy$$
$$\vdots$$
$$a_{nm} = \frac{D}{a^4}\iint_A \left[\frac{\partial^4\bar{w}_n}{\partial x^4} + 2\frac{\partial^4\bar{w}_n}{\partial x^2\,\partial y^2} + \frac{\partial^4\bar{w}_n}{\partial y^4}\right]\bar{w}_m(x,y)dxdy \qquad (10)$$

Similarly, for the external load, we have:

$$b_1 = \iint_A q\bar{w}_1(x,y)dxdy$$
$$b_2 = \iint_A q\bar{w}_2(x,y)dxdy$$
$$b_3 = \iint_A q\bar{w}_3(x,y)dxdy$$
$$\vdots$$
$$b_n = \iint_A q\bar{w}_n(x,y)dxdy \qquad (11)$$

In matrix form, the above formulation gives:

$$\begin{bmatrix} a_{1,1} & a_{1,2} & \cdot & \cdot & \cdot & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdot & \cdot & \cdot & a_{2,m} \\ & & \cdot & & & \\ & & \cdot & & & \\ & & \cdot & & & \\ a_{n,1} & a_{n,2} & \cdot & \cdot & \cdot & a_{n,m} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix} \frac{q}{D}a^4 \qquad (12)$$

On evaluation of the unknown coefficients, C_1, C_2, . . . and C_n from equation (12), the coefficients are substituted into equation (7) to get the corresponding deflection of the plate. The moments are evaluated from equations 3(a) and 3(b).

4. Analysis of Plate

The deflected middle surface of a uniformly loaded rectangular plate as shown in Fig. 1 can be approximated using a grid work of beams. The plate has two opposite edges clamped and the other two opposite edges simply supported. The poisson's ratio is taken as ν = 0.3.

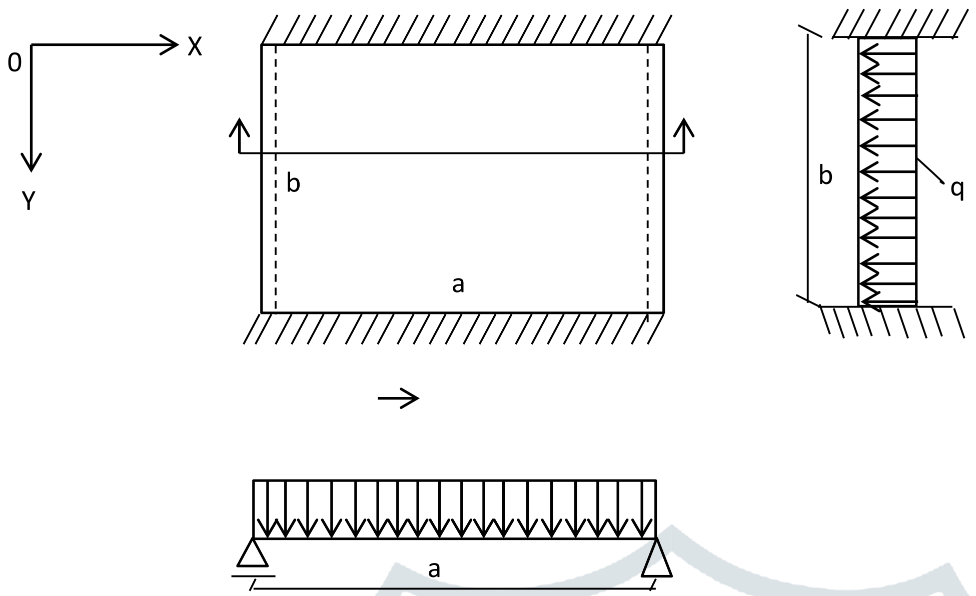

Figure 1: A rectangular plate with two opposite edges clamped and the other two opposite edges simply supported subjected to a uniformly distributed load.

The appropriate deflection function must satisfy at least two prescribed conditions at each boundary point. For the plate shown in Fig. 1, the boundary conditions are:

$$w(x) = \frac{\partial^2 w}{\partial x^2}(x) = 0 \qquad \text{at } x = 0, 1 \tag{13a}$$

$$w(y) = \frac{\partial w}{\partial y}(y) = 0 \qquad \text{at } y = 0, 1 \tag{13b}$$

It is assumed that the deflection function w can be represented in the form of polynomials as follows:

$$w(x) = \sum_{m=0}^{\infty} E_m x^m \tag{14a}$$

$$w(y) = \sum_{n=0}^{\infty} F_n y^n \tag{14b}$$

Where x^m and y^n denote complete sets of independent continuous functions suitable for the representation of the deflected surface. Coefficients $E_m$and $F_n$are determined from the prescribed boundary conditions of the plate while m and n are determined by the type of loading on the plate.

The deflection function is given as the product of the two beam functions in x and y axes, thus:

$$w\,(x, y) = w(x)\,.\,w(y) \tag{15}$$

First approximation

For this approximation, the deflection function is given as follows:

$$w\,(x, y) = C_1 \overline{w}_1 \tag{16}$$

Where C_1 is the unknown coefficient,

$$\overline{w}_1 = C_1(X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) \tag{17}$$

The solution is sought by substituting equation (16) into equations (10) and (11) and evaluating the integrals over the entire area A of the plate. The resulting linear equation is solved for the unknown coefficient, C_1. Then, C_1 is substituted into equation (16) to get the deflection of the plate at any arbitrary point (x,y). The associated moments are determined by substituting corresponding values of deflection into equations (3a) and (3b) and solving accordingly. Different values of deflection and the corresponding moments at the center of the plate are evaluated for aspect ratios $1.0 \leq p \leq 2.0$ and the results are tabulated in Tables 1, 2 and 3 for the deflection, short-span moment and long-term moment respectively.

Second approximation

Here a three-term polynomial for the deflection function is derived as follows:

$$w(x,y) = C_1\overline{w}_1 + C_2\overline{w}_2 + C_3\overline{w}_3 \tag{18}$$

where $\overline{w}_1$ has been defined in equation (17) while

$$\left.\begin{aligned} \overline{w}_2 &= (X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4)X^2 \\ \overline{w}_3 &= (X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4)Y^2 \end{aligned}\right\} \tag{19}$$

Therefore,

$$\begin{aligned} w(x,y) = {} & C_1(X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) + C_2\left(X^3 - 2X^5 + X^6\right)(Y^2 - 2Y^3 + Y^4) + \\ & C_3(X - 2X^3 + X^4)(Y^4 - 2Y^5 + Y^6) \end{aligned} \tag{20}$$

By substituting equation (20) into equations (10) and (11), the resulting 3 x 3 algebraic equation is solved for the unknown coefficients $C_1, C_2,$ and C_3. The determined coefficients are substituted into equation (20) to get the deflection coefficient values at any arbitrary point of the plate. The moment coefficient values are obtained by substituting the deflection values into equations 3 (a-b) and solving accordingly. The results are tabulated in Tables 1, 2 and 3 for the deflection, short-span moment and long-term moment respectively.

Truncated third approximation

The deflection function for this approximation will be represented by a four-term polynomial as follows:

$$w(x,y) = C_1\overline{w}_1 + C_2\overline{w}_2 + C_3\overline{w}_3 + C_4\overline{w}_4 \tag{21}$$

where $\overline{w}_1, \overline{w}_2$ and $\overline{w}_3$ are defined by equations (17) and (19) while

$$\overline{w}_4 = (X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4)X^2Y^2 \tag{22}$$

Therefore,

$$\begin{aligned} w(x,y) = {} & C_1(X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) + C_2\left(X^3 - 2X^5 + X^6\right)(Y^2 - 2Y^3 + Y^4) + \\ & C_3(X - 2X^3 + X^4)(Y^4 - 2Y^5 + Y^6) + C_4(X^3 - 2X^5 + X^6)(Y^4 - 2Y^5 + Y^6) \end{aligned} \tag{23}$$

The unknown coefficients C_1, C_2, C_3 and C_4 are determined by solving the 4 X 4 algebraic equations obtained by substituting equation (23) into equations (10) and (11). The maximum deflections and moments are determined as before by substituting the obtained coefficients into equation (23) for the deflection, and substituting the deflection into equations 3 (a-b) and solving accordingly gives the moments. The results are tabulated in Tables 1, 2 and 3 for the deflection, short-span moment and long-term moment respectively.

Third approximation

For this approximation, the deflection function is written as follows:

$$w(x,y) = C_1\overline{w}_1 + C_2\overline{w}_2 + C_3\overline{w}_3 + C_4\overline{w}_4 + C_5\overline{w}_5 + C_6\overline{w}_6 \tag{24}$$

where $\overline{w}_1, \overline{w}_2, \overline{w}_3$ and $\overline{w}_4$ are defined by equations (17), (19) and (22) while

$$\left.\begin{aligned}\overline{w}_5 &= (X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4)X^4 \\ \overline{w}_6 &= (X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4)Y^4\end{aligned}\right\} \tag{25}$$

Therefore,

$$w(x, y) = C_1(X - 2X^3 + X^4)(Y^2 - 2Y^3 + Y^4) + C_2\left(X^3 - 2X^5 + X^6\right)(Y^2 - 2Y^3 + Y^4) + C_3(X - 2X^3 + X^4)(Y^4 - 2Y^5 + Y^6) + C_4(X^3 - 2X^5 + X^6)(Y^4 - 2Y^5 + Y^6) + C_5(X^5 - 2X^7 + X^8)(Y^2 - 2Y^3 + Y^4) + C_6(X - 2X^3 + X^4)(Y^6 - 2Y^7 + Y^8) \tag{26}$$

Lastly, equation (26) is substituted into equations (10) and (11) and the ensuing 6 X 6 algebraic equation is evaluated for the unknown coefficients in the deflection function. Subsequently, the determined coefficients are put in equation (26) to evaluate the deflection at any point of the plate. The moment coefficient values are in turn determined from equations 3 (a) and 3 (b). The results so obtained are tabulated in Tables 1, 2 and 3 for the deflection, short-span moment and long-term moment respectively.

5. Results and Discussion

Deflection

Table 1 shows the deflection coefficient values for the different approximations considered for plate aspect ratios $1.0 \leq p(b/a) \leq 2.0$ along with the results of the classical solution. The accuracy of the results obtained from the first approximation is satisfactory. The percentage difference when compared with the results of classical solution remained fairly constant from 3.65 (at p = 1.0) to 4.86 (at p = 2.0). This shows minor divergence as the aspect ratio increases from 1.0 to 2.0. These deflection coefficient values are all in upper-bound. Interestingly, the deflection coefficient values for the second approximation and the truncated third approximation are almost the same for the present formulation. The percentage difference with the classical solution gives 0.84 (at p =1) to about 5.20 (at p =2) for both approximations, which shows more divergence than the first approximation. The coefficient values for both approximations are all lower-bound. For the first three approximations, the coefficient values show slight divergence as the aspect ratio increases from 1.0 to 2.0. The third approximation shows a mixed behaviour as the coefficient values are upper-bounded from aspect ratio 1.0 to 1.5 and lower-bounded from 1.6 to 2.0. This means that the coefficient values converged from aspect ratio 1.0 to 1.5 and then diverged from 1.6 to 2.0. Fig. 2 shows the graphical representation of the results of the present formulation for deflection in comparison with the results of the classical solution. It can be observed that the four different approximations show close agreement with the classical solution. The slope of the first approximation however, is the closest to the classical solution of all the four approximations considered.

Table 1: Mid-span (X =0.5, Y =0.5) Deflection Coefficient Values, α, for the Present Study in Comparison with the Classical Solution at Varying Aspect ratio ($W_{max} = (\alpha q a^4/D)$).

Aspect ratio, P	Present Study				Classical Solution
	W_1	W_2	W_3	W_4	W
	First Approximation	Second Approximation	Truncated Third Approximation	Third Approximation	Timoshenko and Woinowsky-Krieger (1970)
1.0	0.00199(3.65%)	0.00190(-0.84%)	0.00190(-0.84%)	0.00219(13.98%)	0.00192
1.1	0.00261(3.98%)	0.00249(-0.93%)	0.00249(-0.92%)	0.00281(12.03%)	0.00251
1.2	0.00330(3.45%)	0.00312(-2.27%)	0.00312(-2.26%)	0.00347(8.68%)	0.00319

1.3	0.00402(3.61%)	0.00378(-2.66%)	0.00378(-2.65%)	0.00413(6.40%)	0.00388
1.4	0.00477(3.70%)	0.00445(-3.35%)	0.00445(-3.33%)	0.00477(3.78%)	0.00460
1.5	0.00551(3.77%)	0.00511(-3.79%)	0.00511(-3.77%)	0.00538(1.41%)	0.00531
1.6	0.00624(3.48%)	0.00575(-4.60%)	0.00575(-4.58%)	0.00595(-1.36%)	0.00603
1.7	0.00695(4.04%)	0.00637(-4.66%)	0.00637(-4.64%)	0.00645(-3.39%)	0.00668
1.8	0.00762(4.10%)	0.00695(-5.05%)	0.00695(-5.03%)	0.00690(-5.79%)	0.00732
1.9	0.00826(4.56%)	0.00749(-5.14%)	0.00750(-5.11%)	0.00727(-7.91%)	0.00790
2.0	0.00885(4.86%)	0.00800(-5.23%)	0.00800(-5.20%)	0.00759(-10.08%)	0.00844

*Values in bracket are the percentage difference between the present study and the classical solution

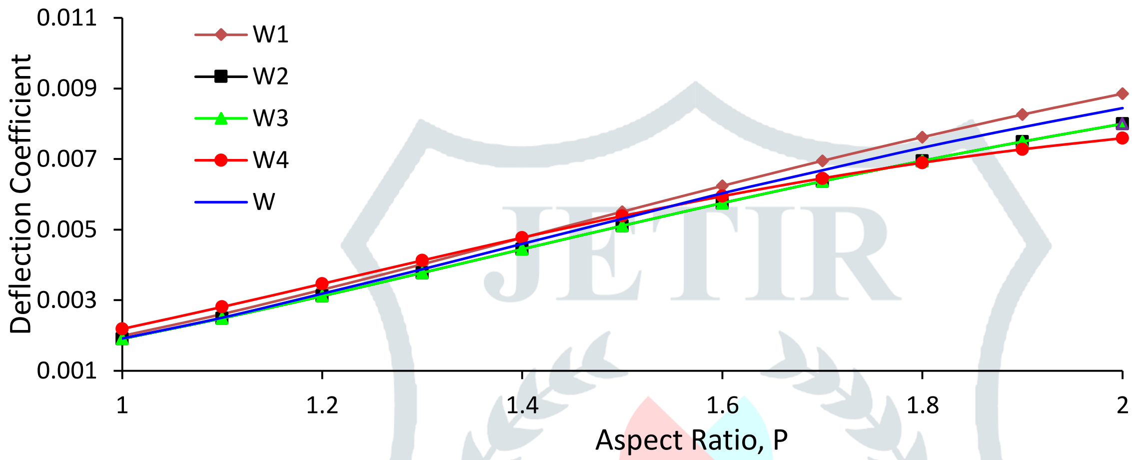

Figure 2: Deflection Coefficients at the Mid-Span for the Present Study in Comparison with the Classical Solution at Varying Aspect Ratio.

Short Span Moment

The response pattern in terms of moment in the short span as shown in Table 2 followed that of the deflection. The moment coefficient values for the first approximation are all in upper-bound; that of the second and truncated third approximations are all in lower-bound while the third approximation is a mix of upper and lower bounded coefficient values. Conventionally, the deflection coefficient values are evaluated to a higher degree of accuracy than the maximum moment values. This is due to the fact that the stress couples are proportional to the second derivatives of the deflection functions. Consequently, for plates having aspect ratios $1.0 \leq p(b/a) \leq 2.0$, the percentage difference range from 17.34 (at p = 1.0) to 10.05 (at p = 2.0) for first approximation. This shows the coefficient values converge as they move from aspect ratio 1.0 to 2.0. The second and truncated third approximations give nearly the same values. They all diverge as they move from aspect ratio 1.0 to 2.0 giving a percentage difference of 6.90 to 46.92 and 6.84 to 46.65 respectively. However, the percentage difference of third approximation coefficient values with the results of the classical solution converged from 71.47 (at p = 1.0) to 0.32 (at p = 1.5) and then diverged from 13.43 (at p = 1.6) to 62.66 (at p = 2.0). This is similar to the pattern shown by the deflection coefficient values. Fig. 3 shows the graphical representation of the results of the present formulation for short span moment in comparison with the results of the classical solution. It can be observed that the slope of the first approximation is the closest to the classical solution of all the four approximations considered.

Table 2: Short Span Moment Coefficient Values, β_x, at Mid-Span (x =0.5, y =0.5) for the Present Study in Comparison with the Classical Solution at Varying Aspect Ratio($(M_x)_{max}) = qa^2\beta_x$).

Aspect ratio, P	Present Study				Classical Solution
	Mx1	Mx2	Mx3	Mx4	Mx
	First Approximation	Second Approximation	Third Truncated Approximation	Third Approximation	Timoshenko and

					Woinowsky-Krieger (1970)
1	0.02863(17.34%)	0.02272(-6.90%)	0.02273(-6.84%)	0.04184(71.47%)	0.02440
1.1	0.03547(15.54%)	0.02673(-12.92%)	0.02676(-12.84%)	0.04823(57.11%)	0.03070
1.2	0.04269(13.54%)	0.03050(-18.88%)	0.03054(-18.78%)	0.05335(41.88%)	0.03760
1.3	0.05007(12.26%)	0.033919(-23.86%)	0.03396(-23.86%)	0.05684(27.44%)	0.04460
1.4	0.05745(11.77%)	0.0368(-28.25%)	0.03695(-28.12%)	0.05852(13.85%)	0.05140
1.5	0.06469(10.58%)	0.03940(-32.65%)	0.03949(-32.50%)	0.05831(-0.32%)	0.05850
1.6	0.07165(10.23%)	0.04148(-36.19%)	0.04159(-36.01%)	0.05627(-13.43%)	0.06500
1.7	0.07826(9.92%)	0.04314(-39.40%)	0.04328(-39.21%)	0.05249(-26.28%)	0.07120
1.8	0.08448(10.00%)	0.04444(-42.13%)	0.04461(-41.91%)	0.04714(-38.62%)	0.07680
1.9	0.09027(9.95%)	0.04542(-44.68%)	0.04562(-44.44%)	0.04040(-50.80%)	0.08210
2	0.09563(10.05%)	0.04613(-46.92%)	0.04636(-46.65%)	0.03245(-62.66%)	0.08690

*Values in bracket are the percentage difference between the present study and the classical solution

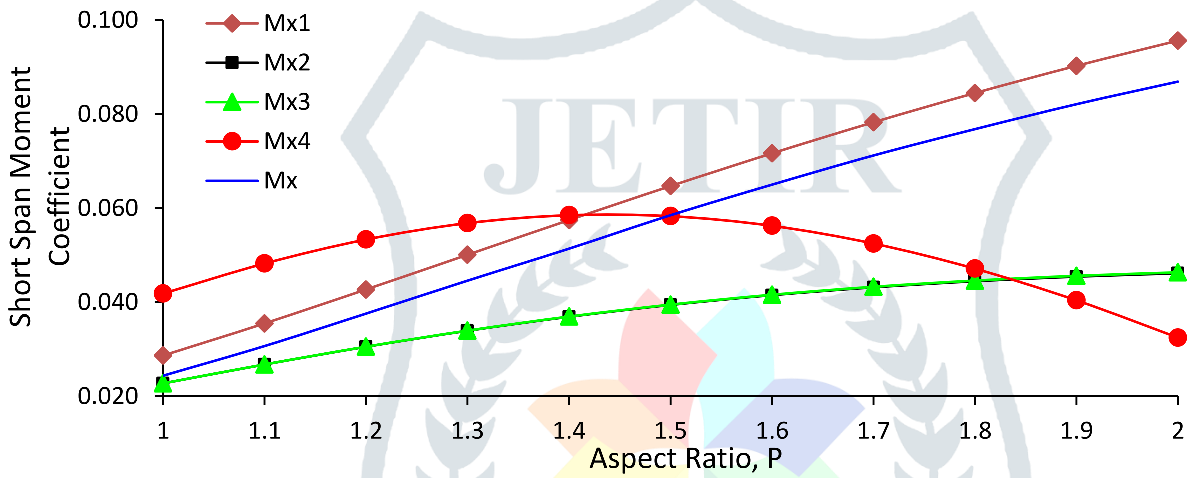

Figure 3: Short Span Moment Coefficient Values at the Mid-Span for the Present Study in Comparison with the Classical Solution at Varying Aspect Ratio.

Long Span Moment Coefficients

Table 3 shows the results of the long span moment coefficients for the present formulation along with the results of the classical solution. For the first approximation, the long span coefficient values are all in upper bound. The coefficient values rise from aspect ratio 1.0 and peak at aspect ratio 2.0 while the classical solutions rise from aspect ratio 1.0 and peak at aspect ratio 1.8. The percentage difference with the classical solution increases from 13.07 (at p =1.0) to 28.52 (at p = 2.0). The second and truncated third approximations, as with preceding results, give almost the same coefficient values. The results are a mix of upper and lower bounded coefficient values. They are upper- bounded from aspect ratio 1.0 to 1.2 and lower bounded from 1.3 to 2.0. The peak coefficient values for both occur at aspect ratio 1.6. The third approximation coefficient values are upper- bounded from aspect ratio 1.0 to 1.5 and lower bounded from 1.6 to 2.0. The peak coefficient values occur at aspect ratio 1.3. As would be expected, these coefficient values show less convergence than the deflection coefficient values because the moment coefficients are a function of the second derivatives of the deflection coefficient values. Fig. 4 shows the graphical representation of the results of the present formulation for long span moment in comparison with the results of the classical solution. It can be observed that the slope of the first approximation is the closest to the classical solution of all the four approximations considered.

Table 3: Long Span Moment Coefficient Values, β_y, at Mid-Span (x =0.5, y =0.5) for the Present Study in Comparison with the Classical Solution at Varying Aspect Ratio ($(M_y)_{max} = qa^2\beta_y$).

Aspect ratio, P	Present Study	Classical Solution

	My1	My2	My3	My4	My
	First Approximation	**Second Approximation**	**Third Truncated Approximation**	**Third Approximation**	**Timoshenko and Woinowsky-Krieger (1970)**
1	0.03754(13.07%)	0.03426(3.18%)	0.03428(3.25%)	0.04299(29.49%)	0.03320
1.1	0.04211(13.50%)	0.037589(1.30%)	0.03761(1.39%)	0.04642(25.12%)	0.03710
1.2	0.04618(15.45%)	0.04023(0.56%)	0.04027(0.67%)	0.04865(21.63%)	0.04000
1.3	0.04970(16.67%)	0.04218(-1.00%)	0.04223(-0.86%)	0.04966(16.58%)	0.04260
1.4	0.05266(17.54%)	0.04347(-2.97%)	0.04354(-2.81%)	0.04949(10.46%)	0.04480
1.5	0.05508(19.74%)	0.04418(-3.95%)	0.04427(-3.77%)	0.04823(4.85%)	0.04600
1.6	0.05701(21.56%)	0.04439(-5.35%)	0.04449(-5.13%)	0.04602(-1.87%)	0.04690
1.7	0.05849(23.14%)	0.04419(-6.96%)	0.04431(-6.72%)	0.04302(-9.44%)	0.04750
1.8	0.05960(24.95%)	0.04368(-8.43%)	0.04381(-8.16%)	0.03936(-17.49%)	0.04770
1.9	0.06039(26.87%)	0.04292(-9.84%)	0.04306(-9.53%)	0.03518(-26.09%)	0.04760
2	0.06092(28.52%)	0.04198(-11.43%)	0.04215(-11.09%)	0.03062(-35.41%)	0.04740

*Values in bracket are the percentage difference between the present study and the classical solution.

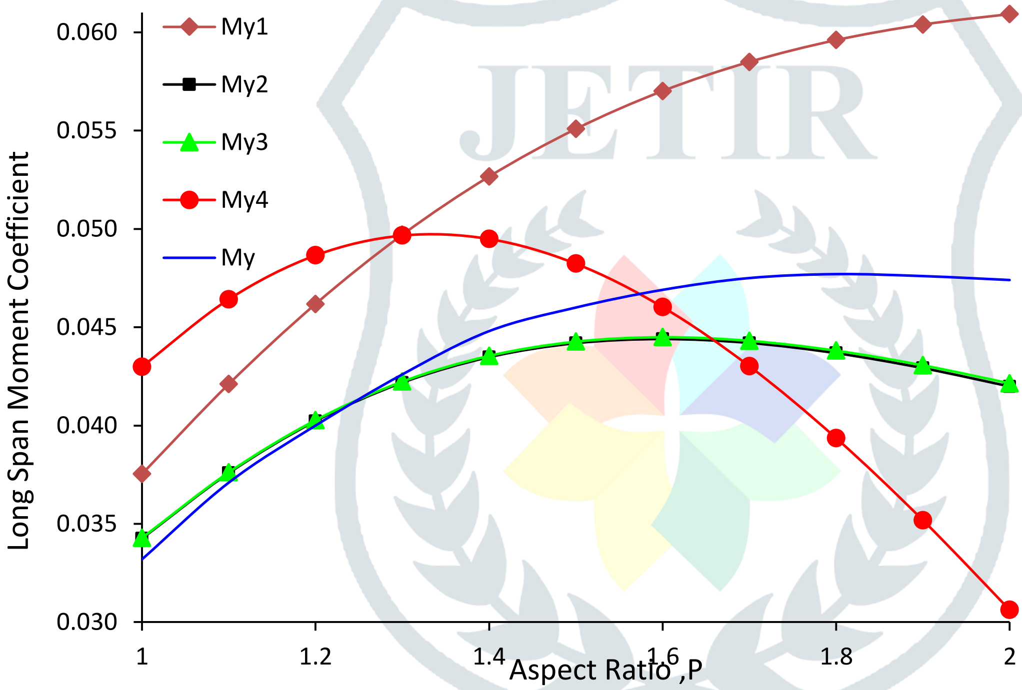

Figure 4: Long Span Moment Coefficient Values at the Mid-Span for the Present Study in Comparison with the Classical Solution at Varying Aspect Ratio.

6. Conclusion

This research presents the maximum deflection and maximum span moment coefficients of a uniformly loaded thin isotropic rectangular plate with two opposite edges simply supported and the other two opposite edges clamped for aspect ratios $1.0 \leq b/a \geq 2.0$ in Galerkin method. The results have been compared with the results in literature and their accuracy and convergence observed. The convergence to the classical solution did not increase as the number of terms of the deflection function increased from the first, to second, to truncated third and to the third approximation. Apart from the first approximation, the second, truncated third and third approximations gave mainly lower-bounded coefficient values in spite of having longer terms in their deflection functions. Secondly, only the slope of the first approximation coefficient values can best match that of the classical solution for both the maximum deflection and the maximum span moments. Thirdly, the average percentage differences between the results of the first approximation and that of the classical solution are 3.9 and 11.9 for the maximum deflection and maximum short span moment coefficients respectively while the percentage differences with both the second and truncated third approximations are about 3.5 and 30.2 for the maximum deflection and maximum short span moment respectively. The third approximation showed the most divergence for the maximum deflection and maximum span moments of the four approximations considered. Therefore, it is concluded that increasing the number of terms of the present formulation beyond the first approximation does not increase the accuracy and convergence of the solutions. Clearly, the multi-term deflection functions corresponding to the second, truncated third and third approximations can be used if only deflection is of essence.

References

[1] Aginam, C.H., Chidolue, C. A. and Ezeagu, C. A. (2012). Application of direct variational method in the analysis of isotropic rectangular plates. ARPN Journal of Engineering and Applied Sciences, 7(9): 1128-1138.

[2] Aginam, C. H., Okonkwo, V. O., Onodagu, P. D., and Okoye, M. O. (2018). Flexural analysis of clamped thin rectangular plates using Galerkin variational method. Saudi Journal of Engineering and Technology (SJEAT), 3(12): 697-704.

[3] Birman, V. (2011). Plate Structures. New York, NY: Springer Netherlands.

[4] Imark, C. E., and Gerdemeli, I. (2007b). The problem of isotropic rectangular plate with four clamped edges. Sadhana, 32(3): 181-186.

[5] Levy, M. (1899). Memoires sur la theories des plaques planes. Journal of Pure and Applied Mathematics, 3, (219).

[6] Mbakogu, F. C., and Pavlovic, M. N. (2000). Bending of clamped orthotropic rectangular plates: a variational symbolic solution. Computers and Structures, 77(2000): 117-128.

[7] Mikhlin, S. G. (1964). Variational methods in mathematical physics. Oxford, United Kingdom: Pergamon press.

[8] Nadai, A. (1925). Die Elastichen Flatten. Berlin, Germany: Verlag- Springer.

[9] Navier, C. L. M. N. (1823). Bulletin des Science de la Societe Philomarhique de Paris.

[10] Osadebe, N. N., and Aginam, C. H. (2011). Bending analysis of isotropic rectangular plate with all edges clamped: variational symbolic solution. Journal of Emerging Trends in Engineering and Applied Sciences (JETEAS), 2(5): 846-852.

[11] Papkovitch, P. F. (1941). Theory of structural design of ships (part 2). Partz Gos Soyuz Izd Sudostr Promyshal. Leningrad (in Russian).

[12] Szilard, R. (2004). Theories and Application of plate analysis. Hoboken, New Jersey: John Wiley & sons, inc.

[13] Taylor, R. L., and Govindjee, S. (2004). Solution of clamped rectangular plate problems. Communication in Numerical Methods in Engineering, 20(10): 757-765.

[14] Timoshenko, S. P., and Woinowsky-Krieger, S. (1959). Theory of plates and shells (2nd ed.). New York, NY: McGraw-Hill Inc.

[15] Timoshenko, S. P., and Woinowsky-Krieger, S. (1970). Theory of plates and shells. Auckland, NewZealand: McGraw-Hill Inc.

[16] Ventsel, E., and Krauthammer, T. (2001). Thin plates and shells: Theory, Analysis and Applications. New York, NY: Marcel Dekker Inc.

[17] Wang, D., and El-Sheikh, A. I. (2005). Large deflection mathematical analysis of rectangular plates. J. Eng. Mech., 131: 809-821.

[18] Wojtaszak, I. A. (1937). The calculation of maximum deflection, moment and shear for uniformly loaded rectangular plate with clamped edges. Trans ASME J Appl Mech, 4 :173-176.