PENTAGONAL FUZZY MATRICES IN THE STUDY OF MEDICAL DIAGNOSIS

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Abstract : The aim of this paper is to diagnosis the diseases by using fuzzy matrices. There are many models of fuzzy matrices to deal with various aspects of medical diagnosis. The pentagonal fuzzy matrices to represent the medical knowledge between the symptoms and diseases. In this paper the above concept is applied to Typhoid, Dengue and Yellow fever.

KEYWORDS: **Medical diagnosis, Pentagonal fuzzy number, Membership function.**

I. INTRODUCTION

Decision making is one of the way to find a best alternative from a set of selected alternatives. In real life problems, decision making is very vague in most of the cases. Fuzzy numbers are used in various real life problems like decision making, control theory, medical sciences and so on. Under the fuzzy environment the decision maker can choose the better option with their own perception. Complement of a fuzzy set was proposed by Neog and Sut (2011) and put forwarded a matrix represents a fuzzy soft set and extended Sanchez's approach for medical diagnosis. Bellman and Zadeh (1970) presented a decision making in fuzzy environment. Meenakshi and Kaliraja (2001) have extended Sanchez's approach for medical diagnosis to an interval valued fuzzy matrix. Many operations were carried out using fuzzy number. Sanchez, (1979) formulated the diagnostic model involving fuzzy matrices representing the medical knowledge between symptoms and diseases. Shimura, (1973) introduced and developed a fuzzy comparison matrix for ranking or ordering of different fuzzy sets. Zadeh, (1965) developed fuzzy systems including fuzzy set theory, fuzzy logic which have a variety of successful applications.

This paper provides pentagonal fuzzy number to diagnosis diseases by various symptoms. In section 2, several definition related to fuzzy number are presented. In section 3, Sanches's approaches was extended to new modified method for medical diagnosis using pentagonal fuzzy matrices. Some endings are given in section 4.

II. PRELIMINARIES

Definition: 2.1

An mxn matrix A = (aij) whose components are unit interval [0,1] is called a **fuzzy matrix.**

Definition: 2.2

A fuzzy number A= $(a_1, a_2, a_3, a_4, a_5)$ is called a **pentagonal fuzzy number matrix.** The points is the domain having ordering $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$, $a_1, a_2, a_3, a_4, a_5 \in R$

Definition: 2.3

i) Addition operation

Let A= $(a_1, a_2, a_3, a_4, a_5)$ and B= $(b_1, b_2, b_3, b_4, b_5)$ to be two pentagonal fuzzy number matrices. Then, A+B= $(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5)$ With $D_i (A+B) \geq \max (d_{iA}, d_{iB})$ i= 1, 2, 3, …….

ii) Subtraction operation

. Let A= $(a_1, a_2, a_3, a_4, a_5)$ and B= $(b_1, b_2, b_3, b_4, b_5)$ to be two pentagonal fuzzy number matrices. Then, A-B= $(a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4, a_5-b_5)$ With $D_i (A-B) \geq \max (d_{iA}, d_{iB})$ i= 1, 2, 3, …….

iii) Multiplication operation

Let $A=(a_1, a_2, a_3, a_4, a_5)$ and $B=(b_1, b_2, b_3, b_4, b_5)$ to be two pentagonal fuzzy number matrix. Then, $AB=(a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5)$ With $D_i(AB) \geq Max\ (d_{iA}, d_{iB})$ i=1,2,3,………

Definition: 2.4

Let $A=(a_1, a_2, a_3, a_4, a_5)$ and $B=(b_1, b_2, b_3, b_4, b_5)$ be two pentagonal fuzzy number matrices of same order. Then the maximum operation on it is given by,

L_{max} = max (A, B) = Sup (a_k, b_k) Where sup (a_k, b_k) = {sup (a_1, b_1), sup (a_2, b_2), sup (a_3, b_3), sup (a_4, b_4), sup (a_5, b_5)} is the K^{th} element of max (A, B).

Definition: 2.5

Let $A= (a_1, a_2, a_3, a_4, a_5)$ and $B= (b_1, b_2, b_3, b_4, b_5)$ be a pentagonal fuzzy number. Then,

$$AM\ (A) = \frac{a1+a2+a3+a4+a5}{5}$$

III. Medical diagnosis and decision making

Fuzzy matrix frame work has been utilized in several different approaches to model the medical diagnostic process and decision making process. A fuzzy decision making is almost important scientific social and economic endeavor. There exist several major approaches with in the theories of fuzzy decision making.

Algorithm

Step1: Consider a pentagonal fuzzy number matrix (F, D) over S, where F is a mapping given by F : D → Ř(S), Ř(S) is a set of all pentagonal fuzzy sets of S. This matrix is denoted by k_0 which is the fuzzy occurrence matrix or symptom disease pentagonal fuzzy number matrix.

Step 2: Consider another pentagonal fuzzy number matrix (F_1, S) over P, where F_1 is a mapping given by $F_1 : S \rightarrow F(P)$. This matrix is denoted by K_r which is the patient symptom pentagonal fuzzy number matrix.

Step 3: Convert the elements of pentagonal fuzzy number into its membership function as follows membership function of (a_k) = $(a_1, a_2, a_3, a_4, a_5)$ is defined $\mu_{ak} = (\frac{a1}{10} < \frac{a2}{10} < \frac{a3}{10} < \frac{a4}{10} < \frac{a5}{10})$ if $0 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq 1$ where, $0 \leq \frac{a1}{10} \leq \frac{a2}{10} \leq \frac{a3}{10} \leq \frac{a4}{10} \leq \frac{a5}{10} \leq 1$. Now, the matrix K_0 and K_r are converted into pentagonal fuzzy membership matrices namely $(K_0)_{mem}$ and $(K_r)_{mem}$.

Step 4: Calculated the following relation matrices

$K_1 = (K_r)_{mem} (*) (K_0)_{mem}$ by using Definition 2.4.

$K_2 = (K_r)_{mem} (*) [Q (-) (K_0)_{mem}]$,Where Q is the pentagonal fuzzy membership matrix in which all entries are (1, 1, 1). $[Q(-) (K_r)_{mem}]$ is complement of $(K_0)_{mem}$

$K_3 = [Q (-) (K_r)_{mem}] (*) (K_0)_{mem}$Where $[Q (-) (K_r)_{mem}]$ is the complement of $(K_r)_{mem}$.

K_2 and K_3 are calculated using subtraction operation and definition 2.4.

$K_4 = \max \{K_2, K_3\}$.It is calculated using definition 2.5

$K_5 = K_1 (-) K_4$. It is calculated using subtraction operation.

Step 5: Calculate K_6 = AM (A_k) and Row i = Maximum of i^{th} row which helps the decision maker to strongly confirm the disease for the patient.

IV. Case study

There are three patients P_1, P_2, P_3. They have symptoms like fever, head ache and vomiting. Let the possible diseases relating to the above symptoms be typhoid, dengue and yellow fever.

Case 1:

Consider the set S= {F, H, V} as the set of parameters represent the symptoms, fever, head ache, vomiting respectively and the set $D = \{d_1, d_2, d_3\}$ where d_1, d_2 and d_3 represent the diseases, typhoid, dengue, yellow fever.

$F(d_1)$ = [{F (10,11,15,17,18)}, {H (2 5,7,9,10)}, {V (7,8.5,9,11,12)}]

$F(d_2)$ = [{F(7,10,11,12.5,14)}, {H (5,8,10,11,13)}, {V (9,10.5,11,12,14)}]

$F(d_3)$ = [{F(5,7,10,11.5,13)}, {H (3,5.6,7,9,11)}, {V (8,10,12,13,14)}]

Thus the pentagonal fuzzy number represents a relation matrix K_0.

$$K_0 = \begin{array}{c} \\ F \\ H \\ V \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (10,11,15,17,18) & (2,5,7,9,10) & (7,8.5,9,11,12) \\ (7,10,11,12.5,14) & (5,8,10,11,13) & (9,10.5,11,12,14) \\ (5,7,10,11.5,13) & (3,5.6,7,9,11) & (8,10,12,13,14) \end{pmatrix} \end{array}$$

Case 2:

If take P = {P_1, P_2, P_3} as the universal set where P_1, P_2 and P_3 represent patients respectively and S = {F, H, V} as the set of parameters. Suppose that,

F_1 (F) = [{P_1(7,8,10.6,11,13)}, {P_2 (4,5,7,8,9)}, {P_3(9,10,12,13.5,15)}]

F_2 (H) = [{P_1(9,10,11.7,12,14)}, {P_2(3,4.5,6,8,9)}, {P_3(8,9.7,10,11,13)}]

F_3 (V) = [{P_1(6,8,9,11,12)}, {P_2(2,4,5.5,6,7.5)}, {P_3(7,8,9,10.7,11)}]

Thus the pentagonal fuzzy number matrix (F_1, S) represents a relation matrix K_r

$$K_r = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} F & H & V \end{array} \\ \begin{pmatrix} (7,8,10.6,11,13) & (4,5,7,8,9) & (9,10,12,13.5,15) \\ (9,10,11.7,12,14) & (3,4.5,6,8,9) & (8,9.7,10,11,13) \\ (6,8,9,11,12) & (2,4,5.5,6,7.5) & (7,8,9,10.7,11) \end{pmatrix} \end{array}$$

Case 3:

$$(K_0)_{mem} = \begin{array}{c} \\ F \\ H \\ V \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (1,1.1,1.5,1.7,1.8) & (0.2,0.5,0.7,0.9,1) & (0.7,0.85,0.9,1.1,1.2) \\ (0.7,1,1.1,1.25,1.4) & (0.5,0.8,1,1.1,1.3) & (0.9,1.05,1.1,1.2,1.4) \\ (0.5,0.7,1,1.15,1.3) & (0.3,0.56,0.7,0.9,1.1) & (0.8,1,1.2,1.3,1.4) \end{pmatrix} \end{array}$$

$$(K_r)_{mem} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} F & H & V \end{array} \\ \begin{pmatrix} (0.7,0.8,1.06,1.1,1.3) & (0.4,0.5,0.7,0.8,0.9) & (0.9,1,1.2,1.35,1.5) \\ (0.9,1,1.17,1.2,1.4) & (0.3,0.45,0.6,0.8,0.9) & (0.8,0.9,1,1.1,1.3) \\ (0.6,0.8,0.9,1.1,1.2) & (0.2,0.4,0.55,0.6,0.75) & (0.7,0.8,0.9,1.07,1.1) \end{pmatrix} \end{array}$$

Case 4:

Computing the following relation matrices.

$$K_1 = (k_r)_{mem} (*) (K_0)_{mem} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \left(\begin{array}{ccc} (0.7,0.8,1.06,1.1,1.3) & (0.5,0.7,1,1.15,1.3) & (0.8,1,1.2,1.3,1.4) \\ (0.9,1,1.17,1.2,1.4) & (0.3,0.56,0.7,0.9,1.1) & (0.8,1,1.2,1.3,1.4) \\ (0.6,0.8,0.9,1.1,1.2) & (0.3,0.56,0.7,0.9,1.1) & (0.7,0.8,0.9,1.09,1.1) \end{array} \right) \end{array}$$

$$K_2 = (K_r)_{mem} (*) (Q (-) (K_0)_{mem}) = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \left(\begin{array}{ccc} (0.5,0.3,0,-0.15,0.3) & (0.8,0.5,0.3,0.1,0) & (0.3,0.15,0.1,-0.1,0.2) \\ (0.5,0.3,0,-0.15,0.3) & (0.8,0.5,0.3,0.1,0) & (0.3,0.15,0.1,-0.1,0.2) \\ (0.5,0.3,0,-0.15,0.3) & (0.8,0.5,0.3,0.1,0) & (0.3,0.15,0.1,-0.1,0.2) \end{array} \right) \end{array}$$

$$K_3 = (Q (-)(K_r)_{mem}) (*)(K_0)_{mem} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \left(\begin{array}{ccc} (0.6,0.5,0.3,0.2,0.1) & (0.6,0.5,0.3,0.2,0.1) & (0.6,0.5,0.3,0.2,0.1) \\ (0.7,0.55,0.4,0.2,0.1) & (0.7,0.55,0.4,0.2,0.1) & (0.7,0.55,0.4,0.2,0.1) \\ (0.8,0.6,0.45,0.4,0.25) & (0.8,0.6,0.45,0.4,0.25) & (0.8,0.6,0.45,0.4,0.25) \end{array} \right) \end{array}$$

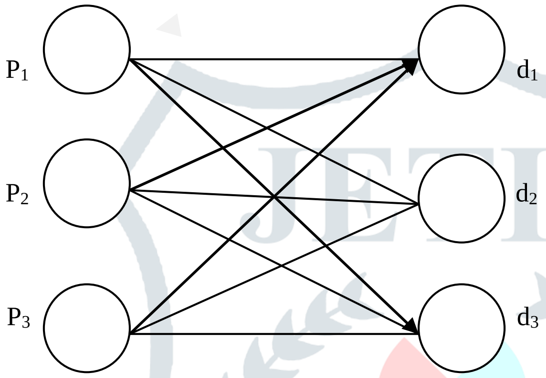
$$K_4 = \mathrm{Max}\ \{K_2, K_3\} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \left(\begin{array}{ccc} (0.6,0.5,0.3,0.2,0.1) & (0.6,0.5,0.3,0.2,0.1) & (0.6,0.5,0.3,0.2,0.1) \\ (0.7,0.55,0.4,0.2,0.1) & (0.7,0.55,0.4,0.2,0.1) & (0.7,0.55,0.4,0.2,0.1) \\ (0.8,0.6,0.45,0.4,0.25) & (0.8,0.6,0.45,0.4,0.25) & (0.8,0.6,0.45,0.4,0.25) \end{array} \right) \end{array}$$

$$K_5 = K_1 - K_2 = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \left(\begin{array}{ccc} (0.1,0.3,0.75,0.9,1.2) & (-0.1,0.2,0.7,0.95,1.2) & (0.2,0.5,0.9,1.1,1.3) \\ (0.2,0.45,0.77,1,1.3) & (-0.4,0.01,0.3,0.7,1) & (0.1,0.42,0.6,0.9,1.2) \\ (-0.2,0.2,0.45,0.7,0.95) & (-0.5,-0.04,0.25,0.5,0.85) & (-0.1,0.2,0.45,0.69,0.85) \end{array} \right) \end{array}$$

Case 5:

	d_1	d_2	d_3	Row i = maximum of I[th] row
P_1	0.65	0.59	0.80	0.80
P_2	0.74	0.32	0.64	0.74
P_3	0.42	0.21	0.41	0.42

This can be represented in the form of a graph namely network as follows:



V. CONCLUSION

The proposed method in decision making effectively reduces the conflict in making decisions. Medicine is one of the fields in which applicability of fuzzy set theory as recognized quite early. Generally the physician knows about the patient from the history, laboratory test result, X-rays and ultra sonic rays etc. This paper present pentagonal fuzzy number for diagnosis the diseases by using various parameter. The result of the analysis shows that the patient P_2 is easily affected by Typhoid, P_3 is also affected by Typhoid and P_1 is affected by Yellow Fever.

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