Study of Wave Propagation in Troposphere on Irregular Terrain Using Finite Elements

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Abstract : Radio coverage in the troposphere and urban areas has been a challenging problem for many years. Researches in wave propagation area have searching for efficient mathematical models for describing the problem of electromagnetic wave propagation in troposphere. Numerous methods are available for predicting electromagnetic wave propagation in the atmosphere. However, the presence of vertical refractivity stratification in the atmosphere complicates the application of some methods. To model refractivity variations in the horizontal as well as vertical direction, geometric optics, coupled-mode analysis or hybrid methods have been employed.

Keywords : troposphere, wave propagation, stratification, geometric optics.

I. INTRODUCTION

Radio coverage in the troposphere and urban areas has been a challenging problem for many years Researchers in wave propagation area have been searching for efficient mathematical models for describing the problem of electromagnetic wave propagation in troposphere. Numerous methods are available for predicting electromagnetic wave propagation in the atmosphere. However, the presence of vertical refractivity stratification in the atmosphere complicates the application of some methods. To model refractivity variations in the horizontal as well as vertical direction, geometric optics, coupled-mode analysis, or hybrid methods have been employed.

In the past, emphasis was given to geometrical optics (GO) techniques and model analysis. Go provide a general geometrical description of ray families, propagation through the atmosphere. Ray tracing methods present many disadvantages; for example the radiowave frequency is not accounted for and it is not always clear whether the ray is trapped by the specific duct structure. An alternative approach for troposheric propagation modeling was developed by Baumgartner and later extended, normally known as Waveguide Model or Coupled Mode Technique. The main disadvantages of coupled mode technique lie in the complexity of the root finding algorithms and large computational demands, especially when higher frequencies and complicated ducting profiles are involved.

The solution of electromagnetic propagation problems in the presence of an irregular terrain is a complicated matter. Three-dimensional variations in refraction and terrain make the full vector problem extremely difficult to solve in a reasonable time. Many existing prediction models are based on simplified Deygout solution for multiple knife-edge diffraction. An important class of propagation models over irregular terrain is based on integral equation formulation which in general can be simplified by using paraxial approximation. if one chooses to simplify the problem by assuming symmetry in one or more of the coordinate directions, the vector problem can be decoupled into scalar problems. However the solution of two dimensional scalar problems is still difficult for realistic environments. Some approximations and numerical schemes for the solution are used to reduce the solution of the full two-way equation to one-way equation. Benefits of one-way propagation are the simple numerical implementation of range dependencies in the medium and the avoidance of prohibitive numerical aspects of solving elliptic equations associated with implementing two range-dependant boundary conditions.

II. Wave propagation modeling by practical form of Helmholtz equation

The paraxial form of Helmholtz equations is:

$$\frac{\partial^2 u(x,z)}{\partial x^2} + j2k_0 \frac{\partial u(x,z)}{\partial x} + \frac{\partial^2 u(x,z)}{\partial z^2} + k_0^2 \left(n^2(x,z) - 1\right) u(x,z) = 0 \quad (1)$$

Where K_0 is the free space wave number, *u(x; z)* is the unknown electric or magnetic field depending on polarization, *n(x:Z)* is the refractive index of troposphere, *x* is the propagation direction and *z* is the transverse direction. Equation (1) is also called wide angel representation of parabolic equation method. In long-distance propagation scenarios the effect of earth's curvature must be considered. In earth flattening transformation refractive index is replaced with modified refractive index *m(x:z)*[2]:

$$m(x,z) = n(x,z)\left(1 + \frac{z}{R}\right) \quad (2)$$

Where *R* is radius of earth. After substituting equation (2) in (1), it becomes,

$$\frac{\partial^2 u(x,z)}{\partial x^2} + j2k_0 \frac{\partial u(x,z)}{\partial x} + \frac{\partial^2 u(x,z)}{\partial z^2} + k_0^2 \left(n^2(x,z) - 1 + \frac{2z}{R}\right) u(x,z) = 0 \quad (3)$$

Wave propagation model based on WPEM as defined in equation (3) is subject to terrain boundary condition, which represents the relationship that must hold between the field *u(x; z)* and terrain. Note from equation (3) that the earth curvature enters only through the 2*z/R* term; if this term is ignored, the equation describes propagation over a flat earth.

Electromagnetic wave propagation above rough surfaces requires a good understanding of various interaction phenomena of the wave with the propagation medium. Numerous methods are able to predict the electromagnetic wave propagation in the troposphere. two kinds of methods are developed in literature, exacts methods and asymptotic methods geometrical optic, parabolic equation, the most popular method is the parabolic equation. In this work a model based on this equation has been implemented to provide a tool for electromagnetic wave propagation prediction. A new method, based on surface generation, is introduced for a better taking into account of the surface's roughness. To conclude, the surface's roughness effects, on electromagnetic wave propagation, are demonstrated threw examples where some results of simulations are showed for different configurations and different surfaces.

III. Boundary and initial conditions:

In two dimensional wave propagation problem, there are two boundaries, one at the starting height, Z =z_{min} which in fact is the earth surface at some height depending on terrain profile, and at the maximum altitude considered, Z = z_{max}. An impedance type boundary condition can be used at lower heights to account for the finite conductivity and permittivity of the surface of the earth. The entrance boundary conditions are expressed by the equation:

$$\left[\frac{\partial u(x,z)}{\partial z} + jk_0 q u(x,z)\right]_{z=z_{max}} = 0$$

where,

$$q = q_v = \frac{jk_0}{\sqrt{\varepsilon_r - j60\sigma\lambda}}$$

$$q = q_h = jk_0\sqrt{\varepsilon_r - j60\sigma\lambda}$$

for vertical and polarization respectively. vr is the complex relative permittivity and ¾ is the conductivity of ground whereas, is the wavelength of radio waves in meters.

When numerical propagation simulations are performed, infinite propagation domains cannot be realized and the size of the propagation domain must be truncated. This is accomplished numerically by implementing absorbing boundary conditions or Perfectly Matched Layer (PML) on the upper boundary at z $=z_{max}$.

IV. Study of finite element formulation problem of wave

The finite element (FE) method is widely used to model wave-like phenomena. For unit cell models with periodic boundary conditions, there are extensive finite element applications; however, the problem of imposing Bloch-periodic boundary conditions within a finite element variational formulation has not received significant attention in finite element monographs or the wider scientific literature. Finite element methods with Bloch-periodic boundary conditions for electromagnetic field computations were treated. Bloch-periodic boundary conditions were incorporated via a matrix transformation, whereas equivalent row and column operations on the system matrices were employed. Mias and Co-workers presented the variational formulation for the Maxwell equations subject to Bloch-periodic boundary conditions. In quantum mechanics, Hermansson and Yevick use higher-order FE basis functions for band-structure calculations. Absolute errors in the four lowest bands for the i-point (wavevector k = 0) energies were reported (similar accuracy for k 6 = 0 was noted), with the FE method requiring many more degrees of freedom than standard Fourier based methods. Ferrari used the form $\psi(x) = u(x)\, e^{ik.x}$ in the construction of a weak formulation for the cell periodic function u(x), allowing periodic rather than Bloch-periodic trial and test spaces. Pask and co-workers considered the required value- and derivative-periodic boundary conditions in a general unit cell to systematically derive the weak formulation and matrix equations for the Poisson and SchrÄodinger equations. Following the formulation of Pask et.al., developed a meshfree formulation allowing full Brilloouin zone (k-point) sampling and general unit cells for crystalline solids.

V. Conclusion

A method to model electromagnetic wave propagation in troposphere on irregular terrain in the presence of height dependant refractivity is presented using finite element analysis. In this work, the Helmholtz equation applied on radiowave propagation properly manipulated and simplified using pade approximation is solved using finite element method. Paraxial form of Helmholtz equation is also called wide angle formulation of parabolic equation (WPEM) and is used because of its accuracy and behavior on large propagation angles. By using this method, horizontal and vertical tropospheric characteristics are assigned to every element, and different refractivity an d terrain profiles can be entered at different stages. We also consider wave propagation on an urban street to demonstrate the effectiveness of our method.

In this work, roughness effects, in electromagnetic wave propagation above natural surfaces, are introduced using a new method based on surface generation and a roughness parameter. Different surfaces are characterized by the means of permittivity. Finally some results are presented for different surfaces (ground, sea, snow cover) and frequencies (UHF-band, X-band).

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