NUMERICAL SIMULATION OF NON-LINEAR NEWTONIAN BLOOD FLOW THROUGH AN AXI-SYMMETRIC STENOSED ARTERY UNDER PERIODIC BODY ACCELERATION

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Abstract : The unsteady and incompressible flow under the influence of periodic body acceleration in presence of cosine-shaped stenosed artery is investigated with the help of numerical simulation. The arterial segment is simulated by a cylindrical tube filled with a viscous incompressible Newtonian fluid described by the Navier-Stokes equation. The nonlinear equation is solved numerically with the proper boundary conditions and pressure gradient that arise from the normal functioning of the heart. The effect of Reynolds number is also discussed. Results are discussed in comparison with the existing models.

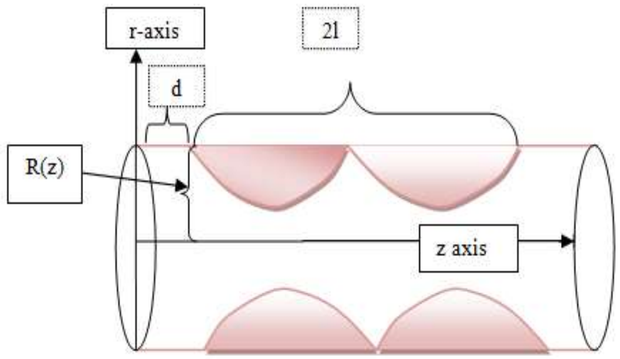
IndexTerms - Incompressible Flow, Navier-Stokes Equation, Reynolds Number.

I. INTRODUCTION

Now days the investigation of blood flow in a stenosed artery is very important because of the fact that most of the diseases in the blood vessels such as heart attacks and strokes are related to blood flow and the physical behaviors of vessel wall. One of the leading causes of the death in the world is due to heart diseases, and the most commonly heard name is atherosclerosis. Atherosclerosis is a type of arteriosclerosis. Atherosclerosis (or arteriosclerotic vascular disease) is a condition where the arteries become narrowed and hardened due to an excessive build up of plaque around the artery wall. The disease disrupts the flow of blood around the body, posing serious cardiovascular complications. Arteries contain what is called an endothelium, a thin layer of cells that keeps the artery smooth and allows blood to flow easily. Atherosclerosis starts when the endothelium becomes damaged, allowing LDL cholesterol to accumulate in the artery wall. Over time this results in plaque being built up, consisting of bad cholesterol (LDL cholesterol). The formation of plaque in the coronary arteries, cerebral circulation can leads to heart attacks and strokes. Atherosclerosis involves an accumulation of low-density lipoprotein in the wall of large arteries, typically where the wall shear rate is low and oscillatory[12]. Modelling blood flow through arterial multi-stenosis is very challenging. Accuracy of the simulation depends mainly on suitable numerical approach, realistic model geometry and boundary conditions.
Recently many investigators have focused their attention on blood flow through stenosed arteries with single stenosis by Riahi et al[2], Chakravarty and Mondal[3], Lee and Xu[6], pointed out that the mathematical model becomes more accurate in the presence of an overlapping stenosis instead of a mild one. Ang and Majumdar[5] studied asymmetric arterial blood flow with numerical solution in three dimension, Ikbal[4] have worked on unsteady response of non-newtonian in magnetic field without considering periodic body acceleration effect. Kohler et al. [13] studied the wall shear stress with the help of magnetic resonance imaging (MRI) measurements of the velocity field and comparing them with simulation outputs. Stroud et al. [8] have studied a 2D plaque model using modeling and simulation while Fischer et al.[11] worked on numerical method for the computational study of arterial blood flow with turbulence. The asymmetric flows in a symmetric sudden expansion channel have been studied using experimental and numerical techniques by Fearn et al. [14] and Durst et al. [15]. Mahapatra et al. [16] investigated unsteady laminar separated flow through constricted channel using finite-difference technique in staggered grid distribution and suggested that the critical value of Reynolds number depend on the area reduction and the length of the constriction. S. Chakravarty et al.[18] solved blood flow model with body acceleration but they do not consider the non-linear terms in the model. Blood shows a non-Newtonian behavior at low shear rates in tubes of smaller diameters, Taylor [19] suggested that at high shear rates commonly found in larger arteries blood behaves like a Newtonian fluid.

With the above motivation in our mind we have worked on numerical simulations of nonlinear pulsatile unsteady Newtonian blood flow in a rigid cylindrical tube through cosine-shape stenosisunder the influences of periodic body acceleration. It appears a few studies address the issueof non-linear terms present in Navier-Stokes equation govern blood flow with periodic bodyacceleration associated with an atherosclerotic plaque. The numerical solutions are obtained ofthe non-linear model using appropriate finite difference method. A comparison of the axial velocity with other existing model [4] has been studied also.

II. MATHEMATICAL FORMULATION OF THE MODEL



Consider the blood flow to be unsteady and incompressible through an axi-symmetric cylindrical tube. Because of the segment of arteries are assuming in cylindrical shape, the model is formulated in cylindrical co-ordinate system. Let (r, θ, z) be the coordinate of any point in the cylinder where the z axis is taken along the axis of artery while$r$, θ are along the radial and circumferential direction respectively. The geometry of the axi-symmetric stenosis in the cross section of the artery is defined by the function $R(z)$ as shown in Fig.1 can be mathematically derived as

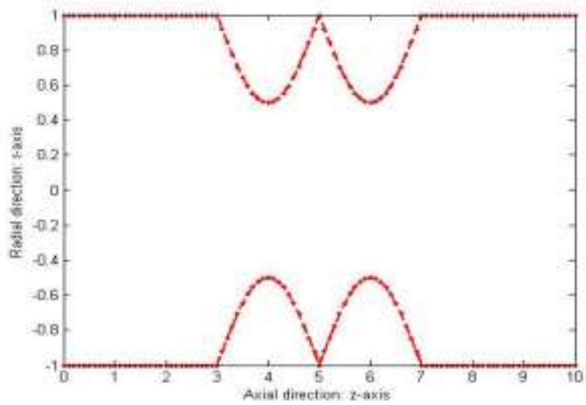


Fig. 1. The geometry of the stenosis is plotted against the axial coordinate axis z for $l = 2,\ d = 3, p = 0.5$.

$$R(z) = \begin{cases} 1 - p\cos\left(\frac{\pi z}{l}\right), & d \le z < l + d \\ 1 + p\cos\left(\frac{\pi z}{l}\right), & l + d \le z \le d + 2l \\ 1, & \text{otherwise} \end{cases}$$

(1)

where $2l$ = length of the stenosis, d = distance of the stenosis from radial axis and p is dimensionless some constant. The fluid flow is governed by the following incompressible Navier Stokes equations in dimensionless form in cylindrical co-ordinate system.

$$\frac{\partial u}{\partial t} = -\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) - \frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2}\right) + a_0\big(cos(\omega t + \varphi)\big) \quad \textbf{(2)}$$

$$\frac{\partial v}{\partial t} = -\left(u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r}\right) - \frac{\partial p}{\partial r} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} - \frac{v}{r^2}\right) \quad \textbf{(3)}$$

and the continuity equation$\frac{\partial u}{\partial z} + \frac{v}{r} + \frac{\partial v}{\partial r} = 0$ **(4)** where u, v are the axial and radial velocity of blood respectively an $Re = \frac{R_0 u_\infty \rho}{\mu}$ be the Reynolds number, where ρ, μ, u_∞ are the density, viscosity and average velocity of the blood, R_0 be the radius of the artery, a_0 is some dimensionless constant. The last term $a_0\big(cos(\omega t + \varphi)\big)$ in x-momentum equation considered as periodic body acceleration. To test the effect of body acceleration the parameter a_0 is used in the model.

According to Burton [17] the pressure gradient $\frac{\partial p}{\partial z}$ for human beings present in (1) can be taken as

$-\frac{\partial p}{\partial z} = A_0 + A_1\cos(\omega t + \varphi)$, $t > 0$ **(5)**

where A_0, A_1 are the constant amplitude of pressure gradient and pulsatile component respectively, $\omega = 2\pi f$, where f is the heart pulse frequency. Again since the lumen radius of artery is small then the pressure term $\frac{\partial p}{\partial r}$ in (3) can be consider as $\frac{\partial p}{\partial r} = 0$.

The initial conditions are$u(r, z, t) = 0$ and $v(r, z, t) = 0$at $t = 0$ **(6)**

and the boundary conditions are $\frac{\partial u}{\partial r}(r, z, t) = 0$, $v(r, z, t) = 0$ when $r = 0$ and on the wall$u(r, z, t) = 0$and $v(r, z, t) = 0$ at $r = R(z)$ **(7)**

III. NUMERICAL SIMULATION: COMPUTATIONAL METHOD

As because of domain is not rectangular so we need to transform the domain into rectangular domain. We have used the following radial transformation $x = \frac{r}{R(z)}$ **(8).** The governing equations (2) and (3) transformed as

$$\frac{\partial u}{\partial t} = -\left(v\frac{1}{R(z)}\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial z} - u\frac{x}{R(z)}\frac{\partial u}{\partial x}\frac{\mathrm{dR}}{\mathrm{dz}}\right) - \frac{\partial p}{\partial z} + \frac{1}{Re}\left\{\frac{1}{R^2(z)}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial u}{\partial x}\right) + \frac{\partial^2 u}{\partial z^2}\right\} - \frac{1}{Re}\left\{2\frac{x}{R(z)}\frac{\mathrm{dR}}{\mathrm{dz}}\frac{\partial^2 u}{\partial x \partial z} + \frac{x}{R(z)}\frac{\partial u}{\partial x}\frac{d^2 R}{\partial z^2} - \left(\frac{\frac{\mathrm{dR}}{\mathrm{dz}}}{\mathrm{R(z)}}\right)^2\left(2x\frac{\partial u}{\partial x} + x^2\frac{\partial^2 u}{\partial x^2}\right)\right\} + a_0\left(cos(\omega t + \varphi)\right)$$

$$\frac{\partial v}{\partial t} = -\left(v\frac{1}{R(z)}\frac{\partial v}{\partial x} + u\frac{\partial v}{\partial z} - u\frac{x}{R(z)}\frac{\partial v}{\partial x}\frac{\mathrm{dR}}{\mathrm{dz}}\right) - \frac{1}{R(z)}\frac{\partial p}{\partial x} + \frac{1}{Re}\left\{\frac{1}{R^2(z)}\left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{x}\frac{\partial v}{\partial x} - \frac{v}{x^2}\right) + \frac{\partial^2 v}{\partial z^2}\right\} - \frac{1}{Re}\left\{2\frac{x}{R(z)}\frac{\mathrm{dR}}{\mathrm{dz}}\frac{\partial^2 v}{\partial x \partial z} + \frac{x}{R(z)}\frac{\partial v}{\partial x}\frac{d^2 R}{\partial z^2} - \left(\frac{\frac{\mathrm{dR}}{\mathrm{dz}}}{\mathrm{R(z)}}\right)^2\left(2x\frac{\partial v}{\partial x} + x^2\frac{\partial^2 v}{\partial x^2}\right)\right\} \quad \textbf{(9)}$$

and the continuity equation becomes

$$\frac{\partial u}{\partial z} + \frac{v}{xR(z)} + \frac{\partial v}{\partial x} - \frac{x}{R(z)}\frac{\partial u}{\partial x}\frac{\mathrm{dR}}{\mathrm{dz}} = 0 \quad \textbf{(10)}$$

and the initial (6) and boundary (7) conditions becomes

$u(x,z,t) = 0$ and $v(x,z,t) = 0$at$t = 0$. **(11)**

$\frac{\partial u}{\partial t}(x,z,t) = 0, v(x,z,t) = 0$ when $x = 0, u(x,z,t) = 0$ and $v(x,z,t) = 0$ at $x = 1$. **(12)**

Here, we describe the methodology used to obtain numerically solution of [(9)-(10)]. The discretization procedure of different terms is as follows. The central difference approximations are used to discretize all the spatial derivatives while the explicit forward finite difference approximation is used for time derivative term present in [(9)- (10)] in the following manner

$$\frac{\partial u}{\partial z} = \frac{(u)_{i+1,j}^n - (u)_{i-1,j}^n}{2\Delta z}, \quad \frac{\partial u}{\partial x} = \frac{(u)_{i+1,j}^n - (u)_{i-1,j}^n}{2\Delta x}$$

$$\frac{\partial u}{\partial t} = \frac{(u)_{i,j}^{n+1} - (u)_{i,j}^n}{\Delta t}, \quad \text{and}$$

$$\frac{\partial^2 u}{\partial^2 z} = \frac{(u)_{i+1,j}^n - 2(u)_{i,j}^n + (u)_{i-1,j}^n}{\Delta z^2} \quad \textbf{(13)}$$

Where$(u)_{i,j}^n = u(x_j, z_i, t_n)$,$z_i = (i-1)\,\Delta z,\, i = 1,2,\dots,N+1;$

$x_j = (j-1)\,\Delta x,\, j = 1,2,\dots.,N+1\,;$

$t_n = (n-1)\,\Delta t,\, j = 1,2,\dots\dots..;$

Similarly we can approximate the all the partial derivatives of v.

The axial velocity $(u)_{i,j}^n$ can be obtained from [(9)- (10)] by applying the above finite difference approximation at any point (z_i, x_j) in the domain of interest at any time t_n with the help of following discritize from initial and boundary conditions [(11)- (12)].

$$(u)_{i,j}^1 = 0,\;\; (v)_{i,j}^1 = 0 \quad \textbf{(14)}$$

$$(u)_{i,1}^n = (u)_{i,2}^n\,, (v)_{i,1}^n = 0\,, \qquad (u)_{i,N+1}^n = (v)_{i,N+1}^n = 0 \quad \textbf{(15)}$$

and input pressure gradient from (5).Thus we obtained the radial velocity $(u)_{i,j}^n$ from (10). Now finally we can determine the volumetric flow rate (Q) and the wall shear stress (τ) with the help of radial and axial velocities from the following equations

$$Q_i^n = 2\pi(R_i^n)^2\int_0^1 x_j(u)_{i,j}^n\,dx_j \quad \textbf{(16)}$$

$$\tau_i^n = \frac{\mu}{R_i^n}\left(\frac{u_{i,N+1} - u_{i,N}}{\Delta n}\right) \quad \textbf{(17)}$$

IV. RESULTS AND DISCUSSION

In this section, we shall talk about the complete analysis of the nonlinear mathematical model for various values of the parameters to study the influence of stenoses and body acceleration on the flow. In our numerical computation, we set dimensionless parameters from [4] and[18] as following:

$l = 2.00, d = 3.00, p = 0.50, \quad Re = 400, \quad 600, 800, a_0 = 0,1\,, A_0 = 0.1, A_1 = 0.2 \times A_0$

The results obtained for axial velocity by solving explicit finite difference scheme with various grid sizes are taken in order to achieve the convergence and stability. We have performed the experiments for grid size 60 × 60, 100 × 100 with $dt = 0.01, 0.001$. The results are found to be very similar in both the cases.

Fig.2(a) illustrates the axial velocity profiles without body acceleration for three different Reynolds number. All curves in Fig.2(a) are coincide with each other when 0 < x < 0.8 and then axial velocity get decreases near the arterial lumen also one can say by observing the Fig.2(b) that the axial velocity increases in the constricted part of the artery as Reynolds number increases.

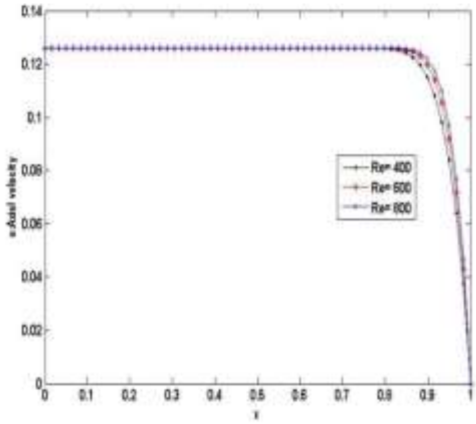


Fig. 2(a) Distribution of axial velocity for different Reynolds number without body acceleration.

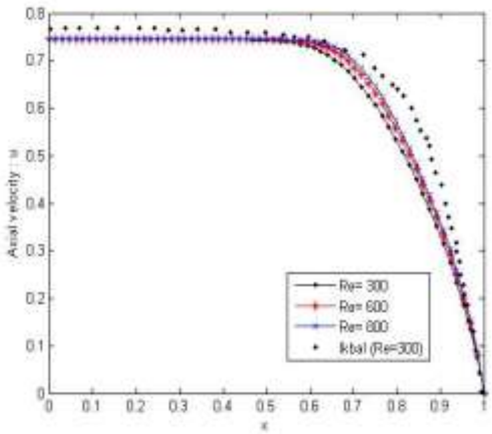


Fig. 2(b) Distribution of axial velocity for different Reynolds number with body acceleration.

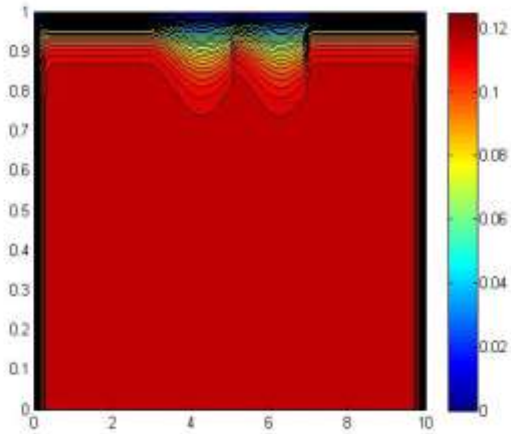


Fig.3(a) Contour distribution of axial velocities for Reynolds number $Re = 400$ withoutbody acceleration

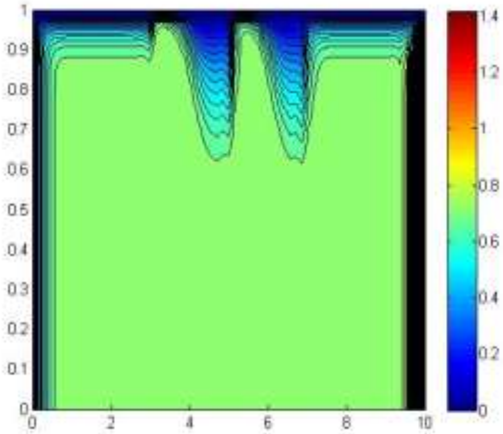


Fig.4(b) Contour distribution of axial velocities for Reynolds number Re=400 with body acceleration

To test the effects body acceleration on axial velocity profile several simulations have been carried out using the contour plot as shown in Fig.3(b). The velocity profile u is shown in different region with different colors representing the value of velocity with the help of color bar. One can see from these plot that velocity profile is divided into different layers due to the constriction of the artery and changes in the plot also can be observed in case of no body acceleration in Fig.3(a) . The Fig.4(a) and Fig.4(b) shows the distribution of flux over the stenosed artery for different Reynolds number. One can conclude that flux decreases near the picks of the stenosis.

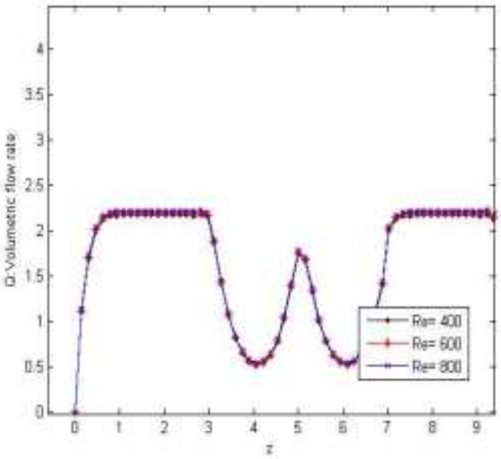


Fig.5(a) The distribution of Flux without body acceleration.

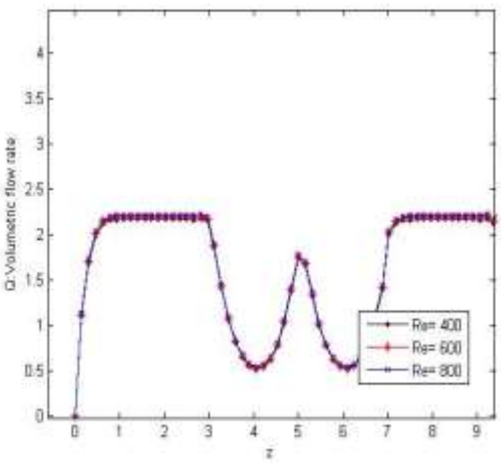


Fig. 6(b) The distribution of Flux with body acceleration.

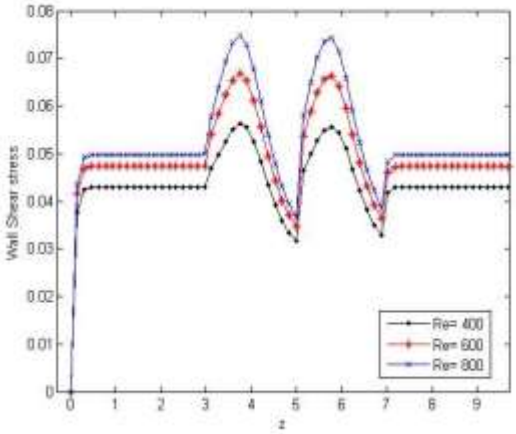


Fig. 7(a)The wall shear stress without body acceleration.

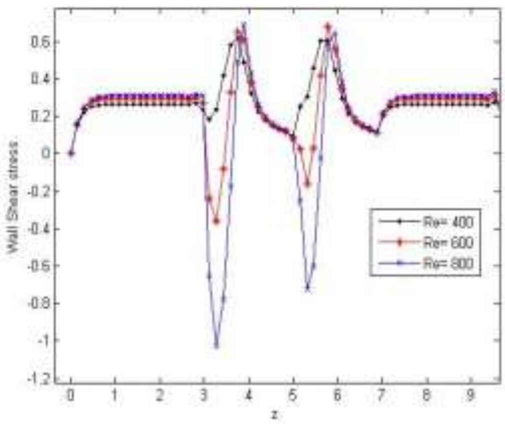


Fig. 8(b) The wall shear stress with body acceleration.

Wall shear stress plays an important role in the creation and propagation of arteriosclerosis. If the wall shear stress is very high then it may damage the arterial wall and is the main cause of the intimal thickening. On the other hand, the plaque formation in an artery creates in the regions of low arterial wall shear stress. Atherosclerotic lesions are associated with low and high wall shear stress. So it is important to study the wall shear stress distribution in the multi-stenosed artery. The Fig.5[a-b] shows the distribution of wall shear stress on the arterial segment for three different Reynolds number. The wall shear stress is increases rapidly near to the peak of the constriction. Here the effects of Reynolds number can be observed from the figure. The wall shear stress increases as Reynolds number increases. The streamlines of the blood flow in the artery with multi-stenosis is found in the transformed rectangular domain with grid 60×60 in the upper half zone and same grid also taken for lower half portion. We have plotted the different types of streamlines in Fig.6 [a-b] at Re=400. All the streamlines follow the straight line path near the axis which gradually get perturbed more towards the wall of the stenosed artery. It is interesting to observe that several flow lines are attracted towards the stenotic wall upstream with the formation of circulation zones while others pass through the constricted region directly following the main stream.

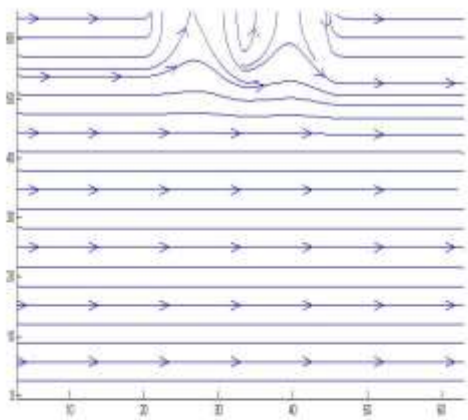


Fig. 9(a)Streamlines of the flow without body acceleration.

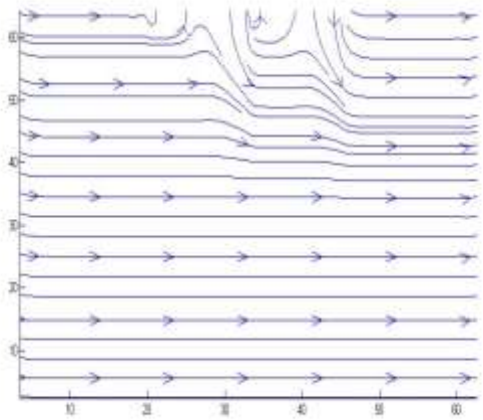


Fig. 10(b) Streamlines of the flow with body acceleration.

V. CONCLUSION

A non-linear mathematical model for blood flow in a cosine shaped stenosed arterial segment has been developed under the influence of body acceleration. The numerical simulation of blood flow is investigated in this study. The flow mechanism has been made, governed by a pulsatile pressure gradient. As the Reynolds number increases, the wall shear stress is increases. The flow velocity diminishes downstream from its value at the onset of the stenosis and further increases upstream towards the non-stenosed region. Volumetric flow rate occur where the arterial narrowing approach maximum. The severity of the stenosis has significant effect on the wall shear stress in such a way that it develops more at the constricted locations than all other sites of the artery. The hemodynamic of the flow in an arterial segment containg muti stenosis have been studied.The results obtained here would help the interested people such as pathologists, medical surgeons and researchers greatly in gaining better insight into blood flow models through the multi-stenosis artery under the influence of body acceleration.

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