STABILITY OF THE CENTRE OF MASS OF A SYSTEM OF TWO SATELLITES CONNECTED BY AN EXTENSIBLE CABLE UNDER THE INFLUENCE OF MAGNETIC FORCE IN THE CIRCULAR ORBIT

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Abstract :- This paper is devoted to study the stability of the equilibrium position of the the centre of mass of the system of two satellites connected by an extensible string under the influence of magnetic force of the earth in circular orbit. We have obtained an equilibrium position which has been shown to be stable in the sense of Liapunov.

Keywords:- Satellites, magnetic force of the earth.

I. Introduction:-

This paper is devoted to examine the stability of an equilibrium point of the centre of mass of the system of two satellites connected by an extensible cable which is supposed to be light and flexible under the influence of magnetic force of the earth in case of circular orbit. First of all we have derived the differential equations of motion of one of the two satellites by using Lagrange's equations of motion of first kind when their centre of mass is moving along a given Keplerian electrical orbit in Nechvills system of coordinates. The general solutions of the differential equations obtained are beyond our reach. Hence in order to facilitate our problem, we put e=0 and so $\rho = \frac{1}{1+e\cos v} = 1$ and hence $\rho' = 0$. Then we get the equations of motion in circular orbit for the centre of mass of the system. Then Jacobi's integral for the pressure in case of two dimensional motion has been obtained. With the help of this Jacobi's integral, we obtain an equilibrium point which has been shown to be stable in the sense of Liapunov.

II. Equations of motion of the system in elliptic orbit -

The equations of motion of one of the two satellites with respect to the centre of mass in Nechvill's coordinates system in elliptic orbit have been obtained in the form:

$$x''-2y'-3\rho x = -\bar{\lambda}_\alpha\left(1-\frac{l_0}{\rho_r}\right)\rho^4 x - A\cos i$$

$$y''+2x' = -\bar{\lambda}_\alpha\left(1-\frac{l_0}{\rho_r}\right)\rho^4 y - \frac{A\rho'}{\rho^2}\cos i$$

and $$z''+z = \bar{\lambda}_\alpha\left(1-\frac{l_0}{\rho_r}\right)\rho^4 z - \frac{A}{\rho}\left[\frac{\rho'}{\rho}\cos(v+w)+\frac{A}{\mu_E}\left(3p^3\rho^3-\mu_E\right)\sin(v+w)\right]\sin i \qquad \ldots\ldots (1)$$

where A cos i = magnetic force parameter

$$\bar{\lambda}_\alpha = \frac{p3\lambda}{\mu l_0}\left(\frac{m_1+m_2}{m_1 m_2}\right)$$

$$\rho = \frac{1}{1+e\cos v}$$; v being true anomaly of the orbit of the centre of mass

$$r = \sqrt{x^2+y^2+z^2}$$

m_1 and m_2 being masses of the two satellites

l_0 = Natural length of the string connection the two satellites.

Here dashes denote differentiation with respect to v.

The condition of constraint is given by

$$x^2+y^2+z^2 \le \frac{l_0^{\,2}}{\rho^2} \qquad \text{………. (2)}$$

Putting ρ=1and ρ'=0 in (1), we get two dimensional equations of motion in the form:

$$\begin{aligned} x''-2y'-3x &= -\bar{\lambda}_\alpha\left(1-\frac{l_0}{r}\right)x - A\cos i \\ y''+2x' &= -\bar{\lambda}_\alpha\left(1-\frac{l_0}{r}\right)y \end{aligned} \qquad \text{………. (3)}$$

Where $r = \sqrt{x^2+y^2}$

The condition of constraint given by (2) takes the form

$$x^2+y^2 \le l_0^{\,2} \qquad \text{………. (4)}$$

We find that the equations of motion given by (3) denot contain time t explicitly. Hence there must exist a Jacobi's integral for the problem.

Multiplying first equation of (3) by 2x and 2nd equation by 2y' and adding them together and then integrating, we get Jacobi's integral for the problem as

$$x'^2+y'^2-3x^2+\bar{\lambda}_\alpha\left(x^2+y^2\right)-2\bar{\lambda}_\alpha l_0\left(x^2+y^2\right)^{\frac{1}{2}}+2xA\cos i = h \qquad \text{………… (5)}$$

Where h is the constant of integration.

III. Equilibrium position of the system: -

We have derived the system of equations (3) for the motion of the system in rotating frame of reference. It has been assumed that the system is moving with effective constraints and connecting cable of the two satellites will always remain tight.

The equilibrium position of the system is obtained by the constant values of the coordinates in the rotating frame of reference.

Let x = x_0 and y = y_0 give the equilibrium position

Where x_0 and y_0 are constants.

Hence $x = 0 \Rightarrow x' = x'' = 0$

and $y = 0 \Rightarrow y' = y'' = 0$

Thus, equations given by (3) take the form:

$$-3x_0 = -\bar{\lambda}_\alpha\left(1-\frac{l_0}{r_0}\right)x_0 - A\cos i$$
$$0 = -\bar{\lambda}_\alpha\left(1-\frac{l_0}{r_0}\right)y_0 \qquad \ldots\ldots\ldots\ldots(6)$$

Where $r_0 = \sqrt{x_0^{\,2} + y_0^{\,2}}$

From (6), we get the equilibrium point as

$$\left[\frac{\bar{\lambda}_\alpha l_0 - A\cos i}{\bar{\lambda}_\alpha - 3}, 0\right] \qquad \ldots\ldots\ldots\ldots (7)$$

It can be easily seen that equilibrium position (7) gives meaningful value of Hook's modulus of elasticity if $\bar{\lambda}_\alpha l_0 - A\cos i$ is positive.

IV. Stability of the equilibrium position of the system: -

We examine the stability of the equilibrium point given by (7) of the system in the sense of Liapunov

For this, let

$$a = x = \frac{\bar{\lambda}_\alpha l_0 - A\cos i}{\bar{\lambda}_\alpha - 3} \text{ and b = y = 0}$$

Let us assume that there are small variations in the coordinates at the given equilibrium point (a,0).

Let δ_1 and δ_2 be small variations in x_0 and y_0 coordinates respectively for a given position of equilibrium. Hence, we get

$$x = a + \delta_1 and\ y = 0 + \delta_2 = \delta_2$$
$$\therefore x' = \delta'_1, x'' = \delta''_1\ and\ y' = \delta'_2, y'' = \delta''_2 \qquad \ldots\ldots\ldots\ldots (8)$$

Using (8) in (3), we get

$$\delta_1'' - 2\delta_2' - 3(a+\delta_1) = -\bar{\lambda}_\alpha\left(1-\frac{l_0}{r_1}\right)(a+\delta_1) - A\cos i$$
$$\delta_2'' + 2\delta_1 = -\bar{\lambda}_\alpha\left(1-\frac{l_0}{r_1}\right)\delta_2 \qquad \ldots\ldots\ldots\ldots (9)$$

Where $r_1^2 = (a+\delta_1)^2 + \delta_2^2$ …………(10)

Multiplying first equation of (8) by 2(a+δ_1') and 2nd equation of (8) by 2δ_2' respectively and adding them together and then integrating

We get Jacobi's integral for the problem as the form

$$(\delta_1')^2 + (\delta_2')^2 - 3(a+\delta_1)^2 + \bar{\lambda}_\alpha\left[(a+\delta_1)^2 + \delta_2^2\right] - 2\bar{\lambda}_\alpha l_0\left[(a+\delta_1)^2 + \delta_2^2\right]^{\frac{1}{2}}$$
$$+ 2A(a+\delta_1)\cos i = h_1 \qquad \ldots\ldots\ldots\ldots (11)$$

Where h_1 is the constant of integration.

To examine the stability of the equilibrium point in the sense of Liapunov, we take Jacobi's integral (11) as Liapunov function v (δ_1,δ_2, δ_1', δ_2') and is obtained by expanding the terms of (11) as

$$
\begin{aligned}
v(\delta_1, \delta_2, \delta_1', \delta_2') = \delta_1'^2 + \delta_2'^2 + \delta_1^2\left[\bar{\lambda}_\alpha - 3\right] & \\
+ \delta_2^2\left[\bar{\lambda}_\alpha - \frac{2\bar{\lambda}_\alpha l_0}{a}\right] & \\
+ \delta_1\left[-6a + 2a\bar{\lambda}_\alpha - 2\bar{\lambda}_\alpha l_0 + 2A\cos i\right] & \qquad \ldots\ldots\ldots\ldots (12) \\
+ \left[a^2\bar{\lambda}_\alpha - 3a^2 - 2a\bar{\lambda}_\alpha l_0 + 2aA\cos i\right] & \\
+ 0(3) & \\
= h_1 &
\end{aligned}
$$

Where 0(3) stands for 3rd and higher order terms in δ_1 and δ_2.

By Liapunov's theorem on stability it follows that the only criterion for given equilibrium position (a, 0) given by (7) to be stable is that v defined by (12) must be positive definite and for this the following three conditions must be satisfied.

(i) $-6a + 2a\bar{\lambda}_\alpha - 2\bar{\lambda}_\alpha l_0 + 2A\cos i = 0$

(ii) $\bar{\lambda}_\alpha - 3 > 0$ ..………... (13)

(iii) $\bar{\lambda}_\alpha - \dfrac{\bar{\lambda}_\alpha l_0}{a} > 0$

We have

$$a = \frac{\bar{\lambda}_\alpha l_0 - A\cos i}{\bar{\lambda}_\alpha - 3} \qquad ..………. (14)$$

But, we have

$$\bar{\lambda}_\alpha l_0 - A\cos i > 0 \qquad ……….. (15)$$

Using (14) and (15), it can be easily seen that all all conditions given in (13) are identically satisfied. Hence, we conclude that the equilibrium point (a, 0) given by (7) is stable in the sense of Liapunov.

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