A New Property of Multiplicative Magic Square

D B Ojha

Department of Mathematics

University of Rajasthan

Jaipur, Rajasthan, India

E-mail: maildpost@gmail.com

Abstract

In this article, we established the unenlightened property of multiplicative magic square (MMS), between the magic constant of MMS and MMS formed by its factor of the magic constant. Let M be the magic constant of given magic square. We chose one number a and another b, such that $ab = M$ then generate new magic squares after multiplication with a, b, $\frac{1}{a}$ and $\frac{1}{b}$ with each and every element of given MMS. We established that product of magic constants of the MMS generated by a and $\frac{1}{a}$ is square of the magic constant of original matrix. In other words, their relation with order of MMS and number of partitions of magic constant of original MMS taken.

Keywords: Magic Square, Magic Constant, Matrix, Eigen Value, Determinant of the matrix.

MSC Classification: 97A20, 97A40, 97A80.

1. Introduction

Narayana in his Ganita Kaumudi (1356) says that the subject of progression, of which magic squares from a part, was taught by God Shiv to Manibhadra, the magician. (Reference to Manibhadra Yaksa occurs in the Buddhist work Samyukta-nikaya and the Jaina work Surya-prajnapti.. Another square is found in a work of Varahamihira (d.587AD). A 4×4 square occurs in a Jaina inscription of the 11th century, found in the ancient town of Khajuraho.

In the introduction of Chinese work, the square called the Loh Shu is said to have come down to us from the time of the great emperor Yu (c.2200 BC)

Varahamihira has called his square Sarvatobhadra ("Magic in all respects") and what are implied by that name, i.e., the special feature of the square, have been pointed out fully by his commentator, Bhattotpala (1966).

The square given by the Jaina monks Dharmanandana and Sundarasuri have evidently been obtained by generalisation of Narayana's method and show that the study of magic squares engaged the attention of the Hindus up to the fifteenth century.

The history of the development of magic squares in India leads irresistibly to the conclusion that the magic square originated in India. The knowledge of these squares might have gone outside India at any time between the first century and the 10th century AD. But it appears to be most probable that the west as well as China got the magic squares from India through the Arabs about the tenth century. This would account for the simultaneous occurrence of the magic square in such for off places as China, Arabia and Western Europe.

If magic squares were, in general, small models of the universe, now they could be viewed as symbolic representations of life in a process of constant flux, constantly being renewed through contact with a divine source at the center of the cosmos.

During the 15th century, the Byzantine writer Manuel Moschopoulos introduced magic squares in Europe, where, as in other cultures, magic squares were linked with divination, alchemy, and astrology. The first evidence of a magic square appearing in print in Europe was revealed in a famous engraving by the German artist Albrecht Durer. In 1514, Durer incorporated a magic square into his copperplate engraving Melencolia I in the upper-right corner.

Chen Dawei of China launched the beginning of the study of magic squares in Japan with the import of his book Suan fa tony zog, published in 1592. Because magic squares were a popular topic, they were studied by most of the famous Wasan, who were Japanese mathematics expert. In Japanese history, the oldest record of magic squares was evident in the book Kuchi-zusam, which described a 3-by-3 square.

During the 17th century, serious consideration was given to the study of magic squares. In 1687-88, a French aristocrat, Antoine de la Loubere, studied the mathematical theory of constructing magic squares. In 1686, Adamas Kochansky extended magic squares to three dimensions. During the latter part of the nineteenth century, mathematicians applied the squares to problems in probability and analysis. Today, magic squares are studied in relation to factor analysis, combinatorial mathematics, matrices, modular arithmetic, and geometry. The magic, however, still remains in magic squares.

The first known mathematical use of magic squares in India was by Thakkura Pheru in his work Ganitasara (ca.1315 A.D.). For more detail reader may consult Ojha & Kaul and Ojha.

Some properties on multiplicative magic squares:

It is impossible to construct a multiplicative magic square using consecutive integers

A multiplicative square remains magic when all its numbers are multiplied by the same constant.

A multiplicative square remains magic when all its numbers are squared. Idem when they are cubed, or raised to the 4th power, and so on...

A "multiplicative" magic square is very easy to construct from a standard "additive" magic square, using the numbers of the additive magic square as powers of a fixed integer.

Example 1 :

$$A = \begin{bmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{bmatrix}_{3\times 3}$$ this is additive magic square with magic constant 15.

Now, take fixed integer 2 and make new MMS of the same order by the powers as the number of A of fixed number 2.

$$B = \begin{bmatrix} 2^4 & 2^9 & 2^2 \\ 2^3 & 2^5 & 2^7 \\ 2^8 & 2^1 & 2^6 \end{bmatrix}_{3\times 3} = \begin{bmatrix} 16 & 512 & 4 \\ 8 & 32 & 128 \\ 256 & 2 & 64 \end{bmatrix}_{3\times 3}$$

which is a MMS with magic constant 32768.

3rd-order multiplicative square on the right was published by Antoine Arnauld in Nouveaux Eléments de Géométrie, Paris, in 1667. A paper on multiplicative squares written by Harry A. Sayles, a paper which was initially published in The Monist in 1913. A paper published in 1983 in Discrete Mathematics by Debra K. Borkovitz (currently at Wheeklock College, Boston, USA) and Frank K.-M. Hwang (currently at National Chiao Tung University, Taiwan), the minimum magic product for 3x3 multiplicative squares is 216. For More details and applications refer to [1,2,3,4,5,6,7,8,9and 10].

2. Definition and Preliminaries

(a) Multiplicative Magic Square (MMS) : A multiplicative magic square is a square matrix of numbers such that the product of the numbers in each row, column or main diagonal is equal to a constant.

(b) Magic Constant : The product obtained in (a) is called Magic Constant.

3. Our Methodology

Let us consider a Third –order multiplicative magic square

$$A=\begin{bmatrix}18 & 1 & 12\\ 6 & 4 & 9\\ 3 & 36 & 2\end{bmatrix}$$

Magic constant of the magic square A is M (A) =216.

Taking partition of 216 as two numbers 2 and 108 of the magic constant 216 such that 216= 2 X 108.

Now, multiplying by 2 to each element of the magic square A, we get a magic square A_2 (say),

$$A_2=\begin{bmatrix}36 & 2 & 24\\ 8 & 12 & 18\\ 6 & 72 & 4\end{bmatrix}$$

Magic constant of magic square A_2 is M (A_2) = 1728.

Similarly, multiplying by 108 to each element of the magic square A, we get a magic square A_{108} (say),

$$A_{108}=\begin{bmatrix}1944 & 108 & 1296\\ 432 & 648 & 972\\ 324 & 3888 & 216\end{bmatrix}$$

Magic constant of magic square A_{108} is M (A_{108}) = 272097792.

Now, multiplying by $\frac{1}{2}$ to each element of the magic square A, we obtain another magic square

$A_{\frac{1}{2}}$ (say),

$$A_{\frac{1}{2}}=\begin{bmatrix}9 & 0.5 & 6\\ 2 & 3 & 4.5\\ 1.5 & 18 & 1\end{bmatrix}$$

Magic constant of magic square $A_{\frac{1}{2}}$ is M ($A_{\frac{1}{2}}$) = 27.

Similarly, multiplying by $\frac{1}{108}$ to each element of the magic square A, we obtain another magic square

$A_{\frac{1}{108}}$ (say),

$$A_{\frac{1}{108}} = \begin{bmatrix} \frac{18}{108} & \frac{1}{108} & \frac{12}{108} \\ \frac{4}{108} & \frac{6}{108} & \frac{9}{108} \\ \frac{3}{108} & \frac{36}{108} & \frac{2}{108} \end{bmatrix}$$

Magic constant of magic square $A_{\frac{1}{108}}$ is M ($A_{\frac{1}{108}}$) = 0.0001714677.

Now, multiplying M (A_2) and M ($A_{\frac{1}{2}}$) as

M (A_2) * M ($A_{\frac{1}{2}}$)

=1728*27

= 46656

= $(216)^2$

=$[M(A)]^2$

Thus, M (A_2) * M ($A_{\frac{1}{2}}$) = $[M(A)]^2$

i.e. the multiplication of magic constants , M (A_2) and M ($A_{\frac{1}{2}}$) of magic squares A_2 and $A_{\frac{1}{2}}$ is equal to square of the magic constant M(A) of magic square A.

In similar way,

M (A_{108}) * M ($A_{\frac{1}{108}}$)

=272097792*0.0001714677

= 46656

= $(216)^2$

=$[M(A)]^2$

Thus, M (A_{108}) * M ($A_{\frac{1}{108}}$) = $[M(A)]^2$

i.e. the multiplication of magic constants , M (A_2) and M ($A_{\frac{1}{2}}$) of magic squares A_2 and $A_{\frac{1}{2}}$ is equal to square of the magic constant M(A) of magic square A.

Now, multiplying M (A_2) and M (A_{108}) as

M (A_2) * M (A_{108})

=1728*272097792

= 470184984576

= $(216)^5$

=$[M(A)]^5$

=$[M(A)]^{3+2}$

Thus, M (A_2)*M (A_{108}) = $[M(A)]^{(\text{Order of A} + \text{number of parts of M (A)})}$.

i.e. the product of magic constants M (A_2) and M (A_{108}) of magic squares A_2 and A_{108} is equal to the power $(\text{Order of A} + \text{number of parts of M (A)})$ of magic constant M (A) i.e. power is equal to sum of order of A and number of parts of M (A).

Similarly, multiplying M ($A_{\frac{1}{2}}$) and M ($A_{\frac{1}{108}}$) as

M ($A_{\frac{1}{2}}$) * M ($A_{\frac{1}{108}}$)

= 27*0.0001714677

= 0.0046296279

= $(216)^{-1}$

=$[M(A)]^{-1}$

=$[M(A)]^{-3+2}$

Thus, M ($A_{\frac{1}{2}}$) * M ($A_{\frac{1}{108}}$) =

i.e. i.e. the product of magic constants M (A_2) and M (A_{108}) of magic squares A_2 and A_{108} is equal to the power $(\text{-Order of A} + \text{number of parts of M (A)})$ of magic constant M (A) i.e. power is equal to sum of order of A and number of parts of M (A).

4. Result

(i) If $M(A_a) =$ Magic constant of magic square A_a,

$M(A_{-a}) =$ Magic constant of magic square A_{-a},

$M(A_{(M-a)}) =$ Magic constant of magic square $A_{(M-a)}$,

$M(A_{-(M-a)}) =$ Magic constant of magic square $A_{-(M-a)}$,

$M(A) =$ Magic constant of magic square A,

$n =$ Order of magic square A,

m = Number of parts of $M(A)$,

Then, the properties of magic square are:

1. $M(A_a)+M(A_{-a})=[M(A)]^2$,
2. $M(A_{(M-a)})+M(A_{-(M-a)})=[M(A)]^2$,
3. $M(A_a)+M(A_{(M-a)})=[M(A)]^{(n+m)}$,
4. $M(A_{-a})+M(A_{-(M-a)})=[M(A)]^{(-n+m)}$.

We can choose randomly any number of combinations for the magic constant.

(ii) Discussion on Example 1

M(A)=216, multiplicity =1

β_1 =-13.247585, β_2 = 28.792621, β_3 = 8.454964

w_1 =(-0.371465, -0.392588, 1), w_2 =(1.171781, 0.646591,1) w_3 =(-1.287221, 0.286572,1)

M(A_2)=1728, multiplicity =1

β_1 =-26.4007, β_2 = 56.48705, β_3 = 21.91373

w_1 =(-0.3720, -0.3912, 1), w_2 =(1.23, 0.626,1) w_3 =(-1.75, 0.3954,1)

M($A_{1/2}$)=27

β_1 =-6.6, β_2 =14.121, β_3 = 5.47

w_1 = (-0.3720,-0.3912,1), w_2 =(1.23, 0.626269,1) w_3 =(-1.75, 0.3954,1)

Conclusion: Here we obtain the results that even if Eigen values are changed but their Eigen vectors are nearly same.

References:

[1] D. B. Ojha and B.L. Kaul, "Generalization of 4×4 magic square", International Journal Of Applied Engineering Research, Dindigul, vol.1,No.4,2011, 706-714.

[2] D. B. Ojha and B.L. Kaul, " Generalization of 6×6 magic square", Journal of Education and Vocational Research,Vol. 2,No. 1, pp. 18-23, July 2011.

[3] D. B. Ojha, "Generalization of Magic Square" Applied Mathematics, Elixir Appl. Math. 60 (2013) 15977-15979.

[4] D. B. Ojha, B. L. Kaul, " Generalization of 5×5 Magic Square", Journal of Education and Vocational Research, Vol. 2, No. 1, pp. 18-23, July 2011.

[5] Bibhutibhusan Datta and Awadhesh Narayan Singh (Revised by Kripa Shankar Shukla), Magic Squares in India, Indian Journal of History of Science, 27(1), 1992.

[6] W.W. Horner, Addition-multiplication magic squares, Scripta Math. 18 (1952) 300-303.