Effect of TCM Scheme on improvement of the BER of GPS Signal

Raj Kumar Goswami and G Sasibhushana Rao
Department of ECE
Andhra University College of Engineering (A), Andhra University, Visakhapatnam-530003
rajkumargoswami@gmail.com

Abstract

Present modulation scheme for GPS employs Binary Phase Shift Keying (BPSK) which uses conventional rectangular (non – return to zero) spreading symbols. With regards to improve the performance of GPS system as a whole the primary focus area has been in the design of the spreading code and selection of the keying rates. Minimal thrust has been given in the areas of selection of the modulation schemes. In the present day scenario there are modulation schemes such as TCM those are available, which can be employed to improve the performance while retaining the basic principle intact. One such modulation scheme has been described in this paper where Rate 1/3, 8-State, 8PSK TCM Scheme has been implemented in place of BPSK that is used by GPS. It has been shown that at the lower SNRs the Bit Error Rate(_BER) has improved by approximately 1 to 2 dB if the tradition BPSK is replaced by the proposed TCM scheme.

I. INTRODUCTION

Present modulation designs in respect of Global Positioning System (GPS) have been restricted to BPSK with rectangular spreading symbols duplicating early modulation designs for digital communications. It is an established fact that when other sources of error diminish, contributions from noise and multipath start to dominate. But bandwidth limitations preclude further improvements that might be obtained using Binary Phase Shift keying (BPSK) modulations with faster keying rates. Also, increasing transmitted power to improve accuracy is expensive and has a limited effect on multipath performance. Receiver design strategies such as very wide-bandwidth receiver front ends and very small early–late spacing in code-tracking discriminators provide diminishing returns at increasing cost. In contrast, modulations designed specifically to mitigate for multipath effects can outperform existing modulation designs while using the same bandwidth. One such modulation scheme is Trellis Coded Modulation (TCM) scheme that can be employed in the application of GPS. In a traditional digital communication, the role of modulator and demodulator is restricted to convert an analog waveform channel to a discrete channel and the role of encoder and decoder is to correct the errors those occurred in this discrete channel. To achieve the higher performance, code rate is lowered and the redundancy in the code increased at the cost of bandwidth expansion. This code then can be integrated with the expanded bandwidth efficient signal set so as to utilize the redundancy resulted from such an expansion in order to avoid bandwidth expansion. The convolutional code when integrated with a bandwidth efficient modulation scheme is termed as TCM [1]. The basic idea of using an error correction coding scheme followed by a suitable modulation scheme has been in existence for long, in fact, multilevel modulation of convolutionally encoded symbols was a known concept before the introduction of TCM. Although the expansion of a signal set provides the redundancy required for coding, it shrinks the distance between the signal points if the average energy is kept constant. The reduction in the distance between the signal points increases the error rate, which should be compensated with coding if the coded scheme is to provide any benefit. Therefore it can be seen that the innovative aspect of TCM is the concept that the convolutional encoding and the modulation should not be treated as separate entities, but rather, as a unique operation. The detection process as a result therefore involve soft, rather than hard-decisions. The use of hard-decision demodulation prior to the decoding in a coded scheme causes an irreversible loss of information, which translates into a loss of SNR. If the maximum likelihood criterion is applied in soft-decision decoding on the AWGN channel, the decision rule of the optimum sequence decoder will depend on the *Euclidean distance*. In other words, the optimum decoder will choose the code sequence that is nearest to the received sequence in terms of Euclidean distance. This TCM schemes can be used for reliable transmission of GPS Signal provided appropriate design criteria [2] is utilized in designing the code.

In this paper, the alternative modulation scheme i.e. TCM scheme has been proposed in place of BPSK for the use of transmission of GPS signal. The design criteria have been formulated by obtaining an insight into the performance of TCM schemes in AWGN channels. This has been followed by developing a set of rules which meet these criteria for the case of a rate 1/3, 8-state, 8-PSK TCM scheme, thus facilitating a systematic design rather than carrying out a brute-force search for the optimum code. The performance of the code constructed using these rules have then been evaluated using the union bound.

II. GPS SIGNAL OVERVIEW

Global Positioning System (GPS) is a modern-day triangulation based positioning scheme. GPS satellites transmit ranging signals that, when received and processed by a GPS receiver, provide distance measurements between the satellites and receiver. The satellites broadcast ranging codes and navigation data on two frequencies using a technique called Code Division Multiple Access (CDMA). There are only two frequencies in use by the system, L1 (1575.42 MHz) and L2 (1227.6 MHz). The fundamental frequency (f_o) is 10.23 MHz, L1 is 154 times f_o and L2 is 120 times f_o. Figure 1 depicts the structure of GPS Satellite signal.

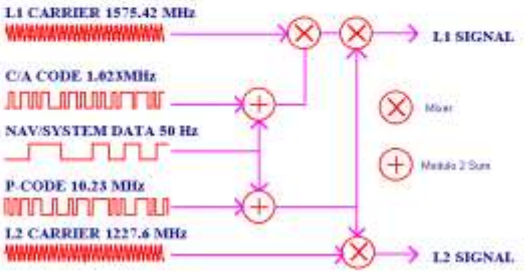


Fig 1 GPS satellite signal

"All transmitted signal elements (carriers, codes, and data) are coherently derived from the same on-board frequency source". This on-board signal frequency is at 10.23MHz, denoted as f_o. The broadcast L1 signal frequency is 154* f_o and the L2 signal is at 120* f_o. When the C/A or P code is modulated onto the carrier wave, it is done in a manner called BPSK modulation. This means that on each of the code chip transitions of the signal, from +1 to -1, there is a potential change in phase of the carrier. This has been depicted in Figure 2.

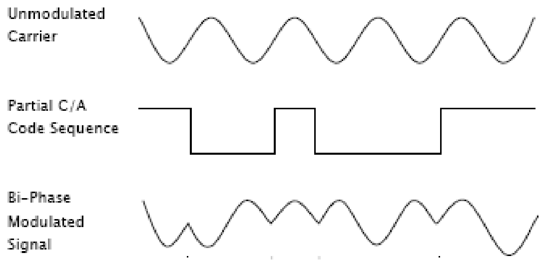


Fig 2 BPSK Modulation of code onto carrier

Because a GPS receiver requires multiple signals from separate satellites, the GPS signals use a multiple access technique to keep signals from interfering with each other. The technique used in GPS is called Direct Sequence Spread Spectrum (DSSS) modulation, which is a type of Code Division Multiple Access (CDMA). This method modulates the carrier in such a way that multiple signals simultaneously occupy the same frequency spectrum, while still maintaining orthogonality in "code space," thus, they can be readily distinguished from one another at the receiver. In a DSSS system, there are three basic components and they are the carrier frequency, the data modulation, and the pseudorandom spreading modulation. Figure 3 shows these three basic components in a very simplified diagram of a DSSS transmitter. The spreading modulation is a pseudorandom sequence of ones and zeros which applies additional BPSK modulation to the data modulated carrier. The spreading modulation frequency is typically at least an order of magnitude higher than the data modulation frequency. The spreading modulation "codes" the signal in the transmitter so that it can only be decoded with the same spreading sequence locally generated in the receiver.

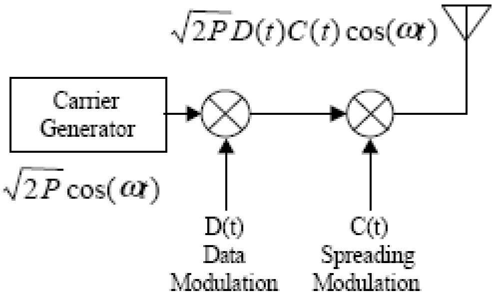


Fig 3 Basic spread spectrum transmitter

The carrier is data modulated using Phase Shift Keying (PSK); where data bit values are used to shift the carrier signal phase as shown in Figure 4.

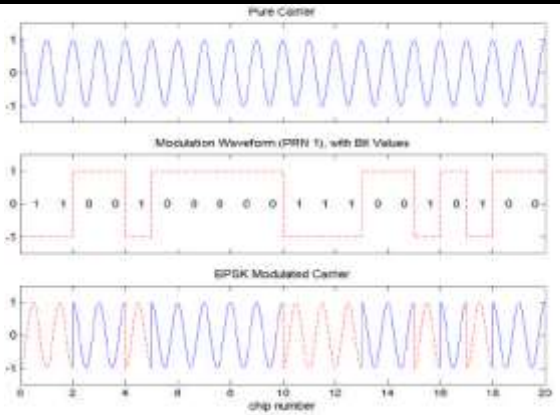


Fig 4 Example of BPSK modulation

III. TCM OVERVIEW

A general block diagram of a TCM scheme is shown in Fig. 5. Input bits are encoded by a trellis encoder to produce a sequence of signals s_l = $(s_1, s_2 \ldots s_l)$, where each signal s_i is a two-dimensional vector chosen from an MPSK signal set and l denotes the current time index. Using complex notation, we can represent each of the signals, s_i, by a point in a complex plane. The coded signals are interleaved to spread the bursts of errors caused by a slowly varying fading process. The sequence of interleaved signals is denoted by $\boldsymbol{b}_l$ $= (b_1, b_2 \ldots b_l)$. The in-phase and quadrature components of the interleaved coded signals are pulse-shaped for eliminating ISI and then used to modulate a carrier for transmission over the channel. The channel corrupts the transmitted signal by introducing a fading gain and an additive white Gaussian noise term.

$$c_i = a_i \cdot e^{j\phi_i} \qquad (2)$$

At the receiver, the in-phase and quadrature components of the received signal are demodulated and quantized for soft-decision decoding. The *channel estimator* provides an estimate of the channel gain that, in turn, can be used in the decoding process to improve the performance of the coded system. The estimate of the channel gain is referred as Channel State Information (CSI).

$$r_i = a_i s_i + n_i \qquad (3)$$

The output sequence of the demodulator $\boldsymbol{v}_l$ = (v_1,v_2, …v_l) and the CSI sequence $\hat{\boldsymbol{C}}'_l$ = ($\hat{C}'_1$, $\hat{C}'_2$, … $\hat{C}'_l$) are deinterleaved to produce the sequences $\boldsymbol{r}_l$ = (r_1,r_2, …r_l) and $\hat{\boldsymbol{C}}_l$ = ($\hat{C}_1$, $\hat{C}_2$, … $\hat{C}_l$), respectively. The two sequences $\boldsymbol{r}_l$ and $\hat{\boldsymbol{C}}_l$ are the inputs to TCM decoder which performs maximum likelihood (ML) decoding.

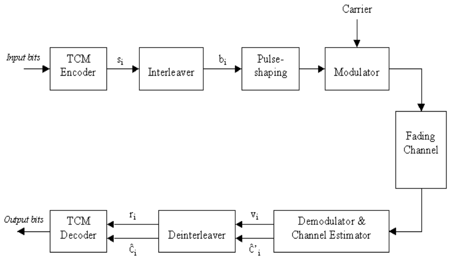


Fig. 5 Block diagram of TCM Scheme.

A discrete time model for the system of Fig. 5 is shown in Fig. 6. Using this model the received signal at time *i* can be written as

$$r_i = c_i . s_i + n_i \qquad (1)$$

where n_i is a sample of a zero-mean complex Gaussian noise process with variance $\sigma^2_n = N_0/2$ and the complex channel gain c_i is a sample of a complex Gaussian process with variance σ^2_c. Using phasor notation, the complex channel gain c_i is expressed as

where, a_i and ϕ_i are the amplitude and the phase processes, respectively.

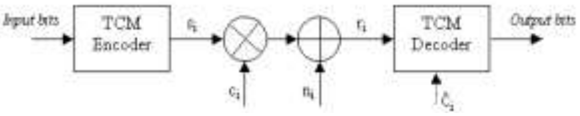


Fig. 6 Baseband system model of Fig. 5

It is assumed that the receiver performs coherent detection, and hence the channel phase shift is compensated by the receiver and therefore (1) can be further simplified as where a_i is the amplitude of the fading process. For a channel with only a diffused multipath component, the fading amplitude is usually modeled to be Rayleigh distributed with a probability density function (PDF)

$$p_A(a) = 2ae^{-a^2}, \qquad a \geq 0. \qquad (4)$$

When there is a single dominant, nonfading component in the received signal, along with multipath fading, the amplitude a_i is Rician distributed, with a PDF

$$p_A(a) = 2a(1+K)e^{-(K+a^2(1+K))} I_0(2a\sqrt{K(1+K)}),$$
$$a \geq 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

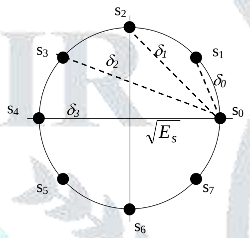
where *K* is the Rician parameter defined as the ratio of the energies of the received signal in the direct and diffused multipath components and $I_0(.)$ is the zero-order modified Bessel function of the first kind. Note that both the PDF's (4) and (5) are normalized, i.e., the fading amplitude, *a*, has a mean-squared value of unity. A normalized probability density function is chosen so that the measured signal energy at the receiver represents the average signal energy per channel symbol, E_s. The fading channel under consideration is a multipath dominated frequency-selective fading channel, to which the above model would not conform. However, It is assumed that an appropriate equalization/channel-compensation mechanism would be employed to overcome the effect of time-varying multipath prior to the TCM-decoding of the received symbols. Also, further assumed that the interleaving is ideal, i.e., an interleaver depth is infinite. These imply that the fading amplitudes are statistically independent and provide a memoryless channel model for the performance analysis. Though fading channel has been discussed in this section, it is done for the sake of completeness, when there is no multipath then the channel is limited to AWGN Channel. In this paper the design has been focused for AWGN channel.

IV. PROPOSED DESIGN

In this section a design has been proposed for designing and constructing rate 1/3, 8-state 8- PSK, TCM codes, optimum for AWGN channels. Using heuristics, Ungerboeck has designed a rate 2/3, 8-state, 8PSK TCM that provides an effective length, *L*, of 2 and $d^2_{free} = 4.586E_s$. In this paper following guidelines, have been proposed to design rate 1/3, 8-state, 8PSK TCM schemes optimized for the AWGN channel.

The signals associated with transitions between states of consecutive stages are represented by a matrix of dimension 8 x 8, whose ij^{th} element represents the signal associated with the path from state *i*, at stage *k*, to state *j*, at stage *k+1*, of the trellis. Also, the elements of the i^{th} row indicate signals associated with path diverging from state *i* and the elements of the j^{th} column show signals associated with paths reemerging at state *j*.

Using set partitioning, the 8PSK signal set can be partitioned into two subsets, viz., $A_0 = \{s_0, s_2, s_4, s_6\}$ and $A_1 = \{s_1, s_3, s_5, s_7\}$ with intra-set distances δ_1 and δ_3, as shown in the signal constellation diagram of Fig. 7.



$$\delta_0^2 = (2-\sqrt{2})E_s \approx 0.586E_s$$
$$\delta_1^2 = 2E_s$$
$$\delta_2^2 = (2+\sqrt{2})E_s \approx 3.414E_s$$
$$\delta_3^2 = 4E_s$$

Fig. 7 8PSK signal constellation.

The trellis diagram for the proposed design is shown in figure 8.

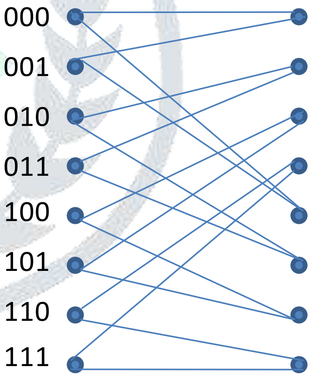


Fig. 8 Trellis diagram of the rate 1/3, 8-state, 8PSK TCM

After having gone through the GPS signal structure and the concept of TCM, block diagram shown in figure 9 depicts the proposed modification of figure 3, post implementation of proposed TCM scheme in place of BPSK that is used presently in the GPS.

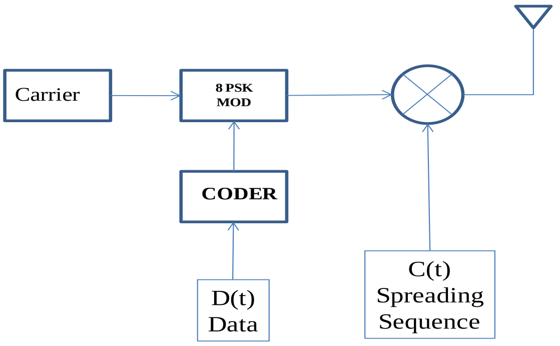


Fig. 9 The proposed implementation

V. PERFORMANCE ANALYSIS

The performance of the TCM scheme has been evaluated by using the union bound. An upper bound on the bit error probability can be obtained by taking into consideration the effect of all possible error events of all lengths. Unfortunately, in enumerating such error events the generating function approach cannot be employed due to the intractable form of the exact expression for $P_2(\boldsymbol{s}_l, \hat{\boldsymbol{s}}_l)$. However, the bit error probability can be estimated reasonably well by considering a small number of dominant error events rather than by accounting for the error event paths of all lengths. Hence we can approximate the bit error probability as

$$P_b \approx \frac{1}{n}\sum_{l=1}^{\lambda}\sum_{s_l \neq \hat{s}_l} \bar{n}_l \, P_2(s_l, \hat{s}_l) \qquad (6)$$

where n_l is the average number of bit errors associated with the error event $(\boldsymbol{s}_l, \hat{\boldsymbol{s}}_l)$, n is the number of information bits per symbol and λ is a limit imposed on the actual length of the error events to be considered.

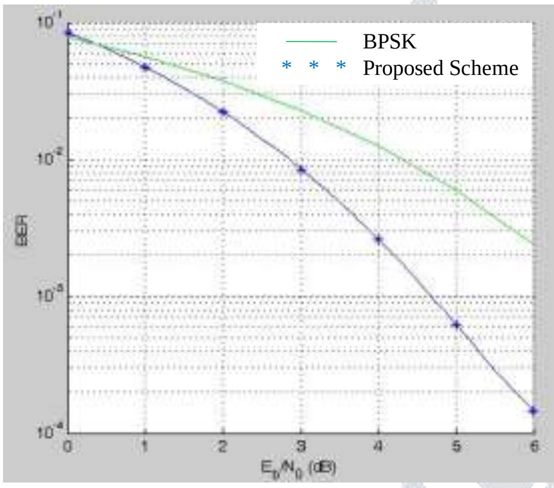


Fig. 10 Comparison of the performance bound

Fig. 10 shows a comparison of the performance bound of the BPSK scheme that is used by the existing GPS system and the proposed Rate 1/3, 8-State, 8-PSK scheme. It is seen that the proposed scheme designed here exhibits about 1-2 dB advantage over the existing BPSK scheme.

VI CONCLUSIONS

It is concluded that by replacing the existing BPSK modulation scheme with the proposed TCM scheme, 1-2 dB gain can be achieved especially at low SNRs. It has been achieved by integrating the convolutional code with an expanded bandwidth signal set and thereby utilizing the redundancy. It is seen that a higher performance is achieved without lowering the code rate. With the design of the TCM scheme for AWGN channel that has been proposed in this paper, it is concluded that the proposed 8-state, rate 1/3 TCM scheme exhibits about 1-2 dB advantage over the existing BPSK scheme at smaller values of SNR's. Not only this, the convolutional code also gives an added advantage of secrecy as the receiver must know about the code that has been implemented in the transmitter in order to decode the signal. In this paper though we have designed and evaluated the TCM scheme for the AWGN Channel, however in future this work can be extended to the design of the code for the fading channel.

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