IMPORTANCE OF GOAL PROGRAMMING FOR SOLVING FRACTIONAL PROGRAMMING PROBLEM IN FUZZY SET

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Abstract

Goal Programming method is more suitable to find the optimal solutions of the problems having L-R number fuzzy coefficients to various field of production planning problem, transportation problem, and other real world multi-objective programming problems. Bellman and Zadeh (1970) gave the concept that the constraints and goals in such situations may be viewed as fuzzy sets in present study we observe that in real life optimization problems the decision making becomes further complicated in situations when multiple objectives are conflicting, non commensurate and imprecise in nature. Thus method based on goal programming problem need the additional information from decision makers for priority structure of various goals and their aspiration levels in view of resolving this difficulty of setting appropriate priority and aspiration levels to various objectives.

Keyword: - Goal Programming, Fuzzy sets, Optimization, Transportation Problem.

Introduction

The modeling of a real life optimization problem needs to address several objective functions and hence become a multi-objective programming problem in a natural way. The goal programming developed by Charnes and Cooper (1968) emerged as powerful tool to solve such multi-objective programming problems. Since commencement of goal programming technique, it has been enriched by many research workers such as Lee (1972), Ignizio (1976, 1982) and many more. The development of fuzzy set by Zadeh (1965) motivated Zimmermann (1978) to give another approach of solving multi objective programming as fuzzy programming. Thus a new dimension of goal programming was introduced as fuzzy goal programming by Narsimhan (1980, 1981) and Ignigio (1982). However, one of the major problems which is faced by decision makers is the modeling of ill conditioned optimization problems or the problems where the coefficients are imprecise and vague. Thus the classical mathematical programming methods of optimization failed to model such problems. Bellman and Zadeh (1970) gave the concept that the constraints and goals in such situations may be viewed as fuzzy sets in present study we observe that in real life optimization problems the decision making becomes further complicated in situations when multiple objectives are conflicting, non commensurate and imprecise in nature. Thus method based on goal programming problem need the additional information from decision makers for priority structure of various goals and their aspiration levels in view of resolving this difficulty of setting appropriate priority and aspiration levels to various objectives.

α-cut of a LR-fuzzy number:–

Let $\tilde{A}$ be a LR – fuzzy number denoted by $\tilde{A}$ = (a, b, β γ) then its α-cut is defined as $(\tilde{A}_{LR})_{\alpha} = [\tilde{A}^{U}_{\alpha}, \tilde{A}^{U}_{\alpha}] = [(a-\beta)+\beta\alpha, (b+\gamma)-\gamma\alpha]$ which is a crisp interval.

Multi-objective goal programming formulation with α-cut of the fuzzy number:–

Let us consider a multi-objective optimization problem with n decision variables, m constraints and k objective functions,

$$\max Z(X) = \{\tilde{C}_1X, \tilde{C}_2X, \tilde{C}_3X, \ldots\ldots, \tilde{C}_kX\}$$

$$\text{s. t.} \quad \tilde{A}_iX_j \ (\lesssim \approx \gtrsim)\ \tilde{b}_i \qquad i = 1, 2, 3, \ldots, m$$

$$X_j \geq 0 \qquad j = 1, 2, 3, \ldots, n \qquad (1)$$

where X = {$x_1, x_2, x_3, \ldots, x_n$}, $\tilde{C}_k$(k = 1, 2, ... , K) and $\tilde{b}_i$ (i = 1, 2, 3, ...,m) are n dimensional and m dimensional vectors respectively, $\tilde{A}$ is a x n matrix with fuzzy parameter and $\tilde{b}_i$ and $\tilde{C}_k$ and fuzzy number. Since the above problem (1) have fuzzy coefficients which have possibilistic distribution in a uncertain intervals and hence may be approximated in terms of its α-cut intervals.

Let $\tilde{A}_\alpha$ be α-cut interval of fuzzy number $\tilde{A}$ defined by the definition (2.2) $\tilde{A}_\alpha = [\tilde{A}^{L}_{\alpha}, \tilde{A}^{U}_{\alpha}]$.

Where $\tilde{A}^{L}_{\alpha}$, and $\tilde{A}^{U}_{\alpha}$ are the lower and upper bound of the α-cut interval $\tilde{A}_\alpha$ of fuzzy number. Since $\tilde{C}_k$ the coefficients of the objects function are fuzzy numbers, α-cut interval of $\tilde{C}_k$ can be defined as:-

$$(\tilde{C}_k)_\alpha = [(\tilde{C}_k)^{L}_{\alpha}, (\tilde{C}_k)^{U}_{\alpha}] \qquad (2)$$

where $[(\tilde{C}_k)^{L}_{\alpha}$ and $(\tilde{C}_k)^{U}_{\alpha}]$ is given as in definition of α-cut interval. Thus $(\tilde{C}_k)_\alpha$ can be represented as a closed interval $[(\tilde{C}_k)^{L}_{\alpha}, (\tilde{C}_k)^{U}_{\alpha}]$, such that $\tilde{C}_k \in [(\tilde{C}_k)^{L}_{\alpha}, (\tilde{C}_k)^{U}_{\alpha}]$

Now the lower and upper bound for the respective α-cut intervals of objective function are defined as:-

$$[(Z_k(x)_\alpha]^U = \sum_{j=1}^{n} (\tilde{C}_k)^{U}_{\alpha} X_j \qquad (3)$$

$$[(Z_k(x)_\alpha]^L = \sum_{j=1}^{n} (\tilde{C}_k)^{L}_{\alpha} X_j \qquad (4)$$

In the ext step, we construct a membership function for the maximization type objective function Z_k (X), that is replaced by the upper bound of its α-cut interval i.e.

$$[(Z_k(x)_\alpha]^U = \sum_{j=1}^{n} (\tilde{C}_k)^{U}_{\alpha} X_j \qquad (5)$$

similarly to construct a membership function for maximization type objective function Z_k (X), can be replaced by the lower bound of its α-cut interval that is –

$$[(Z_k(x)_\alpha]^L = \sum_{j=1}^{n} (\tilde{C}_k)^{L}_{\alpha} X_j \qquad (6)$$

and the constraint inequalities

$$\sum_{j=1}^{n} (\tilde{A}_{ij})\ X_j \lesssim \tilde{B}_i \qquad i = 1, 3, \ldots, m_1 \qquad (7)$$

$$\sum_{j=1}^{n} (\tilde{A}_{ij})\ X_j \gtrsim \tilde{B}_i \qquad i = m_1+1, m_1+2, \ldots, m_2 \qquad (8)$$

can be written in terms of α-cut values as

$$\sum_{j=1}^{n} (\tilde{A}_{ij})^{U}_{\alpha} X_j \geq (\tilde{\beta}_i)^{L}_{\alpha} \qquad i = 1, 2, \ldots, m_1 \qquad (9)$$

$$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{L} X_j \leq (\tilde{\beta}_i)_{\alpha}^{U} \qquad i = m_1+1, m_1+2, \ldots, m_2 \qquad (10)$$

And the fuzzy equality constraint

$$\sum_{j=1}^{n} (\tilde{A}_{ij}) X_j \approx (\tilde{B}_i) \qquad i = m_2+1, m_2+2, \ldots, m \qquad (11)$$

can be transformed into two inequalities as

$$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{L} X_j \leq (\tilde{\beta}_i)_{\alpha}^{U} \qquad i = m_2+1, m_2+2, \ldots, m \qquad (12)$$

$$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{U} X_j \geq (\tilde{\beta}_i)_{\alpha}^{L} \qquad i = m_2+1, m_2+2, \ldots, m \qquad (13)$$

thus the undertaken maximization problem is transformed in to the following multi objective linear programming problem (MOLPP) as

$$\max [(Z_k(x))_{\alpha}]^U = \sum_{j=1}^{n} (\tilde{C}_k)_{\alpha}^{U} X_j \quad k = 1, 2, 3, \ldots, K$$

subject to

$$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{U} X_j \geq (\tilde{B}_i)_{\alpha}^{L} \qquad i = 1, 2, \ldots, m_1$$

$$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{L} X_j \leq (\tilde{B}_i)_{\alpha}^{U} \qquad i = m_1+1, m_1+2, \ldots, m_2$$

$$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{L} X_j \leq (\tilde{B}_i)_{\alpha}^{U} \qquad i = m_2+1, m_2+2, \ldots, m$$

$$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{U} X_j \geq (\tilde{\beta}_i)_{\alpha}^{L} \qquad i = m_2+1, m_2+2, \ldots, m$$

$$X_j \leq 0. \qquad i = 1, 2, 3, \ldots, m \qquad (14)$$

Now consider the transformation of objectives to fuzzy goals by means of assigning an aspiration level to each of them. Thus applying the goal programming approach, the problem (14) can be transformed in to fuzzy goals by taking certain aspiration level and introducing under deviational variables to each of the objective functions. In proposed method the above maximization type objective function, is transformed as

$$\frac{\sum_{j=1}^{n} (\bar{C}_k)_{\alpha}^{U} Xj - 1_k}{gk - 1_k} + d_k^{-} \geq 1. \qquad (15)$$

Where $d_k^{-} \geq 0$, is under deviational variables and g_k is aspiration level the k[th] goal and the highest acceptable level for the k[th] goal and the lowest acceptable level 1_k are ideal and anti-ideal solutions and are computed as for appropriate values of $\alpha \in [0, 1]$

$$g_k = \max \sum_{j=1}^{n} (\tilde{C}_k)_{\alpha}^{U} X_j \qquad k = 1, 2, 3, \ldots, K \qquad (16)$$

$$l_k = \min \sum_{j=1}^{n} (\tilde{C}_k)_{\alpha}^{L} X_j \qquad k = 1, 2, 3, \ldots, K \qquad (17)$$

now using min-sum goal programming method, the above fuzzy goal programming problem is converted in to single objective linear programming problem as follows.

Fin $x \in X$ so as to

$$\min Z = \sum_{j=1}^{n} w_k \, d_k^{-}$$

subject of

$$\frac{\sum_{j=1}^{n} (\bar{C}_k)_{\alpha}^{U} Xj - 1_k}{gk - 1_k} + d_k^{-} \geq 1.$$

$$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{U} X_j \geq (\tilde{B}_i)_{\alpha}^{L} \qquad i = 1, 2, \ldots, m_1$$

$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{L} X_j \leq (\tilde{B}_i)_{\alpha}^{U}$ $\quad$ i = $m_1+1, m_1+2, \ldots, m_2$

$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{L} X_j \leq (\tilde{B}_i)_{\alpha}^{U}$ $\quad$ i = $m_2+1, m_2+2, \ldots, m$

$\sum_{j=1}^{n} (\tilde{A}_{ij})_{\alpha}^{U} X_j \geq (\tilde{\beta}_i)_{\alpha}^{L}$ $\quad$ i = $m_2+1, m_2+2, \ldots, m$

$X_j \leq 0.$ $\quad$ j = 1, 2, 3, …. , n

$$d_k^{-} \geq 0. \tag{18}$$

Here, Z represents the achievement function and the weights w_k attached to the under deviational variables d_k^{-} an

$$w_k = \begin{cases} \dfrac{1}{g_k - l_k} & \text{for maximizing case} \\ \dfrac{1}{u_k - g_k} & \text{for minimizing case} \end{cases} \tag{19}$$

Fractional goal programming formulation with α–cut of the fuzzy parameters

Let us consider a fractional optimization problem with n decision variables and m constraints as

$$\text{maximize } Z_k(X) = \frac{\tilde{C}_k X + \tilde{\alpha}_k}{\tilde{d}_k X + \tilde{\beta}k} \qquad k = 1, 2, 3, \ldots, k$$

Subject to

$$\tilde{A}_i X_j \ (\leq, \approx \geq) \ \tilde{b}_i \qquad i = 1, 2, 3, \ldots, m \tag{20}$$

where X = {x_1, x_2, x_3, ….. , x_n}, and $\tilde{b}_i$ (i = 1, 2, 3, …. , m) are n dimensional and m dimensional vectors respectively, $\tilde{A}$ is a m × n matrix fuzzy parameter $\tilde{C}$, $\tilde{d}$, $\tilde{\alpha}$ $\tilde{\beta}$ and $\tilde{b}_i$ are fuzzy numbers.

It is also to assume that $\tilde{d}_k X + \tilde{\beta}_k > 0, \forall x \in X$.

Since, above problem (20) have fuzzy coefficients which have possibilistic distribution in an uncertain intervals and hence the problem can be written in terms of its α-cut intervals.

Now the lower and upper bound for the respective α-cut intervals of the objective function are defined as:-

$$[(Z_k(X)_\alpha]^L = \frac{(\tilde{C}_k)_\alpha^L X + (\tilde{\alpha}_k)_\alpha^L}{(\tilde{d}_k)_\alpha^U X + (\tilde{\beta}_i)_\alpha^U} \tag{21}$$

$$[(Z_k(X)_\alpha]^U = \frac{(\tilde{C}_k)_\alpha^U X + (\tilde{\alpha}_k)_\alpha^U}{(\tilde{d}_k)_\alpha^L X + (\tilde{\beta}_i)_\alpha^L} \tag{22}$$

In the next step, we construct a membership function for the maximization type objective functions Z_k (X), and can be replaced by the upper bound of its α-cut interval i.e.

$$[(Z_k(X)_\alpha]^U = \frac{(\tilde{C}_k)_\alpha^U X + (\tilde{\alpha}_k)_\alpha^U}{(\tilde{d}_k)_\alpha^L X + (\tilde{\beta}_k)_\alpha^L} \tag{23}$$

$$\mu_k(X) = \begin{cases} 1 & Z_k(X) \geq g_k \\ \dfrac{[(Z_k(K))_\alpha]^U - l_k}{g_k - l_k} & l_k \leq Z_k(X) < g_k \\ 0 & Z_k(X) < l_k \end{cases} \tag{24}$$

Similarly we construct a membership function for the maximization type objective functions Z_k (X), and can be obtained by replacing the upper bound by lower bound of its α-cut interval as

$$[(Z_k(X)_\alpha]^L = \frac{(\tilde{C}_k)^L_\alpha X + (\tilde{\alpha}_k)^L_\alpha}{(\tilde{d}_k)^U_\alpha X + (\tilde{\beta}_i)^U_\alpha} \qquad (25)$$

$$\mu_k(X) = \begin{cases} 1 & Z_k(X) \le g_k \\ \dfrac{u_k - [(Z_k(K))_\alpha]^L}{u_k - g_k} & g_k < Z_k(X) \le u_k \\ 0 & Z_k(X) > u_k \end{cases} \qquad (26)$$

and the constraint inequalities and equalities are transformed as defined in the equation (7)to (11) and (13).

Now the undertaken maximization problem is transformed in to the following linear programming problem (LPP) as:-

$$\text{maximize } [(Z_k(X)_\alpha]^U = \frac{(\tilde{C}_k)^U_\alpha X + (\tilde{\alpha}_k)^U_\alpha}{(\tilde{d}_k)^L_\alpha X + (\tilde{\beta}_k)^L_\alpha}$$

subject to

$\sum_{j=1}^{n} (\tilde{A}_{ij})^U_\alpha X_j \ge (\tilde{\beta}_i)^L_\alpha$ $\quad$ i = 1, 2, ... , m_1

$\sum_{j=1}^{n} (\tilde{A}_{ij})^L_\alpha X_j \le (\tilde{B}_i)^U_\alpha$ $\quad$ i = m_1+1, m_1 + 2, ... , m_2

$\sum_{j=1}^{n} (\tilde{A}_{ij})^L_\alpha X_j \le (\tilde{B}_i)^U_\alpha$ $\quad$ i = m_2+1, m_2 + 2, ... , m

$\sum_{j=1}^{n} (\tilde{A}_{ij})^U_\alpha X_j \ge (\tilde{\beta}_i)^L_\alpha$ $\quad$ i = m_2+1, m_2 + 2, ... , m

$X_j \ge 0.$ $\quad$ j = 1, 2, 3, ... , n $\quad$ (27)

Further consider the conversion of objectives to fuzzy goals by means of assigning an aspiration level to the objective function. Thus applying the goal programming method, the problem (23) can be transformed into fuzzy goal by taking certain aspiration levels and introducing under deviational variables to the objective function. In proposed method the above maximization type objective function, is transformed as

$$\frac{[(Z_k(X))_\alpha]^U - l_k}{g_k - 1_k} + d_k^- \ge 1. \qquad (28)$$

Where d_k^- $\ge$ 0, is under deviational variables and g_k is aspiration level the k[th] objective goal and the highest acceptable level for the objective goal and the lowest acceptable level 1_k are ideal and anti-ideal solutions and are computed as for appropriate values of $\alpha \in [0, 1]$

$$g_k = \max \frac{(\tilde{C}_k)^U_\alpha X + (\tilde{\alpha}_k)^U_\alpha}{(\tilde{d}_k)^L_\alpha X + (\tilde{\beta}_{ki})^L_\alpha} \qquad (29)$$

$$l_k = \min \frac{(\tilde{C}_k)^L_\alpha X + (\tilde{\alpha}_k)^L_\alpha}{(\tilde{d}_k)^U_\alpha X + (\tilde{\beta}_k)^U_\alpha} \qquad (30)$$

Now using min-sum goal programming method, the above fuzzy goal programming problem is converted in to single objective linear programming problem as follows.

Find $x \in X$ so as to

man Z = $\sum_{j=1}^{n} w_k d_k^-$

subject to

$$\frac{[(Z_k(X))_\alpha]^U - l_k}{g_k - 1_k} + d_k^- \geq 1.$$

$\sum_{j=1}^{n} (\tilde{A}_{ij})_\alpha^U X_j \geq (\tilde{\beta}_i)_\alpha^L$ i = 1, 2, ... , m_1

$\sum_{j=1}^{n} (\tilde{A}_{ij})_\alpha^L X_j \leq (\tilde{B}_i)_\alpha^U$ i = m_1+1, m_1 + 2, ... , m_2

$\sum_{j=1}^{n} (\tilde{A}_{ij})_\alpha^L X_j \leq (\tilde{B}_i)_\alpha^U$ i = m_2+1, m_2 + 2, ... , m

$\sum_{j=1}^{n} (\tilde{A}_{ij})_\alpha^U X_j \geq (\tilde{\beta}_i)_\alpha^L$ i = m_2+1, m_2 + 2, ... , m

$X_j \geq 0.$ j = 1, 2, 3, ... , n

$d_k^- \geq 0.$ (31)

Here, Z represents the achievement function and the weights w_i attached to the under deviational variable d_k^-, and are defined as in equation (19).

CONCLUSION:–

This method is more suitable to find the optimal solutions of the problems having L-R number fuzzy coefficients to various field of production planning problem, transportation problem, and other real world multi-objective programming problems.

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