PERFORMANCE ANALYSIS SINGLE TCM SCHEME WITH MULTI-TCM SCHEME (MULTIPLICITY FACTOR 2) FOR RAYLEIGH CHANNELS

Raj Kumar Goswami and G Sasibhushana Rao
Department of ECE
Andhra University College of Engineering (A), Andhra University, Visakhapatnam-530003
rajkumargoswami@gmail.com

Abstract
In designing of any digital communication system, coding and modulation are generally treated as two independent functions. The role of modulator and demodulator is restricted primarily to convert an analog waveform channel to a discrete channel and the role of encoder and decoder is to correct the errors those occurred in this discrete channel. To achieve the higher performance, code rate is lowered and the redundancy in the code increased at the cost of bandwidth expansion. This code can be integrated with the expanded bandwidth efficient signal set so as to utilize the redundancy resulted from such an expansion in order to avoid bandwidth expansion. The convolutional code when integrated with a bandwidth efficient modulation scheme is termed as Trellis Coded Modulation (TCM) [1]. In constructing conventional TCM for fading channels the effective length of the code is increased by increasing the number of encoder states and preventing parallel transitions. In this paper we examine an alternate approach for increasing the effective length of the code by assigning more than one symbol per trellis branch. This technique, proposed by Simon and Divsalar [2], is referred to as multiple trellis coded modulation (MTCM). Finally the performance of both the scheme is compared.

I. Introduction

Error control coding is used to improve the performance of a digital communication system. Traditionally, coding and modulation have been considered as two independent functions in the design of a digital communication system. The modulator and demodulator are usually devised to convert an analog waveform channel to a discrete channel and the encoder and decoder are designed to correct the errors that occurred in the discrete channel. Higher performance is achieved by lowering the code rate and increasing the redundancy in the code at the cost of bandwidth expansion by an amount equal to the reciprocal of the code rate. To avoid bandwidth expansion, one may integrate a code with an expanded bandwidth efficient signal set and utilize the redundancy resulting from such an expansion. The integration of a convolutional code with a bandwidth efficient modulation scheme is called *Trellis Coded Modulation* (*TCM*) [1].

The basic idea of using an error correction coding scheme followed by a suitable modulation scheme such that there is no bandwidth expansion is not new. Multilevel modulation of convolutionally encoded symbols was a known concept before the introduction of TCM. Although the expansion of a signal set provides the redundancy required for coding, it shrinks the distance between the signal points if the average energy is kept constant. The reduction in the distance between the signal points increases the error rate, which should be compensated with coding if the coded scheme is to provide any benefit. Thus it can be seen that the innovative aspect of TCM is the concept that convolutional encoding and modulation should not be treated as separate entities, but rather, as a unique operation. The detection process will, as a result, involve soft, rather than hard-decisions. The use of hard-decision demodulation prior to the decoding in a coded scheme causes an irreversible loss of information, which translates into a loss of SNR. If the maximum likelihood criterion is applied in soft-decision decoding on the AWGN channel, the decision rule of the optimum sequence decoder will depend on the *Euclidean distance*. In other words, the optimum decoder chooses the code sequence that is nearest to the received sequence in terms of Euclidean distance. The consequence of this is that, in an AWGN channel, the parameter governing the performance of the transmission system using TCM is, in fact, the *free Euclidean distance* between the transmitted signal sequences and not the *free Hamming distance* of the convolutional code.

In this paper, we consider design alternatives for an appropriate TCM scheme for the HF channel. We start by obtaining an insight into the performance of TCM schemes in fading channels, leading to a set of design criteria. This is followed up by developing a set of rules which help to meet these criteria for the case of a rate 2/3, 8-state, 8PSK TCM scheme. The performance of the code constructed using these rules is then evaluated using the union bound. Finally we also investigate whether it is possible to improve on this performance via the design and use of the so-called Multiple Trellis Coded Modulation (MTCM), proposed by Simon and Divsalar [2] vis-à-vis the associated complexity.

II. System model

A general block diagram of a TCM scheme on a fading channel is shown in Fig. 1. Input bits are encoded by a trellis encoder to produce a sequence of signals $\boldsymbol{s}_l$ = $(s_1, s_2 \ldots s_l)$, where each signal s_i is a two-dimensional vector chosen from an MPSK signal set and l denotes the current time index. Using complex notation, we can represent each of the signals, s_i, by a point in a complex plane. The coded signals are interleaved to spread the bursts of errors caused by a slowly varying fading process. The sequence of interleaved signals is denoted by $\boldsymbol{b}_l$ = $(b_1, b_2 \ldots b_l)$. The in-phase and quadrature components of the interleaved coded signals are pulse-shaped for eliminating ISI and then used to modulate a carrier for transmission over the channel. The channel corrupts the transmitted signal by introducing a fading gain and an additive white Gaussian noise term.

At the receiver, the in-phase and quadrature components of the received signal are demodulated and quantized for soft-decision decoding. The *channel estimator* provides an estimate of the channel gain that, in turn, can be used in the decoding process to improve the performance of the coded system. The estimate of the channel gain is referred as Channel State Information (CSI).

The output sequence of the demodulator $\boldsymbol{v}_l = (v_1, v_2, \ldots v_l)$ and the CSI sequence $\boldsymbol{c}'_l = (c'_1, c'_2, \ldots c'_l)$ are deinterleaved to produce the sequences $\boldsymbol{r}_l = (r_1, r_2, \ldots r_l)$ and $\boldsymbol{c}_l = (c_1, c_2, \ldots c_l)$, respectively. The two sequences $\boldsymbol{r}_l$ and $\boldsymbol{c}_l$ are the inputs to TCM decoder which performs maximum likelihood (ML) decoding.

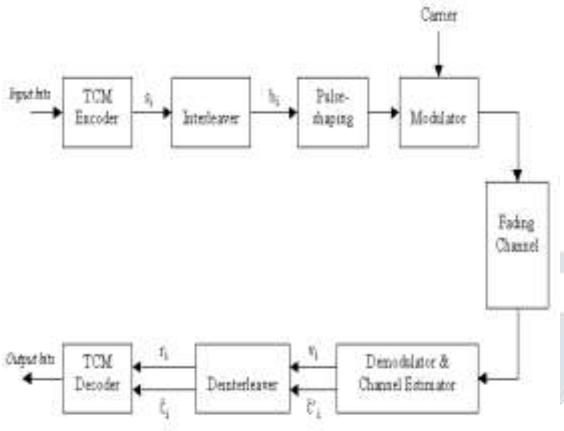

Fig. 1 System block diagram.

Using this model the received signal at time *i* can be written as

$$r_i = c_i . s_i + n_i \qquad (1)$$

where n_i is a sample of a zero-mean complex Gaussian noise process with variance $\sigma^2_n = N_0/2$ and the complex channel gain c_i is a sample of a complex Gaussian process with variance σ^2_c.

Next, we assume that the receiver performs coherent detection, and hence the channel phase shift is compensated by the receiver. In this case, (1) can be simplified as

$$c_i = a_i \cdot e^{j\phi_i} \qquad (2)$$

where, a_i and ϕ_i respectively are the amplitude and the phase processes of the fading channel.

It has been assumed that an appropriate equalization or channel compensation mechanism would be employed to overcome the effect of time-varying multipath prior to the TCM-decoding of the received symbols. Also, it is further assumed that the interleaving is ideal, i.e., an interleaver with infinite depth is used. These imply that the fading amplitudes are statistically independent and provide a memoryless channel model for the performance analysis.

III. Code design of a rate 2/3, 8-state, 8PSK TCM scheme

In this section we formulate a set of rules which would help us to design and construct rate 2/3, 8-state TCM codes which are optimum for fading channels. These rules build up on similar rules that have been proposed earlier in the context of optimal 4-state, rate 2/3 TCM codes for fading channels [3].

Using heuristics, Ungerboeck has designed a rate 2/3, 8-state, 8PSK TCM that provides an effective length, *L*, of 2 and $d^2_{free} = 4.586E_s$, which was optimized for the AWGN channel [3]. The minimum product of squared branch distances along the error events of actual length being 2, $d^2_p(L)$, for this code equals 8. We shall represent the signals associated with transitions between states of consecutive stages by a matrix $\boldsymbol{B}$ of dimension 8 x 8, whose ij^{th} element will represent the signal associated with the path from state *i*, at stage *k*, to state *j*, at stage *k+1*, of the trellis. Also the elements of the i^{th} row will indicate signals associated with path diverging from state *i* and the elements of the j^{th} column will show signals associated with paths reemerging at state *j*. Using set partitioning, the 8PSK signal set can be partitioned into two subsets, viz., $A_0 = \{s_0, s_2, s_4, s_6\}$ and $A_1 = \{s_1, s_3, s_5, s_7\}$ with intra-set distances δ_1 and δ_3, as shown in the signal constellation diagram of Fig. 2.

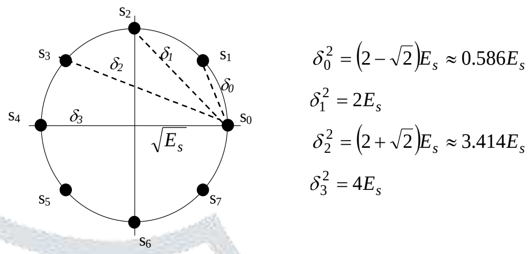

Fig. 2 8PSK signal constellation.

To assign signal points to the elements of matrix $\boldsymbol{B}$, the following rules are utilized.

I. A signal can occur only once in a given row or column.

II. As the number of paths emerging from a state (for a rate 2/3 code) is only 4, and there are 8 states, all transitions are not possible. A signal can be associated with a transition path between two states, only if the LSB of the label of the initial state is the same bit, $z \in \{0,1\}$, as the MSB of the destination state. For a given value of *z*, all the signals associated should strictly be from one of the two sets A_0 or A_1.

III. The distance between a pair of signals associated with a given row or a column is δ_1 or δ_3.

IV. Performance analysis of TCM

Using the union bound, an upper bound on the bit error probability can be obtained taking into consideration the effect of all possible error events of all lengths. The bit error probability can be estimated reasonably well by considering a small number of dominant error events rather than by accounting for the error event paths of all lengths. Hence we can approximate the bit error probability as

$$P_b \approx \frac{1}{n} \sum_{l=1}^{\lambda} \sum_{s_l \neq \hat{s}_l} \bar{n}_l \, P_2(s_l, \hat{s}_l) \qquad (6)$$

Where $\bar{n}_l$ is the average number of bit errors associated with the error event $(\boldsymbol{s}_l, \hat{\boldsymbol{s}}_l)$, *n* is the number of information bits per symbol and λ is a limit imposed on the actual length of the error events to be considered. To evaluate the bit error probability of the rate 2/3, 8-state, 8PSK TCM scheme constructed above we limit our calculation to the error events with λ=4. Fig. 3 shows a comparison of the performance bound of the 4-state TCM scheme constructed as per the rules proposed by Jamali and Tho Le Ngoc [3] with that of the 8-state TCM scheme constructed as per the aforementioned rules. The performance bound was evaluated for a Rayleigh fading channel with CSI. It is seen that the 8-state TCM code designed here exhibits about 3dB advantage over the 4-state TCM code of [3] at large SNR's.

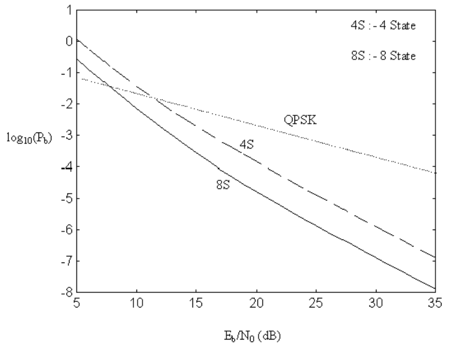

Fig. 3 Performance bound of rate 2/3, 8-state, 8PSK TCM schemes on Rayleigh fading channel with CSI.

V. MTCM

In the preceding sections we considered conventional trellis codes constructed for fading channels. In these schemes, one symbol is assigned to each branch of the code trellis. So, as inferred earlier, for the case of conventional trellis coding for fading channels, from an error probability performance standpoint it is undesirable to design the code to have parallel paths in its trellis diagram [4]. This is because the presence of parallel paths will limit the effective length of the code to be 1. In constructing conventional TCM for fading channels the effective length of the code is increased by increasing the number of encoder states and preventing parallel transitions. In this section we examine an alternative approach for increasing the effective length of the code by assigning more than one symbol per trellis branch. This technique, proposed by Simon and Divsalar [2], is referred to as *multiple trellis coded modulation* (MTCM).

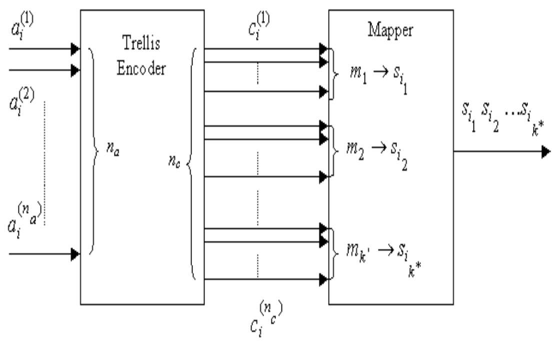

Fig. 4 A general block diagram of a MTCM scheme.

A general block diagram of a MTCM encoder is shown in Fig. 4. A group of n_a binary input bits is encoded by a trellis encoder. The n_c binary output bits of the encoder are grouped into k^* m_i-tuples and each m_i-tuple is mapped to a signal point from a 2^{m_i}-ary signal set. The parameter k^* is called the multiplicity of the code and represents the number of coded symbols assigned to each branch in the trellis diagram. The throughput of such a scheme is n_a/k^* which, in general, may not be an integer-valued number unless n_a is chosen as an integer multiple of the multiplicity k^*. With such a choice the MTCM scheme can be compared to a suitable uncoded scheme, in as far as throughput is concerned.

We shall focus our attention on one such special case, where $n_a = mk^*$ and $n_c = (m+1)k^*$. In this case mk^* input bits are encoded to $(m+1)k^*$ bits and k^* groups of $(m+1)$ bits are mapped to signals of a signal set with cardinality 2^{m+1}. The throughput of such a scheme is m. It can be observed that the MTCM schemes, in spite of parallel transitions in their trellis, can provide effective lengths of more than one. The effective length of the MTCM schemes can be improved by increasing the multiplicity and the number of states, provided appropriate signal allocation is done in the code design procedure. Like conventional TCM schemes, the main goal in designing an MTCM scheme for fading channels is to maximize the effective length of the code, L, and the minimum product distance $d_p^2(L)$. The set partitioning for the construction of an MTCM code optimized for fading channels is explained in detail in [4].

VI. Code design of a rate 4/6, 8-state, 8PSK MTCM scheme

In the light of the above discussion and the design procedure illustrated in [4], a heuristic code design is presented for a rate 4/6 (i.e. an effective rate 2/3), 8-state, multiplicity 2, 8PSK MTCM. The design procedure consists of selecting a suitable trellis diagram and assigning the signals from a partitioned set, which is the Cartesian product of A = $\{s_0, s_1 \ldots s_7\}$ with itself. The number of possible paths leaving each state is 2^4 = 16. Since there are only 8 states, each transition between states has two parallel paths. Here, the partitioned sets constructed for a rate 4/6, 4-state, 8PSK MTCM [3] can be referred to. One more level of partitioning of these sets, such that the corresponding signal points are placed maximally apart, will lead us to the required signal sets to be associated with each transition between states for a rate 4/6, 8-state MTCM. Thus the corresponding signal points in the parallel paths are placed at a squared Euclidean distance of $4E_s$ each. The sets of 8PSK symbol pairs for these transitions are shown with the corresponding state transition matrix in Fig. 5, where S0, S1 …S7 indicate the 8 states of the trellis and the 8PSK symbols $s_0, s_1 \ldots s_7$ are represented by their indices 0,1 …7 respectively. It can be noted that all the parallel paths have a distinct pair of 8PSK symbols that differ in both positions.

	S0	S1	S2	S3	S4	S5	S6	S7
S0	A	B	C	D	E	F	G	H
S1	I	J	K	L	M	N	O	P
S2	B	A	D	C	F	E	H	G
S3	J	I	L	K	N	M	P	O
S4	E	F	G	H	A	B	C	D
S5	M	N	O	P	I	J	K	L
S6	F	E	H	G	B	A	D	C
S7	N	M	P	O	J	I	K	L

$$A=\begin{pmatrix}0&0\\4&4\end{pmatrix}\ B=\begin{pmatrix}2&2\\6&6\end{pmatrix}\ C=\begin{pmatrix}0&2\\4&6\end{pmatrix}\ D=\begin{pmatrix}2&4\\6&0\end{pmatrix}\ E=\begin{pmatrix}0&4\\4&0\end{pmatrix}\ F=\begin{pmatrix}2&6\\6&2\end{pmatrix}\ G=\begin{pmatrix}0&6\\4&2\end{pmatrix}\ H=\begin{pmatrix}2&0\\6&4\end{pmatrix}$$

$$I=\begin{pmatrix}1&1\\5&5\end{pmatrix}\ J=\begin{pmatrix}3&3\\7&7\end{pmatrix}\ K=\begin{pmatrix}1&3\\5&7\end{pmatrix}\ L=\begin{pmatrix}3&5\\7&1\end{pmatrix}\ M=\begin{pmatrix}1&5\\5&1\end{pmatrix}\ N=\begin{pmatrix}3&7\\7&3\end{pmatrix}\ O=\begin{pmatrix}1&7\\5&3\end{pmatrix}\ P=\begin{pmatrix}3&1\\7&5\end{pmatrix}$$

Fig. 5 The state transition matrix along with the assigned sets of 8PSK symbol pairs for the designed MTCM scheme.

The effective length of this MTCM scheme is 2, which is determined by the parallel transitions. However, parallel transitions are not the only error events having effective length of 2. By inspection, examples of error events with more than one trellis branch, i.e. actual length greater than 2, whose effective lengths are 2 can be found. All the error events with effective length 2 have the squared product distance

$$d^2_p(2) = \delta_2 \cdot \delta_2 = 4E_s \cdot 4E_s = 16E^2_s$$

Thus the minimum squared product distance also would be $d^2_p(L)=16E^2_s$ but the number of error events with L=2 and $d^2_p(L)$, i.e. $\alpha(L,d^2_p(L))$ is observed to be 5. This is likely to have an adverse effect on the performance, even though the $d^2_p(L)$ is significantly larger compared to the conventional trellis code of the previous section.

VII. Performance analysis of MTCM

Using the same expression for union bound, an upper bound on the bit error probability can be obtained by taking into consideration the effect of all possible error events of all lengths. Again, here we limit our calculation to the error events with λ=4 for evaluation of the bit error probability for this MTCM scheme. Fig. 6 shows the performance bound of the rate 4/6, 8-state, multiplicity 2, 8PSK MTCM code constructed above for the case of a Rayleigh fading channel.

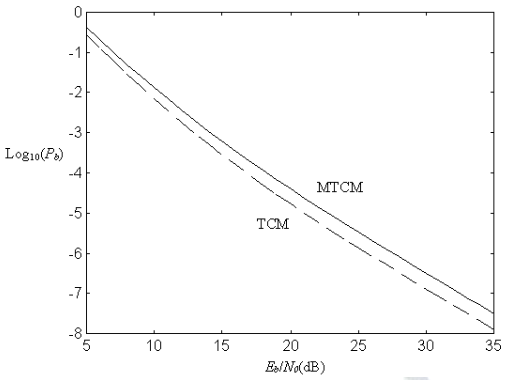

Fig. 6 Performance bound of rate 4/6, 8-state, multiplicity 2, 8PSK MTCM scheme on Rayleigh fading channel with CSI.

VIII. Conclusion

Simon and Divsalar [2] proposed the technique of multiple trellis coded modulation, where in more than one channel symbol per trellis branch is transmitted. The concept was put forth as a means to increase the free Euclidean distance, d^2_{free}. They then proposed the extension of this idea [1, 4] to design codes achieving larger diversities than those achievable with conventional trellis codes of the same effective code rate and number of trellis states with parallel transitions. It can be observed that the MTCM schemes are a generalized form of trellis coded schemes whereby the case of unity multiplicity, i.e. k^*=1, corresponds to the conventional TCM schemes.

On comparison of the two schemes discussed in this paper, in terms of the upper bounds of their performance, both the schemes seem to be almost equally good. There seems to be no improvement in the performance in increasing the multiplicity to 2, as was expected initially. The fact that there are no parallel transitions in the rate 2/3, 8-state TCM scheme designed in section III does not help the MTCM scheme, of the same effective rate (i.e. 4/6) and same number of states (i.e. 8), in increasing the achievable diversity. The most important reason for this is that the effective length of the code remains the same, i.e. 2, in both the cases. Also, on the one hand there is an increase in the minimum squared product distance from $8E^2_s$ to $16E^2_s$, while on the other the number of error events with effective length equal to the effective length of the code and the squared product distance equal to the minimum squared product distance increases from 1 to 5. So it can be expected that the MTCM scheme studied in section V may not offer any advantage over the corresponding conventional scheme discussed in section III. Moreover, as compared to the conventional TCM scheme, a clear-cut disadvantage that stays with the MTCM scheme is its much higher implementation complexity.

Thus we see that for a digital communication system requiring an 8-state TCM scheme with effective rate 2/3, the scheme designed in section III, i.e. the conventional TCM scheme would be more appropriate rather than the rate 4/6 (effectively 2/3), multiplicity 2 MTCM scheme.

XI. References

1. Dariush Divsalar and Marvin K. Simon, "The Design of Trellis Coded MPSK for Fading Channels: Performance Criteria," *IEEE Trans. on Communications*, Vol. 36, No. 9, pp. 1004-1012, Sept. 1988.

2. Dariush Divsalar and Marvin K. Simon, "Multiple Trellis Coded Modulation," *IEEE Trans. on Communications*, Vol. 36, No. 4, pp. 410-419, April 1988.

3. Jamali Hamidreza S. and Tho Le Ngoc, *Coded Modulation Techniques for Fading Channels*, Boston: Kluwer Academic Publishers, 1994.

4. Dariush Divsalar and Marvin K. Simon, *"The Design of Trellis Coded MPSK for Fading Channels: Set Partitioning for Optimum Code Design," IEEE Trans. on Communications*, Vol. 36, No. 9, pp. 1013-1021, Sept. 1988.

5. Ezio Biglieri, Dariush Divsalar, Peter J. Mclane and Marvin K. Simon, *Introduction to Trellis-Coded Modulation With Applications*, Maxwell Macmillan International Editions, 1991.

6. Ralf Mehlan and Heinrich Meyr, "Combined Equalization/Decoding of Trellis Coded Modulation on Frequency-Selective Channels," *Coded Modulation and Bandwidth Efficient Transmission: Proceedings of the Fifth Tirrenia International Workshop on Digital Communications-Tirrenia-Italy*, Elsevier, 1992.

7. J. Du and B. Vucetic, “New M-PSK Trellis Codes for fading channels,” *Electronics Letters*, Vol. 26, No.16, pp. 1267-1269, Aug. 1990.

8. Stephen C. Cook, "*Advances in High-Speed HF Radio Modem Designs*," Presented at the HF95 Conference, Faro, Sweden, 1995.

9. Xiaodong Wang, H. Vincent Poor “*Wireless Communication Systems: Advanced Techniques for Signal Reception*” Prentice Hall PTR 2003