AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH INFLATION

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ABSTARCT

In this study paper we developed an inventory model under the assumption that a retailer trades the new product as well as gather used products from the consumers. An economic order quantity model has been established considering price dependent quadratic demand. Models of this chapter are divided into two sections. In all the models of this chapter shortages are allowed and partially backlogged and the backlogging rate is constant.

Key Words: Deteriorating Items, Inflation**.**

1. INTRODUCTION

1. INTRODUCTION

In today's world, one of the greatest contests is to reserve our limited natural resources and reduce the unused material. Due to rising environmental concern among consumers, the industrial sectors are also stimulating environment-friendly methods to attract new consumers. Consumers also prefer to buy from the enterprises with green image. That is why salvage of used materials and items has got more attention in past span. Collection of used products, as paper, bottle, and battery, is a well-known idea in recent economies. Reuse, remanufacturing and recycling of cars and electronic devices, and disposal of injurious waste are very recent research field. The listed deeds include a very broad area, and it seems to have different organization problems. Reverse logistics comprises not only the material flow from trader to consumer, but also the material flow of used products from consumer to the trader, in order to reduce the drain of environment. By the use of reverse logistics, there are direct and indirect welfares inside of economic advantages for a firm. Direct gains are the possibility of profit increase that means a weakening of use of raw materials, decrease of costs of waste disposal, and value added through reuse. Indirect advantages are the "green" image of a firm which is a facet of affordability for initiatives. Many practical experiences have supported that environmental receptive working of firms results in a stable consumer relationship. It is a competitive advantage of firms that increases in profit chances. A strict jurisdictive regulation is a new argument for practical application of reverse logistic practices, which serves as a method for environmental fortification. The United States and the European Union are leading in environmental regulation, which requires the organizations to keep the law. In the practice this intended activity enhancements in competitive advantages of firms.

Chung, K.J., Huang, T.S. (2007) [1] considered a generic facility location model to discuss the effects of return flows on logistic networks. Ishii, H., Nose, T. (1996) [3] have developed a multi-echelon closed loop supply chain network model for a multi-period and multi-products. They have studied the case of battery recycling where old battery material is used in the production of new batteries. Lee, C.C., Ma, C.Y. (2000) [6] developed an optimal returned policy for a reverse logistics inventory model with backorders. Zhou, Y.W. (2003) [13] have provided a very nice review of the recent research papers in which they reviewed a total of 382 papers to construct a good framework of past, and they identified the gaps where future work was required. Rong, M., Mahapatra, N.K., Maiti, M. (2008) [8] derived a reverse logistic model for deteriorating items with preservation technology investment and learning effect in an inflationary environment

Deterioration is the change, damage, decay, spoilage, fading, desuetude, pilferage, and loss of efficacy or loss of peripheral value of a commodity that results in decreasing usefulness from the original one. Most products such as medicine, blood, fish, alcohol, gasoline, vegetables and radioactive chemicals have finite shelf life, and start to deteriorate once they are refilled. Lee, C.C., Hsu, S.L. (2009) [5] provided an inclusive introduction about the deteriorating items inventory management research status, this paper reviews the recent studies in relevant fields. Wee, H.M., Yu, J.C.P., Law, S. T. (2005) [12] developed an inventory model for items with two parameter weibull distribution deterioration and backlogging. Mandal, N.K., Roy, T.K., Maiti, M. (2006) [7]

derived an inventory models with ramp type demand for deteriorating items with partial backlogging and time varying holding cost. Teng, J.T., Chen, J., Goyal, S. K. (2009) [10] developed two storage inventory models with trade credits, inflation in bulk release.

In recent time, pricing is an important tactical issue because it directly distresses the demand. The demand is said to be elastic if, when the price goes up, the generated revenue goes down. Thus, this is a very important verdict for any manager to make. Whitin (1957) [11] was the first to study price-dependent demand. S.K. Ghosh, S. Khanra, K.S. Chaudhuri(2011) [9] studied an inventory model for price and frequency of advertisement dependent demand. They have provided a general model by using general-type of deterioration and holding cost rates. Jaggi *et al.* (2010) [4] also used selling price dependent demand in their study. Their model features a two ware-house inventory model with non-instantaneous deterioration and under the effects of trade credit. Dye, C.Y., Ouyang, L.Y., Hsieh, T.P. (2007) [2] also considered price dependent demand in their study. In this study, we have assumed that a retailer sells the new product to the customers as well as collects and sells the used products. The optimal pricing, the ordering quantity for a new product and the optimal quantity of a used product are discussed, where customer demands is sensitive to time and the retail price. The total profit is maximized with respect to selling price and cycle time.

This chapter is an extension of "Retailer's optimal pricing and replenishment policy for new product and optimal take-back quantity of used product" (Yugoslav Journal of Operations Research, 2018). In this chapter we extend the previous work by introducing deterioration and variable holding cost under the effect of inflation with shortages which are partially backlogged. Deterioration is taken into consideration is Weibull distribution deterioration with two parameters. Replenishment rate is infinite. Shortages are permissible and a constant fraction of shortages is presumed to be backlogged. The chapter is organized in two sections. Section 3.1 pertains to the development and analysis of inventory model without inflation. Section 3.2 devoted with the development and analysis of inventory model with inflation. Numerical examples are presented for both the cases to explain the model. Sensitivity and concavity analysis are carried out with respect to the parameters of the model.

2.2 Assumptions and Notation

The mathematical model of the proposed model is based on the following assumptions:

- It is a single item inventory system.
- The replenishment is instantaneous.
- The Lead time is negligible or zero.
- Shortages are partially backlogged at a constant rate.
- Holding cost assumed as a variable function of time and is given by $h(t)=(h_1+th_2)$ for new products, Where $h_1,h_2\geq 0$ and $h'(t)=(h_1'+th_2')$ for used product, Where $h_1',h_2'\geq 0$.
- The Demand rate of new product R(p,t) is considered as, $R(p,t)=\alpha\left(1+\alpha_1 t-\alpha_2 t^2\right)-\beta p$ where, $\alpha>0$ denotes the scale demand and $\alpha_1,\alpha_2>0$. The parameter $\beta>0$ denotes the price elasticity.
- The return rate of used product is considered as $R_u(p,t)=a(1-bt)-p(1-p_0)$ where, a,b > 0 and p_0 are the parameters associated with price for the used product.
- The deterioration of time as follows by weibull parameter (two) distribution $\theta_1(t)=\gamma_1\delta_1 t^{\delta_1-1}$ for new products and $\theta_2(t)=\gamma_2\delta_2 t^{\delta_2-1}$ where , $0<\gamma_1<1, 0<\gamma_2<1$ are the scale parameters and $\delta_1>0,\delta_2>0$ are the shape parameters.

The notations used in developing the model are as follows:

A Ordering cost for retailer ($/order)

C Purchase cost per item (constant), ($/order)

h_1, h_2 Holding cost parameters for new product.

h_1', h_2' Holding cost parameters for used product.

Q The replenishment quantity for new product

Q_u The quantity of used product

r The inflation rate.

B Fraction of shortages to be backlogged.

S_0 Shortage cost per unit short per unit time.

S_1 Opportunity cost per unit short.

D_1 Deterioration cost per unit deteriorates for new product.

D_2 Deterioration cost per unit deteriorates for used product.

T The replenishment time (years)

τ The point of time when collection of used products starts (years)

p Selling price per item (a decision variable) ($/unit)

t_1 Time epoch at which inventory level of new products become zero (a decision variable).(days)

R(p, t) Demand rate for new product at $t \geq 0$ (units)

$R_u(p,t)$ Demand rate for used product at $t \geq 0$ (units)

$\pi(p,T)$ Total profit of the retailer during cycle time (in $)

2.3 Mathematical Model

We now proceed to develop and analyze the model.

In this section, we present the general formulations and solutions to the inventory models for a new product as well as for the used product. For the new product the inventory is consumed due to time and price dependent demand. Suppose Q is the ordering quantity to be sold during cycle time [0,T].

The inventory level for the retailer decreases by reason of demand and deterioration effect during $[0, t_1]$. Inventory level becomes zero at $t = t_1$ and then shortages take place. The happening shortages are partially backlogged.

The governing differential equation for inventory level $I_1(t)$ at any time t, where $0 \leq t \leq t_1$ is given by

$$\frac{dI_1(t)}{dt} + \gamma_1 \delta_1 t^{\delta_1 - 1} I_1(t) = -R(p,t); \qquad 0 \leq t \leq t_1 \qquad ...(2.1)$$

With the boundary condition $I_1(t_1) = 0$ and $I(0) = Q$,the solution of differential equation (2.1) is given by

$$I_1(t) = -\alpha(t + \alpha_1 \frac{t^2}{2} - \alpha_2 \frac{t^3}{3} + \gamma_1 \frac{t^{\delta_1+1}}{\delta_1+1} + \alpha_1\gamma_1 \frac{t^{\delta_1+2}}{\delta_1+2} - \alpha_2\gamma_1 \frac{t^{\delta_1+3}}{\delta_1+3})e^{-\gamma_1 t^{\delta_1}}$$

$$+\beta p(t + \gamma_1 \frac{t^{\delta_1+1}}{\delta_1+1})e^{-\gamma_1 t^{\delta_1}} + Qe^{-\gamma_1 t^{\delta_1}} \qquad ...(2.2)$$

By using the boundary condition $I_1(t_1) = 0$, the inventory level $I_1(t)$ and the ordering quantity Q are given by

$$I_1(t)=[\alpha(t_1-t)+\frac{\alpha\alpha_1}{2}(t_1^2-t^2)-\frac{\alpha\alpha_2}{3}(t_1^3-t^3)+\frac{\alpha\gamma_1}{\delta_1+1}(t_1^{\delta_1+1}-t^{\delta_1+1})+\frac{\alpha\alpha_1\gamma_1}{\delta_1+2}(t_1^{\delta_1+2}-t^{\delta_1+2})$$

$$-\frac{\alpha\alpha_2\gamma_1}{\delta_1+3}(t_1^{\delta_1+3}-t^{\delta_1+3})-\beta p((t_1-t)+\frac{\gamma_1}{\delta_1+1}(t_1^{\delta_1+1}-t^{\delta_1+1}))]e^{-\gamma_1 t^{\delta_1}} \quad ...(2.3)$$

$$Q=\alpha(t_1+\frac{\alpha_1 t_1^2}{2}-\frac{\alpha_2 t_1^3}{3}+\frac{\gamma_1 t_1^{\delta_1+1}}{\delta_1+1}+\frac{\alpha_1\gamma_1 t_1^{\delta_1+2}}{\delta_1+2}-\frac{\alpha_2\gamma_1 t_1^{\delta_1+3}}{\delta_1+3})-\beta p(t_1+\frac{\gamma_1 t_1^{\delta_1+1}}{\delta_1+1}) \quad ...(2.4)$$

The amount of backlogged shortage during the interval $[t_1,T]$ satisfies the differential equation

$$\frac{dI_2(t)}{dt}=-BR(p,t); \qquad t_1\le t\le T \quad ...(2.5)$$

with the boundary condition $I_2(t_1)$=0, the solution of the differential equation (2.5) is given by

$$I_2(t)=\alpha\beta[(t_1-t)+\frac{\alpha_1}{2}(t_1^2-t^2)-\frac{\alpha_2}{3}(t_1^3-t^3)]-\beta pB(t_1-t) \quad ...(2.6)$$

Now, for the used product during the period $[\tau,T]$, the inventory level is affected by the return rate of the used product so the governing differential equation for inventory level $I_u(t)$ at any time t, where $\tau\le t\le T$,is given by

$$\frac{dI_u(t)}{dt}+\gamma_2\delta_2 t^{\delta_2-1}I_u(t)=-R_u(p,t); \qquad \tau\le t\le T \quad ...(2.7)$$

With $I_u(\tau)=Q_u$ and $I_u(T)=0$. The solution of the differential equation (2.7) is given

$$I_u(t)=[a((T-t)-\frac{b}{2}(T^2-t^2)+\frac{\gamma_2}{\delta_2+1}(T^{\delta_2+1}-t^{\delta_2+1})-\frac{\gamma_2 b}{\delta_2+2}(T^{\delta_2+2}-t^{\delta_2+2}))$$

$$-p(1-p_0)(T-t)-p(1-p_0)\frac{\gamma_2}{\delta_2+1}(T^{\delta_2+1}-t^{\delta_2+1})]e^{-\gamma_2 t^{\delta_2}} \quad ...(2.8)$$

by from the boundary conditions, the quantity of used product Q_u is given by

$$Q_u=[a((T-\tau)-\frac{b}{2}(T^2-\tau^2)+\frac{\gamma_2}{\delta_2+1}(T^{\delta_2+1}-\tau^{\delta_2+1})-\frac{\gamma_2 b}{\delta_2+2}(T^{\delta_2+2}-\tau^{\delta_2+2}))$$

$$-p(1-p_0)(T-\tau)-p(1-p_0)\frac{\gamma_2}{\delta_2+1}(T^{\delta_2+1}-\tau^{\delta_2+1})]e^{-\gamma_2\tau^{\delta_2}} \quad ...(2.9)$$

2.3.1 The Model without Inflation

Now to calculate total profit, we calculate all the constituents for both new product and used product. The constituents of profit function of the inventory system for new product are as follows.

1. SR_n = Sales revenue from new product = $\frac{1}{T}\int_0^T[pR(p,t)]dt$

$$SR_n=p\alpha(1+\frac{\alpha_1 T}{2}-\frac{\alpha_2 T^2}{3})-\beta p^2 \quad ...(2.10)$$

2. PC_n = Purchase cost = $\frac{CQ}{T}$

$$PC_n=\frac{C}{T}[\alpha(t_1+\frac{\alpha_1 t_1^2}{2}-\frac{\alpha_2 t_1^3}{3}+\frac{\gamma_1 t_1^{\delta_1+1}}{\delta_1+1}+\frac{\alpha_1\gamma_1 t_1^{\delta_1+2}}{\delta_1+2}-\frac{\alpha_2\gamma_1 t_1^{\delta_1+3}}{\delta_1+3})-\beta p(t_1+\frac{\gamma_1 t_1^{\delta_1+1}}{\delta_1+1})] \quad ...(2.11)$$

3. OC_n = Ordering cost $=\frac{A}{T}$ …(2.12)

4. $HC_n =$ Holding cost= $\frac{1}{T}\int_0^{t_1}[(h_1+h_2t)\,I_1(t)]dt$

$$HC_n=\frac{h_1}{T}[\alpha(\frac{t_1^2}{2}-\frac{\gamma_1t_1^{\delta_1+2}}{(\delta_1+1)(\delta_1+2)})+\alpha\alpha_1(\frac{t_1^3}{3}-\frac{\gamma_1t_1^{\delta_1+3}}{(\delta_1+1)(\delta_1+3)})-\alpha\alpha_2(\frac{t_1^4}{4}-\frac{\gamma_1t_1^{\delta_1+4}}{(\delta_1+1)(\delta_1+4)})+\alpha\gamma_1(\frac{t_1^{\delta_1+2}}{(\delta_1+2)}-\frac{\gamma_1t_1^{2\delta_1+2}}{(\delta_1+1)(2\delta_1+2)})+\alpha\alpha_1\gamma_1(\frac{t_1^{\delta_1+3}}{(\delta_1+3)}-\frac{\gamma_1t_1^{2\delta_1+3}}{(\delta_1+1)(2\delta_1+3)})-\alpha\alpha_2\gamma_1(\frac{t_1^{\delta_1+4}}{(\delta_1+4)}-\frac{\gamma_1t_1^{2\delta_1+4}}{(\delta_1+1)(2\delta_1+4)})-\beta p((\frac{t_1^2}{2}-\frac{\gamma_1t_1^{\delta_1+2}}{(\delta_1+1)(\delta_1+2)})+\gamma_1(\frac{t_1^{\delta_1+2}}{(\delta_1+2)}-\frac{\gamma_1t_1^{2\delta_1+2}}{(\delta_1+1)(2\delta_1+2)}))+\frac{h_2}{T}[\alpha(\frac{t_1^3}{6}-\frac{\gamma_1t_1^{\delta_1+3}}{(\delta_1+2)(\delta_1+3)})+\alpha\alpha_1(\frac{t_1^4}{8}-\frac{\gamma_1t_1^{\delta_1+4}}{(\delta_1+2)(\delta_1+4)})-\alpha\alpha_2(\frac{t_1^5}{10}-\frac{\gamma_1t_1^{\delta_1+5}}{(\delta_1+2)(\delta_1+5)})+\alpha\gamma_1(\frac{t_1^{\delta_1+3}}{2(\delta_1+3)}-\frac{\gamma_1t_1^{2\delta_1+3}}{(\delta_1+2)(2\delta_1+3)})+\alpha\alpha_1\gamma_1(\frac{t_1^{\delta_1+4}}{2(\delta_1+4)}-\frac{\gamma_1t_1^{2\delta_1+4}}{(\delta_1+2)(2\delta_1+4)})-\alpha\alpha_2\gamma_1(\frac{t_1^{\delta_1+5}}{2(\delta_1+5)}-\frac{\gamma_1t_1^{2\delta_1+5}}{(\delta_1+2)(2\delta_1+5)})-\beta p((\frac{t_1^3}{6}-\frac{\gamma_1t_1^{\delta_1+3}}{(\delta_1+2)(\delta_1+3)})+\gamma_1(\frac{t_1^{\delta_1+3}}{2(\delta_1+3)}-\frac{\gamma_1t_1^{2\delta_1+3}}{(\delta_1+2)(2\delta_1+3)})) \quad ...(2.13)$$

5. $DC_n =$ Deterioration cost $=\frac{D_1}{T}\int_0^{t_1}\gamma_1\delta_1t^{\delta_1-1}I_1(t)\,dt$

$$DC_n=\frac{D_1\gamma_1\delta_1}{T}[\alpha(\frac{t_1^{\delta_1+1}}{\delta_1(\delta_1+1)}-\frac{\gamma_1t_1^{2\delta_1+1}}{2\delta_1(2\delta_1+1)})+\alpha\alpha_1(\frac{t_1^{\delta_1+2}}{\delta_1(\delta_1+2)}-\frac{\gamma_1t_1^{2\delta_1+2}}{2\delta_1(2\delta_1+2)})-\alpha\alpha_2(\frac{t_1^{\delta_1+3}}{\delta_1(\delta_1+3)}-\frac{\gamma_1t_1^{2\delta_1+3}}{2\delta_1(2\delta_1+3)})+\alpha\gamma_1(\frac{t_1^{2\delta_1+1}}{\delta_1(2\delta_1+1)}-\frac{\gamma_1t_1^{3\delta_1+1}}{2\delta_1(3\delta_1+1)})+\alpha\alpha_1\gamma_1(\frac{t_1^{2\delta_1+2}}{\delta_1(2\delta_1+2)}-\frac{\gamma_1t_1^{3\delta_1+2}}{2\delta_1(3\delta_1+2)})-\alpha\alpha_2\gamma_1(\frac{t_1^{2\delta_1+3}}{\delta_1(2\delta_1+3)}-\frac{\gamma_1t_1^{3\delta_1+3}}{2\delta_1(3\delta_1+3)})-\beta p((\frac{t_1^{\delta_1+1}}{\delta_1(\delta_1+1)}-\frac{\gamma_1t_1^{2\delta_1+1}}{2\delta_1(2\delta_1+1)})+\gamma_1(\frac{t_1^{2\delta_1+1}}{\delta_1(2\delta_1+1)}-\frac{\gamma_1t_1^{3\delta_1+1}}{2\delta_1(3\delta_1+1)}) \quad ...(2.14)$$

6. SC_n =Shortage cost $=\frac{S_0}{T}\int_{t_1}^{T}(-I_2(t))dt$

$$SC_n=\frac{S_0}{T}[-\alpha\beta((t_1T-\frac{T^2}{2}-\frac{t_1^2}{2})+\frac{\alpha_1}{2}(t_1^2T-\frac{T^3}{3}-\frac{2t_1^3}{3})-\frac{\alpha_2}{3}(t_1^3T-\frac{T^4}{4}-\frac{3t_1^4}{4}))+\beta pB(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2})] \quad ...(2.15)$$

7. $LSC_n =$ Lost Sale Cost= $\frac{S_1}{T}\int_{t_1}^{T}(1-B)R(p,t)dt$

$$LSC_n=\frac{S_1(1-B)}{T}[\alpha((T-t_1)+\frac{\alpha_1}{2}(T^2-t_1^2)-\frac{\alpha_2}{3}(T^3-t_1^3))-\beta p(T-t_1)] \quad ...(2.16)$$

The constituents of profit function for the used product are as below.

1. $SR_u =$ Sales revenue from used product= $\frac{1}{T}\int_{\tau}^{T}[p(1-p_0)R_u(p,t)]dt$

$$SR_u = \frac{1}{T}[ap(1-p_0)(T-\tau-\frac{b}{2}(T^2-\tau^2)) - p^2(1-p_0)^2(T-\tau)] \qquad ...(2.17)$$

2. $PC_u =$ Purchase cost = $\left(\frac{C(1-d)Q_u}{T-\tau}\right)$

$$PC_u = \frac{C(1-d)}{(T-\tau)}[a((T-\tau)-\frac{b}{2}(T^2-\tau^2)+\frac{\gamma_2}{\delta_2+1}(T^{\delta_2+1}-\tau^{\delta_2+1})-\frac{\gamma_2 b}{\delta_2+2}(T^{\delta_2+2}-\tau^{\delta_2+2}))$$
$$-p(1-p_0)(T-\tau)-p(1-p_0)\frac{\gamma_2}{\delta_2+1}(T^{\delta_2+1}-\tau^{\delta_2+1})]e^{-\gamma_2\tau^{\delta_2}} \qquad ...(2.18)$$

3. $HC_u =$ Holding cost= $\frac{1}{T}\int_{\tau}^{T}[(h_1'+h_2't)I_u(t)]dt$

$$HC_u = \frac{h_1'}{T}\{a[(\frac{T^2}{2} - \frac{\gamma_2 T^{\delta_2+2}}{(\delta_2+1)(\delta_2+2)} - T\tau + \frac{\gamma_2 T\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^2}{2} - \frac{\gamma_2\tau^{\delta_2+2}}{(\delta_2+2)}) - \frac{b}{2}(\frac{2T^3}{3} -$$
$$\frac{2\gamma_2 T^{\delta_2+3}}{(\delta_2+1)(\delta_2+3)} - T^2\tau + \frac{\gamma_2 T^2\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^3}{3} - \frac{\gamma_2\tau^{\delta_2+3}}{(\delta_2+3)}) + \frac{\gamma_2}{(\delta_2+1)}(\frac{(\delta_2+1)T^{\delta_2+1}}{(\delta_2+2)}$$
$$- \frac{\gamma_2 T^{2\delta_2+2}}{(2\delta_2+2)(\delta_2+1)} - T^{\delta_2+1}\tau + \frac{\gamma_2 T^{\delta_2+1}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+2}}{(\delta_2+2)} - \frac{\gamma_2\tau^{2\delta_2+2}}{(2\delta_2+2)}) - \frac{b\gamma_2}{(\delta_2+2)}$$
$$(\frac{(\delta_2+2)T^{\delta_2+3}}{(\delta_2+3)} - \frac{(\delta_2+2)\gamma_2 T^{2\delta_2+3}}{(\delta_2+1)(2\delta_2+3)} - T^{\delta_2+2}\tau + \frac{\gamma_2 T^{\delta_2+2}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+3}}{(\delta_2+3)} - \frac{\gamma_2\tau^{2\delta_2+3}}{(2\delta_2+3)})]$$
$$-p(1-p_0)(\frac{T^2}{2} - \frac{\gamma_2 T^{\delta_2+2}}{(\delta_2+1)(\delta_2+2)} - T\tau + \frac{\gamma_2 T\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^2}{2} - \frac{\gamma_2\tau^{\delta_2+2}}{(\delta_2+2)}) - p(1-p_0)\frac{\gamma_2}{(\delta_2+1)}$$
$$(\frac{(\delta_2+1)T^{\delta_2+2}}{(\delta_2+2)} - \frac{\gamma_2 T^{2\delta_2+2}}{(2\delta_2+2)} - T^{\delta_2+1}\tau + \frac{\gamma_2 T^{\delta_2+1}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+2}}{(\delta_2+2)} - \frac{\gamma_2\tau^{2\delta_2+2}}{(2\delta_2+2)})\} + \frac{h_2'}{T}\{a[(\frac{T^3}{6} -$$
$$\frac{\gamma_2 T^{\delta_2+3}}{(\delta_2+2)(\delta_2+3)} - \frac{T\tau^2}{2} + \frac{\gamma_2 T\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^3}{3} - \frac{\gamma_2\tau^{\delta_2+3}}{(\delta_2+3)}) - \frac{b}{2}(\frac{T^4}{4} - \frac{2\gamma_2 T^{\delta_2+4}}{(\delta_2+2)(\delta_2+4)} - \frac{T^2\tau^2}{2} +$$
$$\frac{\gamma_2 T^2\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^4}{4} - \frac{\gamma_2\tau^{\delta_2+4}}{(\delta_2+4)}) + \frac{\gamma_2}{(\delta_2+1)}(\frac{(\delta_2+1)T^{\delta_2+3}}{2(\delta_2+3)} - \frac{(\delta_2+1)\gamma_2 T^{2\delta_2+3}}{(\delta_2+2)(2\delta_2+3)} - T^{\delta_2+1}\frac{\tau^2}{2} +$$
$$\frac{\gamma_2 T^{\delta_2+1}\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^{\delta_2+3}}{(\delta_2+3)} - \frac{\gamma_2\tau^{2\delta_2+3}}{(2\delta_2+3)}) - \frac{b\gamma_2}{(\delta_2+2)}(\frac{(\delta_2+2)T^{\delta_2+4}}{2(\delta_2+4)} - \frac{(\delta_2+2)\gamma_2 T^{2\delta_2+4}}{(\delta_2+2)(2\delta_2+4)}$$
$$-T^{\delta_2+1}\frac{\tau^2}{2} + \frac{\gamma_2 T^{\delta_2+2}\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^{\delta_2+4}}{(\delta_2+4)} - \frac{\gamma_2\tau^{2\delta_2+4}}{(2\delta_2+4)})] - p(1-p_0)(\frac{T^3}{6} - \frac{\gamma_2 T^{\delta_2+3}}{(\delta_2+2)(\delta_2+3)}$$
$$-\frac{T\tau^2}{2} + \frac{\gamma_2 T\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^3}{3} - \frac{\gamma_2\tau^{\delta_2+3}}{(\delta_2+3)}) - p(1-p_0)\frac{\gamma_2}{(\delta_2+1)}(\frac{(\delta_2+1)T^{\delta_2+3}}{2(\delta_2+3)} - \frac{(\delta_2+1)\gamma_2 T^{2\delta_2+3}}{(\delta_2+2)(2\delta_2+3)}$$
$$-T^{\delta_2+1}\frac{\tau^2}{2} + \frac{\gamma_2 T^{\delta_2+1}\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^{\delta_2+3}}{\delta_2+3} - \frac{\gamma_2\tau^{2\delta_2+3}}{(2\delta_2+3)})\} \qquad ...(2.19)$$

4. $DC_u =$ Deterioration cost $= \dfrac{D_2}{T}\int_{\tau}^{T}\gamma_2\delta_2 t^{\delta_2-1}I_u(t)\,dt$

$$DC_u=\frac{D_2\gamma_2\delta_2}{T}\{a[(\frac{T^{\delta_2+1}}{\delta_2(\delta_2+1)}-\frac{\gamma_2T^{2\delta_2+1}}{2\delta_2(2\delta_2+1)}-\frac{T\tau^{\delta_2}}{\delta_2}+\frac{T\gamma_2\tau^{2\delta_2}}{2\delta_2}+\frac{\tau^{\delta_2+1}}{(\delta_2+1)}-\frac{\gamma_2\tau^{2\delta_2+1}}{(2\delta_2+1)})$$
$$-\frac{b}{2}(\frac{2T^{\delta_2+2}}{\delta_2(\delta_2+2)}-\frac{\gamma_2T^{2\delta_2+2}}{2\delta_2(2\delta_2+2)}-\frac{T^2\tau^{\delta_2}}{\delta_2}+\frac{T^2\gamma_2\tau^{2\delta_2}}{2\delta_2}+\frac{\tau^{\delta_2+1}}{(\delta_2+1)}-\frac{\gamma_2\tau^{2\delta_2+2}}{(2\delta_2+2)})+\frac{\gamma_2}{(\delta_2+1)}$$
$$(\frac{(\delta_2+1)T^{2\delta_2+1}}{\delta_2(2\delta_2+1)}-\frac{(\delta_2+1)\gamma_2T^{3\delta_2+1}}{2\delta_2(3\delta_2+1)}-\frac{T^{(\delta_2+1)}\tau^{\delta_2}}{\delta_2}+\frac{T^{(\delta_2+1)}\gamma_2\tau^{2\delta_2}}{2\delta_2}+\frac{\tau^{2\delta_2+1}}{(2\delta_2+1)}-\frac{\gamma_2\tau^{3\delta_2+1}}{(3\delta_2+1)})$$
$$-\frac{b\gamma_2}{(\delta_2+2)}(\frac{(\delta_2+2)T^{2\delta_2+2}}{\delta_2(2\delta_2+2)}-\frac{(\delta_2+2)\gamma_2T^{3\delta_2+2}}{2\delta_2(3\delta_2+2)}-\frac{T^{(\delta_2+2)}\tau^{\delta_2}}{\delta_2}+\frac{T^{(\delta_2+2)}\gamma_2\tau^{2\delta_2}}{2\delta_2}+\frac{\tau^{2\delta_2+2}}{(2\delta_2+2)}$$
$$-\frac{\gamma_2\tau^{3\delta_2+2}}{(3\delta_2+2)})]-p(1-p_0)(\frac{T^{\delta_2+1}}{\delta_2(\delta_2+1)}-\frac{\gamma_2T^{2\delta_2+1}}{2\delta_2(2\delta_2+1)}-\frac{T\tau^{\delta_2}}{\delta_2}+\frac{T\gamma_2\tau^{2\delta_2}}{2\delta_2}+\frac{\tau^{\delta_2+1}}{(\delta_2+1)}$$
$$-\frac{\gamma_2\tau^{2\delta_2+1}}{(2\delta_2+1)})-p(1-p_0)\frac{\gamma_2}{(\delta_2+1)}(\frac{(\delta_2+1)T^{2\delta_2+1}}{\delta_2(2\delta_2+1)}-\frac{(\delta_2+1)\gamma_2T^{3\delta_2+1}}{2\delta_2(3\delta_2+1)}-\frac{T^{(\delta_2+1)}\tau^{\delta_2}}{\delta_2}$$
$$+\frac{T^{(\delta_2+1)}\gamma_2\tau^{2\delta_2}}{2\delta_2}+\frac{\tau^{2\delta_2+1}}{(2\delta_2+1)}-\frac{\gamma_2\tau^{3\delta_2+1}}{(3\delta_2+1)})\}\qquad ...(2.20)$$

Therefore, the total profit is given by $\pi(p,t_1)=(SR_n-HC_n-PC_n-OC_n-DC_n-SC_n-LSC_n)+(SR_u-HC_u-PC_u-DC_u)$

$$\pi(p,t_1)=\frac{1}{T}\int_0^T[pR(p,t)]dt-\frac{CQ}{T}-\frac{A}{T}-\frac{1}{T}\int_0^{t_1}[(h_1+h_2t)I_1(t)]dt-\frac{D_1}{T}\int_0^{t_1}\gamma_1\delta_1t^{\delta_1-1}I_1(t)\,dt$$
$$-\frac{S_0}{T}\int_{t_1}^{T}(-I_2(t))dt-\frac{S_1}{T}\int_{t_1}^{T}(1-B)R(p,t)dt+\frac{1}{T}\int_{\tau}^{T}[p(1-p_0)R_u(p,t)dt-\left(\frac{C(1-d)Q_u}{T-\tau}\right)$$
$$-\frac{1}{T}\int_{\tau}^{T}[(h_1'+h_2't)I_u(t)\,dt-\frac{D_2}{T}\int_{\tau}^{T}\gamma_2\delta_2t^{\delta_2-1}I_u(t)\,dt \qquad ...(2.21)$$

The total profit is a function of two variables p and T. Using the classical optimization technique, we calculate maximum profit for the numerical example provided in the next section.

2.3.1.1 Numerical Example

We use the following example to illustrate the theoretical results developed in this modal. To perform the numerical analysis, data have been taken from the literature in appropriate units. Using MATHEMATICA 8.0 we calculate the total profit.

Example1: We consider an inventory system with the following parameters in appropriate units:

$\beta=6, C=\$30, \alpha=50, \alpha_1=3\%, \alpha_2=4\%, \gamma_1=0.85, \delta_1=2, A=\$100, h_1=\$1.2, h_2=1,$
$D_1=0.01, S_0=1.5, B=0.8, S_1=20, a=10, p_0=0.9, \tau=0.35, b=0.15, h_1'=\$2.5, h_2'=1,$
$\gamma_2=0.9, \delta_2=3, D_2=0.03, d=0.01, r=0.3, T=1\,year$

For the model (I), when the total profit is taken without inflation. The optimal values of decision variables are obtained as $(p^*,t_1^*)=(119.237,3.37122)$.The maximum profit is $\pi_{\max}=\$279271$. The concavity of the profit function is shown in figure 2.1,2.2&2.3.

π

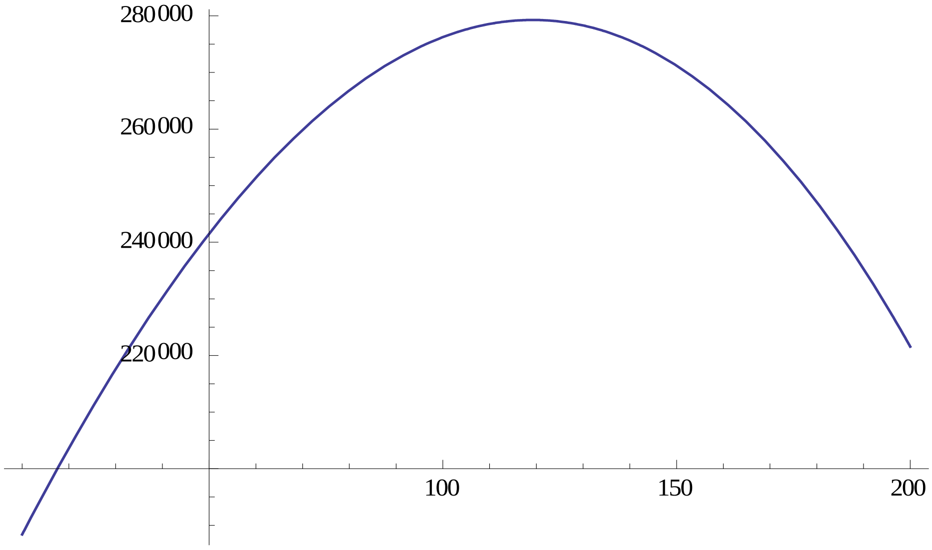

Figure 2. r of the total average profit functions π with respect to p

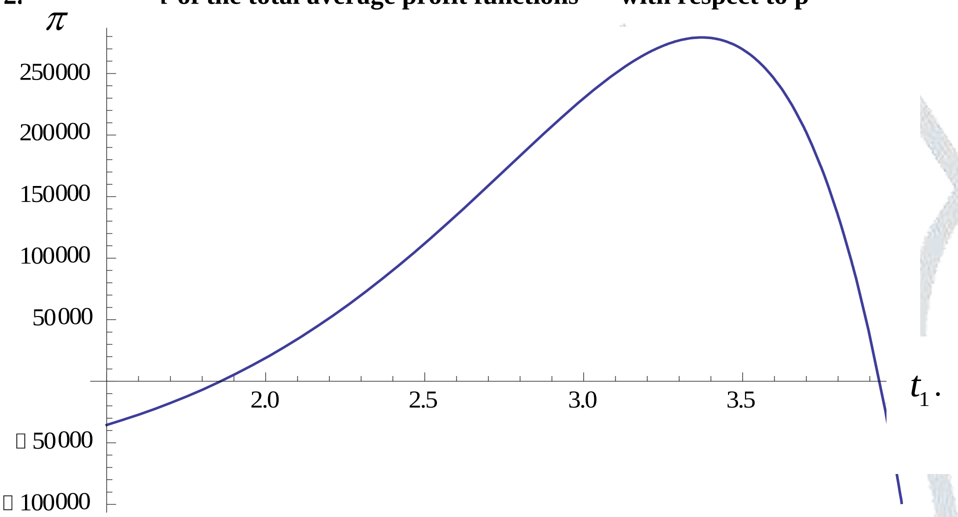

Figure 2.2: Behaviour of the total average profit functions π with respect to t_1.

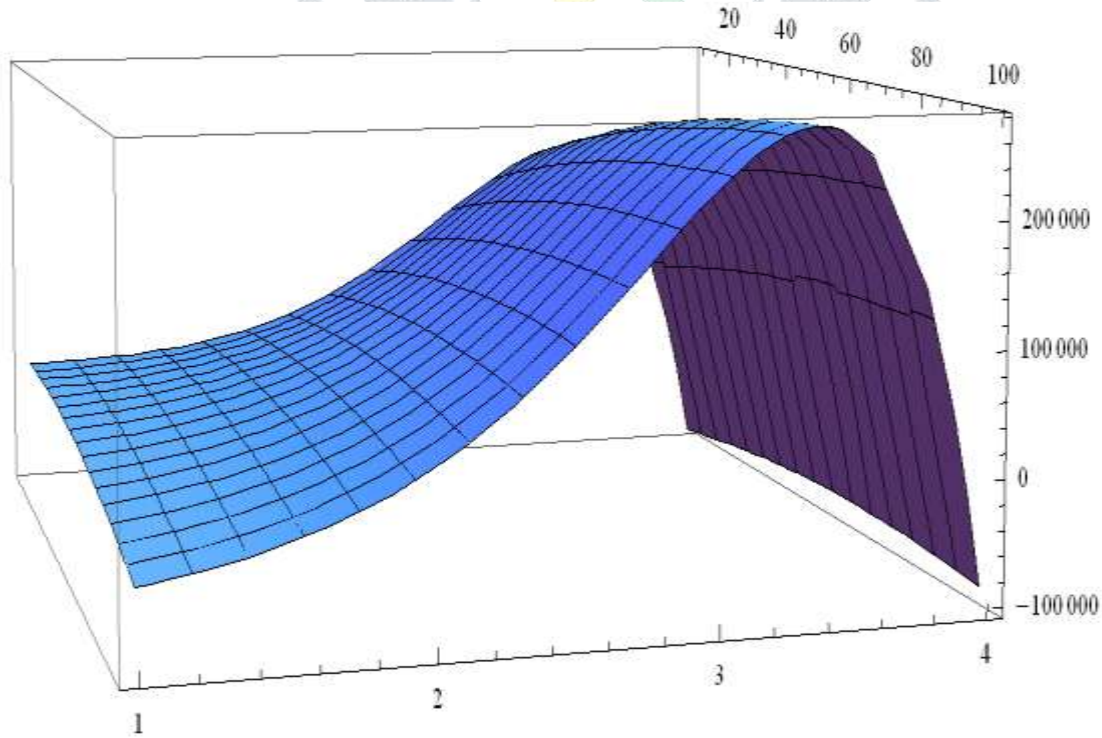

Figure2.3: Concavity of profit function for model (I)

2.3.1.2 Sensitivity Analysis

In the present table (2.1) we study the effects of changes in the values of the system parameters $A, C, h_1, h_2, h_1^{'}, h_2^{'}, p_0, \alpha \, \& \, \beta$ on the total system cost in consideration. The sensitivity analysis is performed by changing each of the parameters by -50%,-40%,-30%,-20%,-10%,10%,20%,30%,40% and 50% taking one parameter at a time and keeping the remaining parameter unchanged. The analysis is based on the results obtained from example 1.

Table 2.1: For the model (I), Sensitivity of the optimal value with respect to the model parameters.

Changing Parameters	Initial values	% % change in parameter V value	% % change in optimal values		
			p^*	t_1^*	π
A	100	+50%	119.237	3.37122	279221
		+40%	1 119.237	3.37122	279231
		+30%	1 119.237	3.37122	279241
		+20%	119.237	3.37122	279251
		+10%	119.237	3.37122	279261
		-10%	119.237	3.37122	279281
		-20%	119.237	3.37122	279291
		-30%	119.237	3.37122	2 279301
		-40%	119.237	3.37122	279311
		-50%	119.237	3.37122	279321
C	30	+50%	197.786	3.69403	739347
		+40%	183.202	-	7
		+30%	163.395	2.29602×10^{106}	7.2389×10^{957}
		+20%	149.96	-	
		+10%	126.581	2.09493×10^{106}	3.17262×10^{957}
				3.51091	430472
		-10%	119.312	-	
		-20%	89.9058	1.7012×10^{106}	4.87178×10^{957}
		-30%	78.2549	-	
		-40%	62.5565	1.32539×10^{106}	5.15197×10^{957}
		-50%	49.8823	3.21197	166589
				-	
				9.66519×10^{106}	3.00454×10^{957}
				3.02498	87612.1
				2.91669	59175.6
h_1	1.2	+50%	112.981	3.26724	241398
		+40%	114.119	3.28679	248097
		+30%	1 115.31	3.30693	255199
		+20%	116.558	3.32769	262737
		+10%	117.965	3.3491	270747
		-10%	122.092	-	

		-20% -30% -40% -50%	122.191 123.782 125.458 127.223	1.52898×10^{106} 3.41761 3.44197 3.46716 3.4932	1.86419×10^{957} 298045 308402 319488 331371
h_2	1	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	116.768 107.975 111.477 113.043 114 116.816 117.095 117.569 119.368 120.573	- 1.34265×10^{106} 3.18805 - 1.39967×10^{106} 3.27189 3.28798 - 1.47042×10^{106} 3.33866 - 1.49773×10^{106} 3.3746 3.39327	8.6340×10^{955} 215621 1.0898×10^{956} 243123 248751 1.5697×10^{956} 267367 1.8524×10^{956} 281426 289022
$h_1^{'}$	2 2.5	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	114.994 114.994 104.337 - 1.058×10^{104} - 7.116×10^{106} 114.993 114.992 114.992 1.078×10^{102} 114.991	3.30446 3.30446 -1.456×10^{106} - 2.2405×10^{103} 1. -1.116×10^{106} 3.30446 3.30446 3.30446 -1.675×10^{101} 3.30446	254656 254655 1.4381×10^{956} 6.9519×10^{930} 1.321×10^{102} 254655 254655 254655 5.075×10^{911} 254655
$h_2^{'}$	1	+50% +40% +30% +20% +10% -10% -20%	114.991 114991 114.991 8.024×10^{102} 1 114.991 114.991	3.30446 3.30446 3.30446 - 1.0056×10^{104} 3.30446 3.30446	2 254655 254655 254655 5.137×10^{936} 294655 254655

		-30% -40% -50%	1 114.991 114.459 114.459 114.991	3.30446 - - 1.4573×10^{106} 3.30446 3.30446	254655 1.448×10^{956} 254654 254654
p_0	0.9	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	1.9145×10^{18} 2.6997×10^{10} 4.3223×10^{17} 156.1 135.884 1.1154×10^{102} -7.44791 5.81578 4.73458 3.94756	-2.418×10^{122} -3.397×10^{114} - 5.4404×10^{121} 3.3016 - 1.723×10^{106} - 6.5610×10^{103} 3.30658 0.00518902 -0.0476088 -0.0842526	1.3438×10^{1102} 2.72×10^{1031} 2.0395×10^{1031} 276408 6.5442×10^{956} 1.099×10^{936} 143038 -651.673 -673.912 -692.5
α	50	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	10.7108 10.4739 10.3457 3.7328×10^{103} 111.341 119.641 114.291 113.942 113.593 113.245	0.0749329 0.0940838 0.121984 - 9.9928×10^{103} 3 3.37526 3.30354 3.30246 3 3.30118 3.29965 3.29777	-650.338 -650.546 -647.029 5.8240×10^{936} 279926 238385 222119 20 205856 189597 173342
β	6 6	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	119.249 183.674 117.859 117.028 116.081 119.984 4.630×10^{102} 9.62913 8.3458 -	3.315 - 1.5098×10^{106} 3.31117 3.30908 3.30684 - 1.44156×10^{106} - 9.97314×10^{103}	316747 1.9914×10^{956} 291855 279433 267032 1.3132×10^{956} 4.768×10^{956}

			4.647×10^{102}	0 0.00559658 -0.0311024 - 1.2384×10^{104}	-494.215 -429.604 3.3474×10^{937}

The study of above table (2.1) reveals the following interesting facts:

- Table 2.1 clearly shows that, when the value of p_0 decreases or increases, the selling price p, the inventory interval t_1 and the optimal total cost $\pi(p,t_1)$ increases or decreases, respectively. However the optimal total cost $\pi(p,t_1)$,the inventory interval t_1 and the selling prices p at 50%,40%,30%, and -10% of p_0 keep abnormal manners
- When the value of A changed there are no effect on the values of p and t_1, while the values of $\pi(p,t_1)$ changed simultaneously.
- The purchase cost C decreases or increases, the selling price p, the inventory interval t_1 and the optimal total cost $\pi(p,t_1)$ decreases or increases, respectively. But the optimal total cost $\pi(p,t_1)$ and the inventory interval t_1 at 40%,30%,10%,-10% and -30% of purchase cost C keep abnormal behaviors.
- Table 2.1 implies that, the holding cost of new product h_1 and h_2 decreases or increases, the selling price p, the inventory interval t_1 and the optimal total cost $\pi(p,t_1)$ increases or decreases, respectively. But the optimal total cost $\pi(p,t_1)$, the selling price p at -10% of h_1 and 50%, 30%,-10%,-30% of h_2 keep abnormal behaviors.
- With the decreases or increases of holding cost of used product $h_1^{'}$, the inventory interval t_1, the selling price p and the optimal total cost $\pi(p,t_1)$,increases or decreases, respectively but the inventory interval t_1, the selling price p and the optimal total cost $\pi(p,t_1)$ has no effect on increasing the value of holding cost $h_2^{'}$. However the optimal total cost $\pi(p,t_1)$,the inventory interval t_1 and the selling prices p at 30%,20%,10% and -40% of $h_1^{'}$ and at 20%,-30% of $h_2^{'}$ keep abnormal manners.

2.3.2 The Model with Inflation

Now to calculate total profit, we calculate all the constituents for both new product and used product under the effect of inflation. The constituents of profit function of the inventory system for new product are as follows.

1. SR_n = Sales revenue from new product = $\frac{1}{T}\int_0^T [pR(p,t)]\, e^{-rt}dt$

$$SR_n = p\alpha[(1+\frac{\alpha_1 T}{2}-\frac{\alpha_2 T^2}{3})-r(\frac{T}{2}+\frac{\alpha_1 T^2}{3}-\frac{\alpha_2 T^3}{4})]-\beta p^2(1-\frac{rT}{2}) \quad ...(2.22)$$

2. PC_n = Purchase cost = $\frac{CQ}{T}$

$$PC_n = \frac{C}{T}[\alpha(t_1+\frac{\alpha_1 t_1^2}{2}-\frac{\alpha_2 t_1^3}{3}+\frac{\gamma_1 t_1^{\delta_1+1}}{\delta_1+1}+\frac{\alpha_1\gamma_1 t_1^{\delta_1+2}}{\delta_1+2}-\frac{\alpha_2\gamma_1 t_1^{\delta_1+3}}{\delta_1+3})-\beta p(t_1+\frac{\gamma_1 t_1^{\delta_1+1}}{\delta_1+1})] \quad ...(2.23)$$

3. OC_n = Ordering cost $=\frac{A}{T}$ …(2.24)

4. $HC_n =$ Holding cost= $\frac{1}{T}\int_0^{t_1}[(h_1+h_2t)\,I_1(t)]e^{-rt}dt$

$$HC_n=\frac{h_1}{T}[\alpha(\frac{t_1^2}{2}-\frac{\gamma_1 t_1^{\delta_1+2}}{(\delta_1+1)(\delta_1+2)})+\alpha\alpha_1(\frac{t_1^3}{3}-\frac{\gamma_1 t_1^{\delta_1+3}}{(\delta_1+1)(\delta_1+3)})-\alpha\alpha_2(\frac{t_1^4}{4}-\frac{\gamma_1 t_1^{\delta_1+4}}{(\delta_1+1)(\delta_1+4)})+\alpha\gamma_1(\frac{t_1^{\delta_1+2}}{(\delta_1+2)}-\frac{\gamma_1 t_1^{2\delta_1+2}}{(\delta_1+1)(2\delta_1+2)})+\alpha\alpha_1\gamma_1(\frac{t_1^{\delta_1+3}}{(\delta_1+3)}-\frac{\gamma_1 t_1^{2\delta_1+3}}{(\delta_1+1)(2\delta_1+3)})-\alpha\alpha_2\gamma_1(\frac{t_1^{\delta_1+4}}{(\delta_1+4)}-\frac{\gamma_1 t_1^{2\delta_1+4}}{(\delta_1+1)(2\delta_1+4)})-\beta p((\frac{t_1^2}{2}-\frac{\gamma_1 t_1^{\delta_1+2}}{(\delta_1+1)(\delta_1+2)})+\gamma_1(\frac{t_1^{\delta_1+2}}{(\delta_1+2)}-\frac{\gamma_1 t_1^{2\delta_1+2}}{(\delta_1+1)(2\delta_1+2)}))+\frac{h_2}{T}[\alpha(\frac{t_1^3}{6}-\frac{\gamma_1 t_1^{\delta_1+3}}{(\delta_1+2)(\delta_1+3)})+\alpha\alpha_1(\frac{t_1^4}{8}-\frac{\gamma_1 t_1^{\delta_1+4}}{(\delta_1+2)(\delta_1+4)})-\alpha\alpha_2(\frac{t_1^5}{10}-\frac{\gamma_1 t_1^{\delta_1+5}}{(\delta_1+2)(\delta_1+5)})+\alpha\gamma_1(\frac{t_1^{\delta_1+3}}{2(\delta_1+3)}-\frac{\gamma_1 t_1^{2\delta_1+3}}{(\delta_1+2)(2\delta_1+3)})+\alpha\alpha_1\gamma_1(\frac{t_1^{\delta_1+4}}{2(\delta_1+4)}-\frac{\gamma_1 t_1^{2\delta_1+4}}{(\delta_1+2)(2\delta_1+4)})-\alpha\alpha_2\gamma_1(\frac{t_1^{\delta_1+5}}{2(\delta_1+5)}-\frac{\gamma_1 t_1^{2\delta_1+5}}{(\delta_1+2)(2\delta_1+5)})-\beta p((\frac{t_1^3}{6}-\frac{\gamma_1 t_1^{\delta_1+3}}{(\delta_1+2)(\delta_1+3)})+\gamma_1(\frac{t_1^{\delta_1+3}}{2(\delta_1+3)}-\frac{\gamma_1 t_1^{2\delta_1+3}}{(\delta_1+2)(2\delta_1+3)}))-\frac{rh_1}{T}[\frac{\alpha t_1^3}{6}+\frac{\alpha\alpha_1 t_1^4}{8}-\frac{\alpha\alpha_2 t_1^5}{10}+\frac{\alpha\gamma_1 t_1^{\delta_1+3}}{2(\delta_1+3)}+\frac{\alpha\alpha_1\gamma_1 t_1^{\delta_1+4}}{2(\delta_1+4)}-\frac{\alpha\alpha_2\gamma_1 t_1^{\delta_1+5}}{2(\delta_1+5)}-\beta p(\frac{t_1^3}{6}+\frac{\gamma_1 t_1^{\delta_1+3}}{2(\delta_1+3)})]-\frac{rh_2}{T}[\frac{\alpha t_1^4}{12}+\frac{\alpha\alpha_1 t_1^5}{15}-\frac{\alpha\alpha_2 t_1^6}{18}+\frac{\alpha\gamma_1 t_1^{\delta_1+4}}{3(\delta_1+4)}+\frac{\alpha\alpha_1\gamma_1 t_1^{\delta_1+5}}{3(\delta_1+5)}-\frac{\alpha\alpha_2\gamma_1 t_1^{\delta_1+6}}{3(\delta_1+6)}-\beta p(\frac{t_1^4}{12}+\frac{\gamma_1 t_1^{\delta_1+4}}{3(\delta_1+4)})] \quad ...(2.25)$$

5. $DC_n =$ Deterioration cost = $\frac{D_1}{T}\int_0^{t_1}\gamma_1\delta_1 t^{\delta_1-1}I_1(t)e^{-rt}dt$

$$DC_n=\frac{D_1\gamma_1\delta_1}{T}[\alpha(\frac{t_1^{\delta_1+1}}{\delta_1(\delta_1+1)}-\frac{\gamma_1 t_1^{2\delta_1+1}}{2\delta_1(2\delta_1+1)})+\alpha\alpha_1(\frac{t_1^{\delta_1+2}}{\delta_1(\delta_1+2)}-\frac{\gamma_1 t_1^{2\delta_1+2}}{2\delta_1(2\delta_1+2)})-\alpha\alpha_2(\frac{t_1^{\delta_1+3}}{\delta_1(\delta_1+3)}-\frac{\gamma_1 t_1^{2\delta_1+3}}{2\delta_1(2\delta_1+3)})+\alpha\gamma_1(\frac{t_1^{2\delta_1+1}}{\delta_1(2\delta_1+1)}-\frac{\gamma_1 t_1^{3\delta_1+1}}{2\delta_1(3\delta_1+1)})+\alpha\alpha_1\gamma_1(\frac{t_1^{2\delta_1+2}}{\delta_1(2\delta_1+2)}-\frac{\gamma_1 t_1^{3\delta_1+2}}{2\delta_1(3\delta_1+2)})-\alpha\alpha_2\gamma_1(\frac{t_1^{2\delta_1+3}}{\delta_1(2\delta_1+3)}-\frac{\gamma_1 t_1^{3\delta_1+3}}{2\delta_1(3\delta_1+3)})-\beta p((\frac{t_1^{\delta_1+1}}{\delta_1(\delta_1+1)}-\frac{\gamma_1 t_1^{2\delta_1+1}}{2\delta_1(2\delta_1+1)})+\gamma_1(\frac{t_1^{2\delta_1+1}}{\delta_1(2\delta_1+1)}-\frac{\gamma_1 t_1^{3\delta_1+1}}{2\delta_1(3\delta_1+1)}))]-\frac{D_1\gamma_1\delta_1 r}{T}[\frac{\alpha t_1^{\delta_1+2}}{(\delta_1+1)(\delta_1+2)}+\frac{\alpha\alpha_1 t_1^{\delta_1+3}}{(\delta_1+1)(\delta_1+3)}-\frac{\alpha\alpha_2 t_1^{\delta_1+4}}{(\delta_1+1)(\delta_1+4)}+\frac{\alpha\gamma_1 t_1^{2\delta_1+2}}{(\delta_1+1)(2\delta_1+2)}+\frac{\alpha\alpha_1\gamma_1 t_1^{2\delta_1+3}}{(\delta_1+1)(2\delta_1+3)}-\frac{\alpha\alpha_2\gamma_1 t_1^{2\delta_1+4}}{(\delta_1+1)(2\delta_1+4)}-\beta p(\frac{t_1^{\delta_1+2}}{(\delta_1+1)(\delta_1+2)}+\frac{\gamma_1 t_1^{2\delta_1+2}}{(\delta_1+1)(2\delta_1+2)})] \quad ...(2.26)$$

6. SC_n =Shortage cost = $\frac{S_0}{T}\int_{t_1}^{T}(-I_2(t))e^{-rt}dt$

$$SC_n = \frac{S_0}{T}[-\alpha\beta((t_1T - \frac{T^2}{2} - \frac{t_1^2}{2}) + \frac{\alpha_1}{2}(t_1^2T - \frac{T^3}{3} - \frac{2t_1^3}{3}) - \frac{\alpha_2}{3}(t_1^3T - \frac{T^4}{4} - \frac{3t_1^4}{4}))$$
$$+\beta pB(t_1T - \frac{T^2}{2} - \frac{t_1^2}{2})] - \frac{S_0 r}{T}[-\alpha\beta((\frac{t_1T^2}{2} - \frac{T^3}{3} - \frac{t_1^3}{6}) + \frac{\alpha_1}{2}(\frac{t_1^2T^2}{2} - \frac{T^4}{4} - \frac{t_1^4}{4})$$
$$-\frac{\alpha_2}{3}(\frac{t_1^3T^2}{2} - \frac{T^5}{5} - \frac{3t_1^5}{10})) + \beta pB(\frac{t_1T^2}{2} - \frac{T^3}{3} - \frac{t_1^3}{6})] \qquad ...(2.27)$$

7. $LSC_n =$ Lost Sale Cost= $\frac{S_1}{T}\int_{t_1}^{T}(1-B)R(p,t)e^{-rt}dt$

$$LSC_n = \frac{S_1(1-B)}{T}[\alpha((T-t_1) + \frac{\alpha_1}{2}(T^2 - t_1^2) - \frac{\alpha_2}{3}(T^3 - t_1^3)) - \beta p(T-t_1)] - \frac{S_1(1-B)r}{T}$$
$$[\alpha(\frac{1}{2}(T^2 - t_1^2) + \frac{\alpha_1}{3}(T^3 - t_1^3) - \frac{\alpha_2}{4}(T^4 - t_1^4)) - \frac{\beta p}{2}(T^2 - t_1^2)] \qquad ...(2.28)$$

The constituents of profit function with inflation for the used product are as below

1. $SR_u =$ Sales revenue from used product= $\frac{1}{T}\int_{\tau}^{T}[p(1-p_0)R_u(p,t)e^{-rt}dt$

$$SR_u = \frac{1}{T}[ap(1-p_0)(T-\tau - \frac{b}{2}(T^2-\tau^2)) - p^2(1-p_0)^2(T-\tau)] - \frac{r}{T}[ap(1-p_0)(\frac{1}{2}(T^2-\tau^2)$$
$$-\frac{b}{3}(T^3-\tau^3)) - p^2(1-p_0)^2\frac{1}{2}(T^2-\tau^2)] \qquad ...(2.29)$$

2. $PC_u =$ Purchase cost $= \left(\frac{C(1-d)Q_u}{T-\tau}\right)$

$$PC_u = \frac{C(1-d)}{(T-\tau)}[a((T-\tau) - \frac{b}{2}(T^2-\tau^2) + \frac{\gamma_2}{\delta_2+1}(T^{\delta_2+1} - \tau^{\delta_2+1}) - \frac{\gamma_2 b}{\delta_2+2}(T^{\delta_2+2} - \tau^{\delta_2+2}))$$
$$-p(1-p_0)(T-\tau) - p(1-p_0)\frac{\gamma_2}{\delta_2+1}(T^{\delta_2+1} - \tau^{\delta_2+1})]e^{-\gamma_2\tau^{\delta_2}} \qquad ...(2.30)$$

3. $HC_u =$ Holding cost= $\frac{1}{T}\int_{\tau}^{T}[(h_1' + h_2't)e^{-rt}I_u(t)]dt$

$$HC_u = \frac{h_1'}{T}\{a[(\frac{T^2}{2} - \frac{\gamma_2 T^{\delta_2+2}}{(\delta_2+1)(\delta_2+2)} - T\tau + \frac{\gamma_2 T\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^2}{2} - \frac{\gamma_2\tau^{\delta_2+2}}{(\delta_2+2)}) - \frac{b}{2}(\frac{2T^3}{3} -$$

$$\frac{2\gamma_2 T^{\delta_2+3}}{(\delta_2+1)(\delta_2+3)} - T^2\tau + \frac{\gamma_2 T^2\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^3}{3} - \frac{\gamma_2\tau^{\delta_2+3}}{(\delta_2+3)}) + \frac{\gamma_2}{(\delta_2+1)}(\frac{(\delta_2+1)T^{\delta_2+1}}{(\delta_2+2)} -$$

$$\frac{\gamma_2 T^{2\delta_2+2}}{(2\delta_2+2)(\delta_2+1)} - T^{\delta_2+1}\tau + \frac{\gamma_2 T^{\delta_2+1}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+2}}{(\delta_2+2)} - \frac{\gamma_2\tau^{2\delta_2+2}}{(2\delta_2+2)}) - \frac{b\gamma_2}{(\delta_2+2)}$$

$$(\frac{(\delta_2+2)T^{\delta_2+3}}{(\delta_2+3)} - \frac{(\delta_2+2)\gamma_2 T^{2\delta_2+3}}{(\delta_2+1)(2\delta_2+3)} - T^{\delta_2+2}\tau + \frac{\gamma_2 T^{\delta_2+2}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+3}}{(\delta_2+3)} - \frac{\gamma_2\tau^{2\delta_2+3}}{(2\delta_2+3)})]$$

$$-p(1-p_0)(\frac{T^2}{2} - \frac{\gamma_2 T^{\delta_2+2}}{(\delta_2+1)(\delta_2+2)} - T\tau + \frac{\gamma_2 T\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^2}{2} - \frac{\gamma_2\tau^{\delta_2+2}}{(\delta_2+2)}) - p(1-p_0)\frac{\gamma_2}{(\delta_2+1)}$$

$$(\frac{(\delta_2+1)T^{\delta_2+2}}{(\delta_2+2)} - \frac{\gamma_2 T^{2\delta_2+2}}{(2\delta_2+2)} - T^{\delta_2+1}\tau + \frac{\gamma_2 T^{\delta_2+1}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+2}}{(\delta_2+2)} - \frac{\gamma_2\tau^{2\delta_2+2}}{(2\delta_2+2)})\} + \frac{h_2'}{T}\{a[(\frac{T^3}{6}$$

$$-\frac{\gamma_2 T^{\delta_2+3}}{(\delta_2+2)(\delta_2+3)} - \frac{T\tau^2}{2} + \frac{\gamma_2 T\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^3}{3} - \frac{\gamma_2\tau^{\delta_2+3}}{(\delta_2+3)}) - \frac{b}{2}(\frac{T^4}{4} - \frac{2\gamma_2 T^{\delta_2+4}}{(\delta_2+2)(\delta_2+4)} - \frac{T^2\tau^2}{2}$$

$$+\frac{\gamma_2 T^2\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^4}{4} - \frac{\gamma_2\tau^{\delta_2+4}}{(\delta_2+4)}) + \frac{\gamma_2}{(\delta_2+1)}(\frac{(\delta_2+1)T^{\delta_2+3}}{2(\delta_2+3)} - \frac{(\delta_2+1)\gamma_2 T^{2\delta_2+3}}{(\delta_2+2)(2\delta_2+3)} - T^{\delta_2+1}\frac{\tau^2}{2} +$$

$$\frac{\gamma_2 T^{\delta_2+1}\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^{\delta_2+3}}{(\delta_2+3)} - \frac{\gamma_2\tau^{2\delta_2+3}}{(2\delta_2+3)}) - \frac{b\gamma_2}{(\delta_2+2)}(\frac{(\delta_2+2)T^{\delta_2+4}}{2(\delta_2+4)} - \frac{(\delta_2+2)\gamma_2 T^{2\delta_2+4}}{(\delta_2+2)(2\delta_2+4)}$$

$$-T^{\delta_2+1}\frac{\tau^2}{2} + \frac{\gamma_2 T^{\delta_2+2}\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^{\delta_2+4}}{(\delta_2+4)} - \frac{\gamma_2\tau^{2\delta_2+4}}{(2\delta_2+4)})] - p(1-p_0)(\frac{T^3}{6} - \frac{\gamma_2 T^{\delta_2+3}}{(\delta_2+2)(\delta_2+3)}$$

$$-\frac{T\tau^2}{2} + \frac{\gamma_2 T\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^3}{3} - \frac{\gamma_2\tau^{\delta_2+3}}{(\delta_2+3)}) - p(1-p_0)\frac{\gamma_2}{(\delta_2+1)}(\frac{(\delta_2+1)T^{\delta_2+3}}{2(\delta_2+3)} - \frac{(\delta_2+1)\gamma_2 T^{2\delta_2+3}}{(\delta_2+2)(2\delta_2+3)}$$

$$-T^{\delta_2+1}\frac{\tau^2}{2} + \frac{\gamma_2 T^{\delta_2+1}\tau^{\delta_2+2}}{(\delta_2+2)} + \frac{\tau^{\delta_2+3}}{\delta_2+3} - \frac{\gamma_2\tau^{2\delta_2+3}}{(2\delta_2+3)})\} - \frac{rh_1'}{T}\{a[(\frac{T^3}{6} - \frac{T\tau^2}{2} + \frac{\tau^3}{3}) - \frac{b}{2}$$

$$(\frac{T^4}{4} - \frac{T^2\tau^2}{2} + \frac{\tau^4}{4}) + \frac{\gamma_2}{\delta_2+1}(\frac{(\delta_2+1)T^{\delta_2+3}}{2(\delta_2+3)} - \frac{T^{(\delta_2+1)}\tau^2}{2} + \frac{\tau^{\delta_2+3}}{(\delta_2+3)}) - \frac{b\gamma_2}{\delta_2+2}(\frac{(\delta_2+2)T^{\delta_2+4}}{2(\delta_2+4)}$$

$$-\frac{T^{(\delta_2+2)}\tau^2}{2} + \frac{\tau^{\delta_2+4}}{(\delta_2+4)})] - p(1-p_0)(\frac{T^3}{6} - \frac{T\tau^2}{2} + \frac{\tau^3}{3}) - p(1-p_0)\frac{\gamma_2}{\delta_2+1}(\frac{(\delta_2+1)T^{\delta_2+3}}{2(\delta_2+3)}$$

$$-\frac{T^{(\delta_2+1)}\tau^2}{2} + \frac{\tau^{\delta_2+3}}{(\delta_2+3)}) - \frac{rh_2'}{T}\{a[(\frac{T^4}{12} - \frac{T\tau^3}{3} + \frac{\tau^4}{4}) - \frac{b}{2}(\frac{2T^5}{15} - \frac{T^2\tau^3}{3} + \frac{\tau^5}{5})$$

$$+\frac{\gamma_2}{\delta_2+1}(\frac{(\delta_2+1)T^{\delta_2+4}}{3(\delta_2+4)} - \frac{T^{(\delta_2+1)}\tau^3}{3} + \frac{\tau^{\delta_2+4}}{(\delta_2+4)}) - \frac{b\gamma_2}{\delta_2+2}(\frac{(\delta_2+2)T^{\delta_2+5}}{3(\delta_2+5)} - \frac{T^{(\delta_2+2)}\tau^3}{3}$$

$$+\frac{\tau^{\delta_2+5}}{(\delta_2+5)})] - p(1-p_0)(\frac{T^4}{12} - \frac{T\tau^3}{3} + \frac{\tau^4}{4}) - p(1-p_0)\frac{\gamma_2}{\delta_2+1}(\frac{(\delta_2+1)T^{\delta_2+4}}{3(\delta_2+4)} - \frac{T^{(\delta_2+1)}\tau^3}{3}$$

$$+\frac{\tau^{\delta_2+4}}{(\delta_2+4)}) \qquad ...(2.30)$$

4. $DC_u =$ Deterioration cost = $\frac{D_2}{T}\int_{\tau}^{T} \gamma_2\delta_2 t^{\delta_2-1} I_u(t) e^{-rt} dt$

$$DC_u = \frac{D_2\gamma_2\delta_2}{T}\{a[(\frac{T^{\delta_2+1}}{\delta_2(\delta_2+1)} - \frac{\gamma_2 T^{2\delta_2+1}}{2\delta_2(2\delta_2+1)} - \frac{T\tau^{\delta_2}}{\delta_2} + \frac{T\gamma_2\tau^{2\delta_2}}{2\delta_2} + \frac{\tau^{\delta_2+1}}{(\delta_2+1)} - \frac{\gamma_2\tau^{2\delta_2+1}}{(2\delta_2+1)})$$

$$-\frac{b}{2}(\frac{2T^{\delta_2+2}}{\delta_2(\delta_2+2)} - \frac{\gamma_2 T^{2\delta_2+2}}{2\delta_2(2\delta_2+2)} - \frac{T^2\tau^{\delta_2}}{\delta_2} + \frac{T^2\gamma_2\tau^{2\delta_2}}{2\delta_2} + \frac{\tau^{\delta_2+1}}{(\delta_2+1)} - \frac{\gamma_2\tau^{2\delta_2+2}}{(2\delta_2+2)}) + \frac{\gamma_2}{(\delta_2+1)}$$

$$(\frac{(\delta_2+1)T^{2\delta_2+1}}{\delta_2(2\delta_2+1)} - \frac{(\delta_2+1)\gamma_2 T^{3\delta_2+1}}{2\delta_2(3\delta_2+1)} - \frac{T^{(\delta_2+1)}\tau^{\delta_2}}{\delta_2} + \frac{T^{(\delta_2+1)}\gamma_2\tau^{2\delta_2}}{2\delta_2} + \frac{\tau^{2\delta_2+1}}{(2\delta_2+1)} - \frac{\gamma_2\tau^{3\delta_2+1}}{(3\delta_2+1)})$$

$$-\frac{b\gamma_2}{(\delta_2+2)}(\frac{(\delta_2+2)T^{2\delta_2+2}}{\delta_2(2\delta_2+2)} - \frac{(\delta_2+2)\gamma_2 T^{3\delta_2+2}}{2\delta_2(3\delta_2+2)} - \frac{T^{(\delta_2+2)}\tau^{\delta_2}}{\delta_2} + \frac{T^{(\delta_2+2)}\gamma_2\tau^{2\delta_2}}{2\delta_2} + \frac{\tau^{2\delta_2+2}}{(2\delta_2+2)}$$

$$-\frac{\gamma_2\tau^{3\delta_2+2}}{(3\delta_2+2)})] - p(1-p_0)(\frac{T^{\delta_2+1}}{\delta_2(\delta_2+1)} - \frac{\gamma_2 T^{2\delta_2+1}}{2\delta_2(2\delta_2+1)} - \frac{T\tau^{\delta_2}}{\delta_2} + \frac{T\gamma_2\tau^{2\delta_2}}{2\delta_2} + \frac{\tau^{\delta_2+1}}{(\delta_2+1)} -$$

$$\frac{\gamma_2\tau^{2\delta_2+1}}{(2\delta_2+1)}) - p(1-p_0)\frac{\gamma_2}{(\delta_2+1)}(\frac{(\delta_2+1)T^{2\delta_2+1}}{\delta_2(2\delta_2+1)} - \frac{(\delta_2+1)\gamma_2 T^{3\delta_2+1}}{2\delta_2(3\delta_2+1)} - \frac{T^{(\delta_2+1)}\tau^{\delta_2}}{\delta_2} + \frac{T^{(\delta_2+1)}\gamma_2\tau^{2\delta_2}}{2\delta_2}$$

$$+\frac{\tau^{2\delta_2+1}}{(2\delta_2+1)} - \frac{\gamma_2\tau^{3\delta_2+1}}{(3\delta_2+1)})\} - \frac{rD_2\gamma_2\delta_2}{T}[a\{(\frac{T^{\delta_2+2}}{(\delta_2+1)(\delta_2+2)} - \frac{T\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+2}}{(\delta_2+2)}) - \frac{b}{2}$$

$$(\frac{2T^{\delta_2+3}}{(\delta_2+1)(\delta_2+3)} - \frac{T^2\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+3}}{(\delta_2+3)}) + \frac{\gamma_2}{(\delta_2+1)}(\frac{T^{2\delta_2+2}}{(2\delta_2+2)} - \frac{T^{\delta_2+1}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{2\delta_2+2}}{(2\delta_2+2)})$$

$$-\frac{b\gamma_2}{(\delta_2+2)}(\frac{(\delta_2+2)T^{2\delta_2+3}}{(\delta_2+1)(2\delta_2+3)} - \frac{T^{\delta_2+2}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{2\delta_2+3}}{(2\delta_2+3)})\} - p(1-p_0)(\frac{T^{\delta_2+2}}{(\delta_2+1)(\delta_2+2)}$$

$$-\frac{T\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{\delta_2+2}}{(\delta_2+2)}) - p(1-p_0)\frac{\gamma_2}{(\delta_2+1)}(\frac{T^{2\delta_2+2}}{(2\delta_2+2)} - \frac{T^{\delta_2+1}\tau^{\delta_2+1}}{(\delta_2+1)} + \frac{\tau^{2\delta_2+2}}{(2\delta_2+2)})] \qquad ...(2.31)$$

Therefore, the total profit is given by

$$\pi(p,T) = (SR_n - HC_n - PC_n - OC_n - DC_n - SC_n - LSC_n) + (SR_u - HC_u - PC_u - DC_u)$$

$$\pi(p,T) = \frac{1}{T}\int_0^T [pR(p,t)]e^{-rt}dt - \frac{CQ}{T} - \frac{A}{T} - \frac{1}{T}\int_0^{t_1}[(h_1 + h_2 t)I_1(t)]e^{-rt}dt -$$

$$\frac{D_1}{T}\int_0^{t_1}\gamma_1\delta_1 t^{\delta_1-1}I_1(t)e^{-rt}dt - \frac{S_0}{T}\int_{t_1}^{T}(-I_2(t))e^{-rt}dt - \frac{S_1}{T}\int_{t_1}^{T}(1-B)R(p,t)e^{-rt}dt +$$

$$\frac{1}{T}\int_\tau^T [p(1-p_0)R_u(p,t)e^{-rt}dt - \left(\frac{C(1-d)Q_u}{T-\tau}\right) -$$

$$\frac{1}{T}\int_\tau^T [(h_1' + h_2' t)I_u(t)e^{-rt}]dt - \frac{D_2}{T}\int_\tau^T \gamma_2\delta_2 t^{\delta_2-1}I_u(t)e^{-rt}dt \qquad ...(2.32)$$

The total profit is a function of two variables p and T. Using the classical optimization technique, we calculate maximum profit for the numerical example provided in the next section.

2.3.2.1 Numerical Example

We use the following example to illustrate the theoretical results developed in this modal. To perform the numerical analysis, data have been taken from the literature in appropriate units. Using MATHEMATICA 8.0 we calculate the total profit.

Example1: We consider an inventory system with the following parameters in appropriate units:

$\beta = 6, C = \$30, \alpha = 50, \alpha_1 = 3\%, \alpha_2 = 4\%, \gamma_1 = 0.85, \delta_1 = 2, A = \$100, h_1 = \$1.2, h_2 = 1,$
$D_1 = 0.01, S_0 = 1.5, B = 0.8, S_1 = 20, a = 10, p_0 = 0.9, \tau = 0.35, b = 0.15, h_1^{'} = \$2.5, h_2^{'} = 1,$
$\gamma_2 = 0.9, \delta_2 = 3, D_2 = 0.03, d = 0.01, r = 0.3, T = 1\, year$

For the model (II), when the total profit is taken with inflation the optimal values of decision variables are obtained as $(p^*, t_1^*) = (148.912, 3.23856)$.The maximum profit is $\pi_{\max} = \$245762$. The concavity of the profit function is shown in figure 2.4, 2.5 & 2.6.

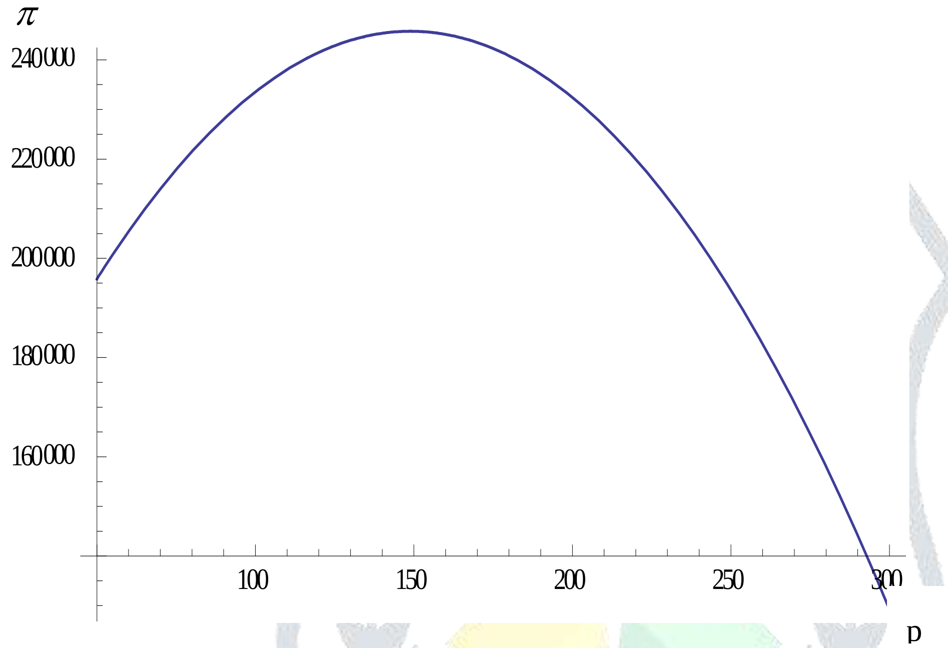

Figure.2.4 the total average profit functions π with respect to p

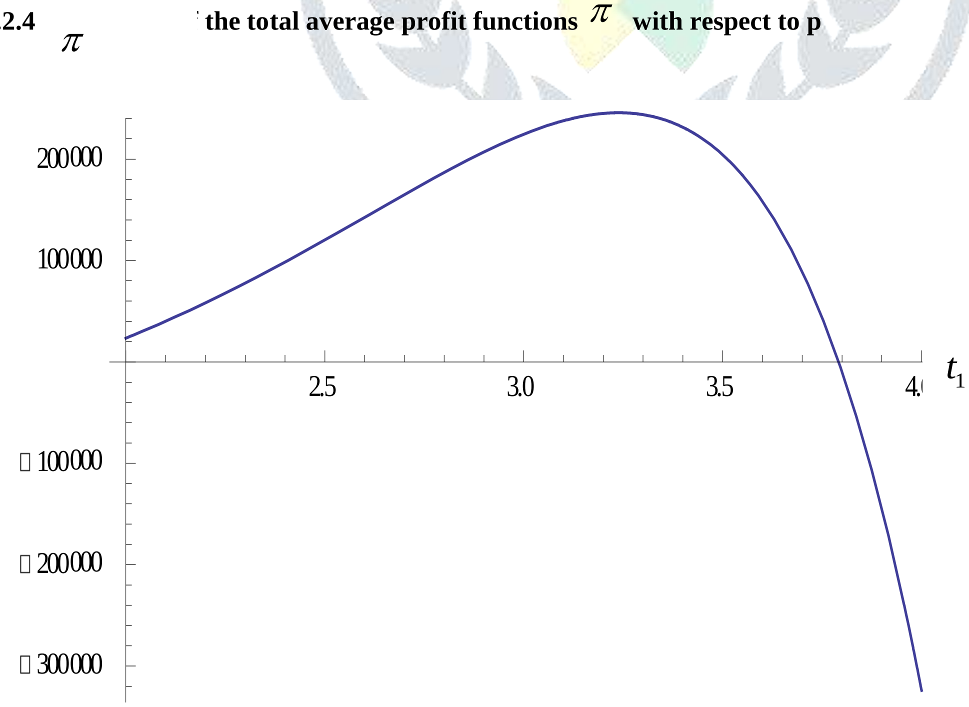

Figure 2.5: Behaviour of the total average profit functions π with respect to t_1.

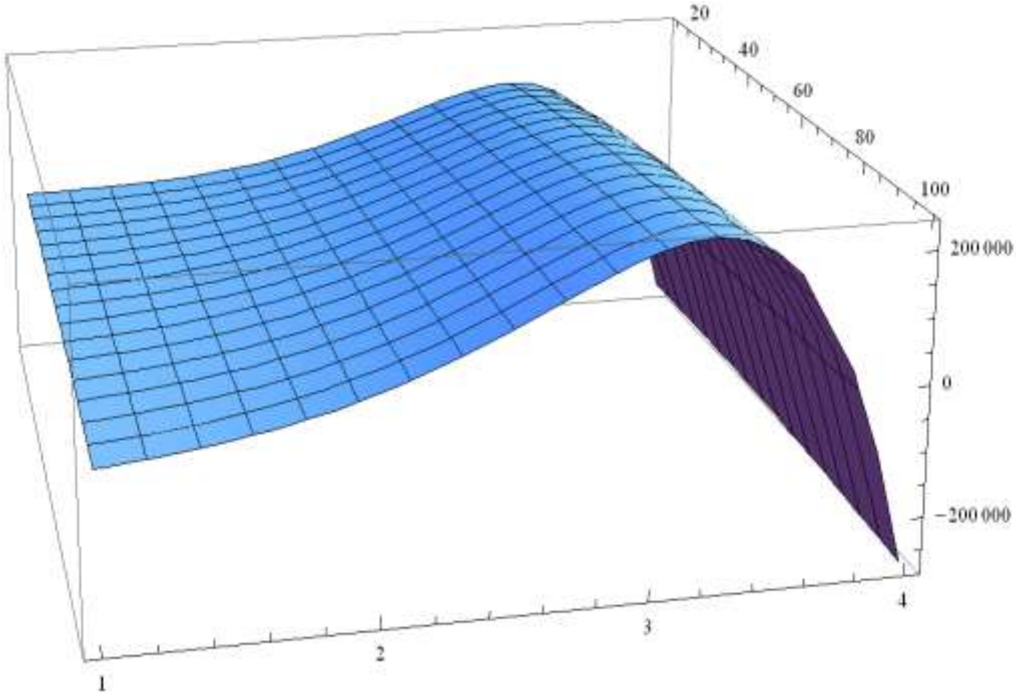

Figure2.6: Concavity of profit function for model (II)

2.3.2.2 Sensitivity Analysis

In the present table we study the effects of changes in the values of the system parameters $A, C, h_1, h_2, h_1^{'}, h_2^{'}, p_0, \alpha \,\&\, \beta$ on the total system cost in consideration. The sensitivity analysis is performed by changing each of the parameters by -50%,-40%,-30%,-20%,-10%,10%,20%,30%,40% and 50% taking one parameter at a time and keeping the remaining parameter unchanged. The analysis is based on the results obtained from example 1.

Table 2.2: For the model (II), Sensitivity of the optimal value with respect to the model parameters.

Changing Parameters	Initial values	% change in parameter value	% change in optimal values		
			p^*	t_1^*	π
		+50%	148.912	3.23856	245712
		+40%	148.912	3.23856	245722
		+30%	148.912	3.23856	245732
		+20%	148.912	3.23856	245742
A		+10%	148.912	3.23856	245752
	100	-10%	148.912	3.23856	245772
		-20%	148.912	3.23856	245782
		-30%	148.912	3.23856	245792
		-40%	148.912	3.23856	245802
		-50%	148.912	3.23856	255812
		+50%	273.256	- 3.4667	
		+40%	247.683	$\times 10^{106}$	2.869×10^{957}
		+30%	221.098	3.50806	592920
		+20%	195.756	3.44621	486649
		+10%	171.685	3.38101	394043

C	30	-10% -20% -30% -40% -50%	127.47 108.72 88.7351 71.5361 59.4823	3.31198 3.160 -2.9176×10^{102} 2.98344 2.88246 -7.14002×10^{102}	314080 188114 5.929×10^{922} 101045 69790.5 1.8662×10^{953}
h_1	1.2	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	138.48 140.365 142.344 144.423 146.61 151.339 153.899 155.222 159.464 159.515	3.12522 3.14647 3.1684 3.19103 3.21441 3.26352 3.28934 -1.9807×10^{106} 3.3437 -2.0546×10^{106}	206953 213744 220978 228695 236940 255218 265369 1.8159×10^{957} 288045 2.525×10^{957}
h_2	1	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	129.126 132.138 132.21 139.754 146.175 154.372 160.597 167.781 176.19 183.758	3.01488 3.05271 -1.7231×10^{106} 3.13767 -1.8236×10^{106} 3.2968 3.36178 3.43492 3.51837 -2.352×10^{106}	176040 186551 6.7942×10^{956} 212005 9.5266×10^{956} 267126 292633 323611 362023 4.1162×10^{957}
h_1'	2.5	+50% +40% +30% +20% +10% -10% -20% -30% -40%	148.914 148.914 148.913 148.913 148.913 145.166 148.911 148.911 148.911	3.23856 3.23856 -1.884×10^{106} 3.23856 3.23856 -1.883×10^{106}	245764 245763 1.1586×10^{957} 245763 245763 1.155×10^{957}

		-50%	148.911	3.23856 3.23856 3.23856 3.23856	245762 245762 245761 245761
$h_2^{'}$	1	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	148.913 148.913 147.904 147.512 147.412 148.912 148.912 148.912 148.912 148.912	3.23856 3.23856 - 1.8840×10^{106} - 1.8840×10^{106} 3.23856 3.23856 3.23856 3.23856 3.23856 - 1.8837×10^{106}	245763 245763 1.1576×10^{957} 1.1554×10^{957} 245762 245762 245762 245762 2 245762 1.1558×10^{957}
p_0	0.9	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	4.90233 -4.306×10^{102} 147.478 148.619 148.678 148.9 148.946 148.952 149.639 149.776	-0.055306 - 7.0165×10^{106} 3.23868 - 1.8760×10^{106} 3.23858 3.23856 3.23858 3.23862 - 1.8699×10^{106} 3.23877	-721.35 1.5950×10^{935} 243807 1.1138×10^{957} 2 245293 246045 246142 246155 1.0820×10^{957} 246345
α	50	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	11.1046 6.578×10^{102} 10.9234 149.842 154.529 148.442 163.383 149.812 147.011 146.528	0.0630009 - 1.137×10^{106} 0.117309 3.24603 - 1.8906×10^{106} 3.23487 - 1.87404×10^{106} -	-580.33 1.6877×10^{937} -581.65 275910 1.3063×10^{957} 230963 8.9230×10^{956}

				1.8665×10^{106} 3.22402 3.22049	7.5732×10^{956} 187628 173526
β	6	+50% +40% +30% +20% +10% -10% -20% -30% -40% -50%	146.388 147.411 151.504 150.728 147.909 149.496 10.0036 10.1317 10.9106 12.353	3.23832 3.23845 3.23850 - 1.8734×10^{106} - 1.8781×10^{106} 3.23857 0.0656606 0.00915995 -0.0252768 -0.0494993	282145 291278 292254 1.1001×10^{957} 1.1250×10^{957} 234389 -507.225 -461.996 -404.569 -331.11

The study of above table (2.2) reveals the following interesting facts:

- Table 2.2 clearly shows that, when the value of p_0 decreases or increases, the selling price p, the optimal total cost $\pi(p,t_1)$ increases or decreases, respectively. However the optimal total cost $\pi(p,t_1)$,the inventory interval t_1 and the selling prices p at 40%,20%, and -40% of p_0 keep abnormal manners
- When the value of A changed there are no effect on the values of p and t_1, while the values of $\pi(p,t_1)$ changed simultaneously.
- The purchase cost C decreases or increases, the selling price p, the inventory interval t_1 and the optimal total cost $\pi(p,t_1)$ decreases or increases, respectively. The optimal total cost $\pi(p,t_1)$ and the inventory interval t_1 at 50%,-20% and -50% of purchase cost C keep abnormal behaviors. The selling price p keeps abnormal behavior at -20% of purchase cost C.
- On increasing or decreasing the value of the holding cost of new product h_1 and h_2, the selling price p, the inventory interval t_1 and the optimal total cost $\pi(p,t_1)$ increases or decreases, respectively. But the optimal total cost $\pi(p,t_1)$, the inventory interval t_1 at -30%,-50% of h_1 and 30%, 10%,-50% of h_2 keep abnormal behaviors.
- With the decreases or increases of holding cost of used product h_1', the inventory interval t_1, the selling price p and the optimal total cost $\pi(p,t_1)$, slightly decreases or increases, respectively but the inventory interval t_1, the selling price p and the optimal total cost $\pi(p,t_1)$,slightly increases on increasing and remains same on decreasing the value of holding cost h_2'. However the optimal total cost $\pi(p,t_1)$, the inventory interval t_1 at 30%,-10% of h_1' and at 30%, 20%,-50% of h_2' keep abnormal manners.

2.4 Observations

In this section, two inventory models have been discussed without inflation and with inflation. Both the two models have been considered here to represent the optimal total cost. From the numerical illustration of the models it is observed that the inventory interval and the selling price do not change with the increment in the ordering cost for retailer, while the optimal total cost decreases. On increasing or decreasing the value of the purchase, the selling price, the inventory interval and the optimal total cost increases or decreases for both the models respectively. The optimal total cost, the selling price and the inventory interval increases with the increment in the holding cost of new product and used product. Furthermore, the results of the models are important for formulating the decisions, when the inventory model is considered without inflation and with inflation.

2.5 Conclusion

In this study, we developed an inventory model under the assumption that a retailer trades the new product as well as gather used products from the consumers. An economic order quantity model has been established considering price dependent quadratic demand. Models of this chapter are divided into two sections. In all the models of this chapter shortages are allowed and partially backlogged and the backlogging rate is constant. In the first section of these models (i.e. the section 2.3.1), an inventory model has been developed without inflation. In the second section of these models (i.e. the section 2.3.2) the model has been developed with inflation. The amount to which inflation has precious the business world is clearly explained through the sensitivity analysis, wherever the effect of inflation is observably shown over the total optimal cost. In these models the demand rate of new products and return rate of used products both are connected to selling price. For both sections the inventory models are considered with variable holding cost and the deterioration rates are taken as two parameter Weibull distribution function of time.

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