Application of Fuzzy Relational Equation by Using Max-Add Composition

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Abstract

In this paper, we discuss about the features and smooth way of active users in some video conferencing services of ''On line teaching and learning process" by using fuzzy Max-Add composition.

1 Introduction

Fuzzy relations are significant concepts in fuzzy theory and have been widely used in many fields such as fuzzy clustering, fuzzy control and uncertainty reasoning. The notion of fuzzy relational equations based upon the max-min composition was first investigated by Sanchez [3]. Fuzzy relational (relation) equations are identities of the form $R \circ S = T$, where R, S and T are fuzzy relations (R is a fuzzy relation between sets X and Y, S is a fuzzy relation between Y and Z and T is a fuzzy relation between X and Z). The maximum-addition composition of fuzzy relations were introduced and studied by Rakesh Kumar Triapathi [2]. In this paper, we discuss about the features and smooth way of active users in some video conferencing services of "On Line teaching and learning process" by using fuzzy Max-Add composition.

2 Preliminaries

Definition 2.1. *[4] Let X and Y be two nonempty sets. A fuzzy relation $\tilde{\mathfrak{R}}$ between X and Y is a fuzzy subset of $X \times Y$ where $\mu_R : X \times Y \to [0,1]$. If $X = Y$, then $\tilde{\mathfrak{R}}$ is called a binary fuzzy relation.*

A fuzzy relation can be represented in the following manner:
Let $A = a_i$, $i = 1,2,\ldots,n$ *and* $B = b_j$, $j = 1,2,\ldots,m$.

A fuzzy relation $\widetilde{\Re} \subseteq A \times B$ *can be represented as* $\widetilde{\Re} = \{(a_i,b_j),R(a_i,b_j)\}$.

Definition 2.2. *[5] Consider two fuzzy relations* $\widetilde{\Re_1}$ *and* $\widetilde{\Re_2}$

$\widetilde{\Re_1}(x,y) \subseteq X \times Y$ *and* $\tilde{R}_2(y,z) \subseteq Y \times Z$.

The max-min composition of $\widetilde{\Re_1}$ *and* $\widetilde{\Re_2}$ *is given as follows:*

$\widetilde{\Re_1} \circ \widetilde{\Re_2} = \{(x,z), \max y \in Y (\min(\mu_{R1}(x,y),\mu_{R2}(y,z)))\}$ *Or*

$\mu_{R_1 \circ R_2}(x,z) = \bigvee_{y \in Y} \{\mu_{R_1}(x,y) \wedge \mu_{R_2}(y,z)\}$ *where* $x \in X$, $y \in Y$ *and* $z \in Z$.

μ_{R1} *and* μ_{R2} *are the membership function of fuzzy relations on fuzzy sets. "* $\wedge$ *" denotes the "* min *" function and "* $\vee$ *" denotes the "* max *" function.*

Definition 2.3. *[1] Consider two fuzzy relations* $\widetilde{\Re_1}$ *and* $\widetilde{\Re_2}$ $\widetilde{\Re_1}(x,y) \subseteq X \times Y$ *and* $\widetilde{\Re_2}(y,z) \subseteq Y \times Z$.

The max-add composition of $\widetilde{\Re_1}$ *and* $\widetilde{\Re_2}$ *is given as follows:*

$\widetilde{\Re_1} \circ \widetilde{\Re_2} = \{(x,z), \max y \in Y\, [(\mu_{R1}(x,y)+\mu_{R2}(y,z))-(\mu_{R1}(x,y)\cdot\mu_{R2}(y,z))]\}$

Or

$$\mu_{R_1 \circ R_2}(x,z) = \underset{y \in Y}{\vee} \left\{\mu_{R_1}(x,y) \binom{\wedge}{+} \mu_{R_2}(y,z)\right\}$$ *where* $x \in X$, $y \in Y$ *and* $z \in Z$.

μ_{R1} *and* μ_{R2} *are the membership function of fuzzy relations on fuzzy sets. "* $\vee$ *" denotes the "* max *" function and "* $\binom{\wedge}{+}$ *" denotes the "algebraic sum" function.*

3 Application of maximum-addition composition

We are choosing three video conferencing services

1. Google meet
2. Google zoom

3. Skype

The following data are determined from the 120 observations of the related users of video conferencing system. By giving membership grades " 1,·75,·5,·25 , or 0", respectively to linguistic terms "must be, strong, neither nor, could be and non", R_o and R_c may be used to draw different methods of users' conclusions of video conferencing system.

This can be explained with the following example:

Let S = set of all sources

T = set of all tools (Video conferencing systems)

U = set of all users.

The sources $S = \{s_1,s_2,s_3,s_4,s_5\}$, where s_1 represents video clarity, s_2 represents audible, s_3 represents discussion, s_4 represents screen size, s_5 represents attachment option.

Also, the set T is $\{t_1,t_2,t_3\}$, where t_1 represents the video conferencing system of google meet, t_2 represents the video conferencing system of google zoom, t_3 represents the video conferencing system of skype. In addition, $U = \{u_1,u_2,u_3,u_4\}$ is the set of users consider in the example.

The matrix for occurrence relation is $R_o = S \times T$ which indicates the level of occurrence of sources $s_i; i = 1,2,...,5$ with disease $t_j; j = 1,2,3$ and is given by

Matrix for occurrence relation is $R_o = S \times T$ indicates the level of occurrence of sources.

$$\widetilde{\Re_0} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 0.9 & 0.2 & 0.7 \\ 0.5 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0.8 \\ 0.9 & 0.1 & 0.2 \\ 0.7 & 0.8 & 0.3 \end{bmatrix} \end{array}$$

Matrix for appearance relation is $R_a = S \times T$ corresponds to the degree of appearance in various sources of video conferencing system.

$$\widetilde{\Re_a} = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 0.0 & 0.8 & 0.3 \\ 0.7 & 0.5 & 0.6 \\ 0.2 & 0.4 & 0.1 \\ 0.5 & 0.6 & 0.9 \\ 1.0 & 0.3 & 0.7 \end{bmatrix} \end{array}$$

Now assume a fuzzy relation $\widetilde{S_a}$ that indicates the degree of appearance of sources s_1, s_2, s_3, s_4, s_5 for four users of u_1, u_2, u_3, u_4 as follows. These indicate the degree to which the sources are appearing in users.

$$\widetilde{\mathfrak{R}_s} = \begin{array}{c c} & \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} & \begin{bmatrix} 0.8 & 1.0 & 0.3 & 0.0 & 0.2 \\ 0.3 & 0.5 & 0.6 & 0.7 & 0.8 \\ 0.1 & 0.2 & 0.4 & 0.1 & 0.9 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.7 \end{bmatrix} \end{array}$$

Using these relations $\widetilde{\mathfrak{R}_a}$, $\widetilde{\mathfrak{R}_0}$ and $\widetilde{\mathfrak{R}_s}$ calculate four different relations as below:

The occurrence indication relation $\widetilde{\mathfrak{R}_1}$ calculated by $\widetilde{\mathfrak{R}_1} = \widetilde{\mathfrak{R}_s} \circ \widetilde{\mathfrak{R}_0}$

The appearance indication relation $\widetilde{\mathfrak{R}_2}$ calculated by $\widetilde{\mathfrak{R}_2} = \widetilde{\mathfrak{R}_s} \circ \widetilde{\mathfrak{R}_a}$,

The non-occurrence indication relation $\widetilde{\mathfrak{R}_3}$ calculated by $\widetilde{\mathfrak{R}_3} = \widetilde{\mathfrak{R}_s} \circ (1 - \widetilde{\mathfrak{R}_0})$.

Finally, the non-symptom indication relation $\widetilde{\mathfrak{R}_4}$ calculated by $\widetilde{\mathfrak{R}_4} = (1 - \widetilde{\mathfrak{R}_s}) \circ \widetilde{\mathfrak{R}_0}$.

The above relations require composition of fuzzy Matrix using max-add rule. The process of fuzzy matrix composition is the best video conferencing system of online teaching and learning method with time consuming. In this paper, by using max-add composition for calculation of $\widetilde{\mathfrak{R}_1} - \widetilde{\mathfrak{R}_4}$. The result obtained in this paper are given below.

The following results obtained manually are given below which provide different users conclusions.

$\widetilde{\mathfrak{R}_1} = \widetilde{\mathfrak{R}_s} \circ \widetilde{\mathfrak{R}_0}$

$$= \begin{array}{c c} & \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} & \begin{bmatrix} 0.8 & 1.0 & 0.3 & 0.0 & 0.2 \\ 0.3 & 0.5 & 0.6 & 0.7 & 0.8 \\ 0.1 & 0.2 & 0.4 & 0.1 & 0.9 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.7 \end{bmatrix} \end{array} \circ \begin{array}{c c} & \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} & \begin{bmatrix} 0.9 & 0.2 & 0.7 \\ 0.5 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0.8 \\ 0.9 & 0.1 & 0.2 \\ 0.7 & 0.8 & 0.3 \end{bmatrix} \end{array}$$

$$= \begin{array}{c c} & \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} & \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 0.97 & 0.96 & 0.92 \\ 0.97 & 0.98 & 0.93 \\ 1.00 & 1.00 & 1.00 \end{bmatrix} \end{array}$$

$\widetilde{\mathfrak{R}_2} = \widetilde{\mathfrak{R}_s} \circ \widetilde{\mathfrak{R}_a}$

$$= \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} 0.8 & 1.0 & 0.3 & 0.0 & 0.2 \\ 0.3 & 0.5 & 0.6 & 0.7 & 0.8 \\ 0.1 & 0.2 & 0.4 & 0.1 & 0.9 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.7 \end{bmatrix} \end{array} \circ \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 0.0 & 0.8 & 0.3 \\ 0.7 & 0.5 & 0.6 \\ 0.2 & 0.4 & 0.1 \\ 0.5 & 0.6 & 0.9 \\ 1.0 & 0.3 & 0.7 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 1.00 & 0.88 & 0.97 \\ 1.00 & 0.93 & 0.97 \\ 1.00 & 1.00 & 1.00 \end{bmatrix} \end{array}$$

$\widetilde{\mathfrak{R}_3} = \widetilde{\mathfrak{R}_s} \circ (1 - \widetilde{\mathfrak{R}_0})$

$$= \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} 0.8 & 1.0 & 0.3 & 0.0 & 0.2 \\ 0.3 & 0.5 & 0.6 & 0.7 & 0.8 \\ 0.1 & 0.2 & 0.4 & 0.1 & 0.9 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.7 \end{bmatrix} \end{array} \circ \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 0.1 & 0.8 & 0.3 \\ 0.5 & 0.7 & 0.8 \\ 0.9 & 0.4 & 0.2 \\ 0.1 & 0.9 & 0.8 \\ 0.3 & 0.2 & 0.7 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 0.96 & 0.97 & 0.94 \\ 0.94 & 0.92 & 0.97 \\ 1.00 & 1.00 & 1.00 \end{bmatrix} \end{array}$$

$\widetilde{\mathfrak{R}_4} = (1 - \widetilde{\mathfrak{R}_s}) \circ \widetilde{\mathfrak{R}_0}$

$$= \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \end{array} \\ \begin{bmatrix} 0.2 & 0.0 & 0.7 & 1.0 & 0.8 \\ 0.7 & 0.5 & 0.4 & 0.3 & 0.2 \\ 0.9 & 0.8 & 0.6 & 0.9 & 0.1 \\ 0.0 & 0.8 & 1.0 & 0.7 & 0.3 \end{bmatrix} \end{array} \circ \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 0.9 & 0.2 & 0.7 \\ 0.5 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0.8 \\ 0.9 & 0.1 & 0.2 \\ 0.7 & 0.8 & 0.3 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 1.00 & 1.00 & 1.00 \\ 0.97 & 0.84 & 0.91 \\ 0.99 & 0.92 & 0.97 \\ 1.00 & 1.00 & 1.00 \end{bmatrix} \end{array}$$

From $\widetilde{\Re_1} - \widetilde{\Re_4}$, following two matrices $\widetilde{\Re_{av}}\ (1,2)$ and $\widetilde{\Re_{av}}\ (3,4)$ using average composition can be obtained.

$$\widetilde{\Re_{av}}\ (1,2) = \frac{\widetilde{\Re_1} + \widetilde{\Re_2}}{2}$$

$$\widetilde{\Re_{av}}\ (1,2) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 1.000 & 1.000 & 1.000 \\ 0.985 & 0.920 & 0.945 \\ 0.985 & 0.955 & 0.950 \\ 1.000 & 1.000 & 1.000 \end{bmatrix} \end{array}$$

$$\widetilde{\Re_{av}}\ (3,4) = \frac{\widetilde{\Re_3} + \widetilde{\Re_4}}{2}$$

$$\widetilde{\Re_{av}}\ (3,4) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 1.000 & 1.000 & 1.000 \\ 0.965 & 0.905 & 0.925 \\ 0.965 & 0.920 & 0.970 \\ 1.000 & 1.000 & 1.000 \end{bmatrix} \end{array}$$

The least value of $\widetilde{\Re_{av}}\ (1,2)$ is 0.92 and the greatest value of $\widetilde{\Re_{av}}\ (3,4)$ is 1.

Therefore $\frac{Greatest\ value\ in\ \widetilde{\Re_{av}}\ (3,4)}{Least\ value\ in\ \widetilde{\Re_{av}}\ (1,2)} < n$

Therefore $\frac{1}{0.92} < n$

Implies $n = 2$

Now the resultant matrix $\widetilde{\Re_m} = \widetilde{\Re_{av}}\ (1,2) - \frac{1}{5}\widetilde{\Re_{av}}\ (3,4)$

$$\widetilde{\Re_m} = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{bmatrix} 0.5000 & 0.5000 & 0.5000 \\ 0.5025 & 0.4675 & 0.4825 \\ 0.5025 & 0.4950 & 0.4650 \\ 0.5000 & 0.5000 & 0.5000 \end{bmatrix} \end{array}$$

4 Conclusion

In this paper from $\widetilde{\Re_1}$ and $\widetilde{\Re_2}$, the users' u_1, u_2, u_4 are satisfied the type of video conferencing system of t_1 that is google meet. In $\widetilde{\Re_1}$ the user u_3 are satisfied the type of video conferencing system of t_2 that is google zoom. Then the resultant matrix $\widetilde{\Re_m}$ all the users u_1, u_2, u_3 , u_4 mostly liked and satisfied the platform of video conferencing system t_1 that is google meet.

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