Soft Multi b-Compact Spaces

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Abstract : In this paper, we introduce the concept of soft multi b-compactness and study some of its basic properties. We investigate behaviour relative to union, intersection and soft multi b-compact subspaces of soft multi b-compact space. Also we characterize the properties of soft multi generalized b-compact spaces.

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I. INTRODUCTION

Concept of compactness is one of the basic and vital concepts of paramount interest for topologists. In 2012, Zorlutuna et al [13] first studied the compactness for soft topological spaces. In 2014, Akdag and Ozkan [1] introduced and studied the concept of soft b-open sets and soft b-continuous functions. In 2016, Akdag Ozkan et al [3] described soft b-compact spaces in soft multi topological spaces.

The aim of this paper is to mention soft multi b-open and soft multi b-closed, soft multi b-compact, soft multi generalized b-compact spaces using the soft multi finite intersection property. We also give equivalent conditions for a soft multi b-compact space. We say that a soft multi b-compact soft multi generalized topological space gives a parametrized family of b-compact generalized topological spaces in the initial universe.

II. PRELIMINARIES

Definition 2.1 [10] Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U and A be a nonempty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A $\rightarrow$ P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in$ A, F(ε) may be considered as the set of ε - approximate elements of the soft set (F,A).

Definition 2.2 [2] Let { U_i: i $\in$ I} be a collection of universes and such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i} : i\in I\}$ be a collection of sets of parameters. Let U = $\prod_{i\in I} P(U_i)$ where P(U_i) denotes the power set of U_i, E = $\prod_{i\in I} E_{U_i}$ and A $\subseteq$ E. A pair (F, A) is called a soft multiset over U (briefly SMS(U, E)), where F is a mapping given by F : A $\rightarrow$ U.

Definition 2.3 [2] A soft multi set (F, A) $\in$ SM S(U, E) is called a soft multi point in (U, E), denoted by $e_{(F,A)}$, if for the element e $\in$ A, F (e) $\neq \emptyset$ and $\forall e' \in$ A $-$ {e} , F (e$'$) $= \emptyset$.

Definition 2.4 [2] A sub family τ of SMS(U, E) is called soft multi topology on (U, E), if the following axioms are satisfied:

   (O$_1$). $\widetilde{\emptyset}$, $\widetilde{E} \in \tau$,

   (O$_2$). the union of any number of soft multisets in τ belongs to τ, ie.,
for any {(F$_k$, A$_k$) : k $\in$ K} $\subseteq \tau \Rightarrow \widetilde{U}_{k\in k}$ (F$_k$, A$_k$) $\in \tau$,

   (O$_3$). If (F, A), (G, B) $\in \tau$, then (F, A)$\widetilde{\cap}$ (G,B) $\in \tau$.

Example 2.5 Let us consider there are three universes U$_1$, U$_2$ and U$_3$. Let
U$_1$ = {h$_1$, h$_2$, h$_3$, h$_4$}, U$_2$ = {c$_1$, c$_2$, c$_3$} and U$_3$ = {v$_1$, v$_2$}. Let { E_{U_1}, E_{U_2}, E_{U_3} } be a collection of sets of decision parameters related to the above universes,

where

E_{U_1} = { $e_{u_1,1}$ = expensive, $e_{u_1,2}$ = cheap, $e_{u_1,3}$ = wooden, $e_{u_1,4}$ = in green surroundings},

E_{U_2} = { $e_{u_2,1}$ = expensive, $e_{u_2,2}$ = cheap, $e_{u_2,3}$ = sporty},

E_{U_3} = { $e_{u_3,1}$ = expensive, $e_{u_3,2}$ = cheap},

Let U = $\prod_{i=1}^{3} P(U_i)$, E = $\prod_{i=1}^{3} E_{U_i}$, and

A_1 = {e_1 = $(e_{u_1,1},\ e_{u_2,1},\ e_{u_3,1})$, e_2 = $(e_{u_1,1},\ e_{u_2,2},\ e_{u_3,1})$}

A_2 = {e_1 = $((e_{u_1,1},\ e_{u_2,1},\ e_{u_3,1})$, e_3 = $(e_{u_1,2},\ e_{u_2,3},\ e_{u_3,1})$}

Suppose that

(F_1, A_1) = {(e_1, ({h_1, h_2 }, {c_1, c_2 }, {v_1 })), (e_2, ({h_3, h_4 }, {c_1, c_3 }, {v_2 }))},

(F_2, A_2) = {(e_1 , ({h_1, h_3 }, {c_2, c_3 }, {v_1, v_2 })), (e_3, ({h_2, h_4 }, {c_1, c_2 }, {v_2 }))},

(F_3, A_3) = (F_1, A_1) $\widetilde{\cup}$ (F_2, A_2)

= {(e_1 , ({h_1, h_2, h_3}, {c_1, c_2, c_3}, {v_1, v_2})), (e_2, ({h_3, h_4)}, {c_1, c_3}, {v_2 })) ,

   (e_3, ({h_2, h_4 }, {c_1, c_2}, {v_2 }))},

(F_4, A_4) = (F_1, A_1) $\widetilde{\cap}$ (F_2, A_2)

= {(e_1 , ({h_1 }, {c_2}, {v_1 })), (e_2 , ({h_3, h_4 }, {c_1, c_3 }, {v_2 })) ,

   (e_3, ({h_2, h_4}, {c_1, c_2 }, {v_2}))},

where $A_3 = A_4 = A_1 \cup A_2$ = { e_1 = $(e_{u_1,1},\ e_{u_2,1},\ e_{u_3,1})$, e_2 = $(e_{u_1,1},\ e_{u_2,2},\ e_{u_3,1})$,

e_3 = $(e_{u_1,2},\ e_{u_2,3},\ e_{u_3,1})$}

Then we observe that the subfamily

τ_1 = { $\widetilde{\emptyset}$, $\widetilde{E}$, (F_1, A_1), (F_2, A_2), (F_3, A_3), (F_4, A_4)}

of SMS(U,E) is a soft multi topology on (U, E), since it satisfies the necessary three axioms (O_1), (O_2) and (O_3) and ((U, E), τ_1) is a soft multi topological space.

Definition 2.6 [2] As every soft multi topology on (U, E) must contain the sets $\widetilde{\emptyset}$ and $\widetilde{E}$, so the family I ={$\widetilde{\emptyset}$, $\widetilde{E}$} forms a soft multi topology on (U, E). This soft multi topology is called indiscrete soft multi topology and the pair ((U, E), I) is called an indiscrete soft multi topological space.

Definition 2.10 [2] Let τ be the soft multi topology on (U, E). A soft multiset (F, A) in SMS(U,E) is a neighbourhood of a soft multi set (G,B) if and only if there exists an τ -open soft multi set (H,C) i.e. (H, C) $\in$ τ such that (G, B) $\subseteq$ (H, C) $\subseteq$ (F, A).

Definition 2.11 [2] Let ((U, E), τ) be a soft multi topological space on (U, E) and (F, A) be a soft multiset in SMS (U, E). Then the union of all soft multi open sets contained in (F, A) is called the interior of (F, A) and is denoted by sm –Int(F, A) and defined by

   sm –Int(F, A) = $\widetilde{\cup}$ {(G, B) : (G, B) is a soft multi open set contained in (F, A)}.

Definition 2.12 [2] Let ((U, E), τ) be a soft multi topological space on (U,E) and (F, A) be a soft multiset in SMS(U,E). Then the intersection of all soft multi closed set containing (F, A) is called the closure of (F, A) and is denoted by sm –Cl(F, A) and defined by

   sm –Cl(F, A) = $\widetilde{\cap}$ {(G, B) : (G, B) is a soft multi closed set containing (F, A)}

Observe first that sm –Cl(F, A) is a soft multi closed set, since it is the intersection of soft multi closed sets. Furthermore, sm –Cl(F, A) is the smallest soft multi closed set containing (F, A).

Definition 2.13 [2] Let ((U, E), τ) be a soft multi topological space on (U, E) and (F, A) be asn soft multi set in SMS(U, E). Then the soft multi topology

   $\tau_{(F,A)}$ = {(F, A) $\widetilde{\cap}$ (G, B) : (G, B) $\in$ τ } is called soft multi subspace topology and ((F, A), $\tau_{(F,A)}$) is called soft multi topological subspace of ((U, E), τ).

III. Soft Multi b-Open and Soft Multi b-Closed Sets

   In this section we introduce soft multi b-open sets in soft multi topological spaces and study some of

their properties.

Definition 3.1 A soft multi set (F,A) in a soft multi topological space (U,E) is called
  (i) soft multi b-open set (briefly smb-open) iff (F,A) $\subseteq$ sm$-$Int(sm$-$Cl(F,A))$\cap$ sm$-$Cl(sm$-$Int(F,A)).
  (ii) soft multi b-closed set (briefly smb-closed) iff (F,A) $\supseteq$ sm$-$Int(sm$-$Cl(F,A))$\cap$ sm$-$Cl(sm$-$Int(F,A)).

Theorem 3.2 If (F,A) be a soft multi set in a soft multi topological space (U,E), then
  (i) (F,A) is a soft multi b-open set iff (F,A)^c is a soft multi b-closed set.
  (ii) (F,A) is a soft multi b-closed set iff (F,A)^c is a soft multi b-open set.

Proof: The proof is obvious from the definition 3.1.

Definition 3.3 Let ((U,E),τ) be a soft multi topological space and (F,A) be a soft multi set over (U,E),
  (i) soft multi b-closure (briefly smb-Cl(F,A)) of a soft multi set (F,A) in (U,E) is defined by smb$-$Cl(F,A) = $\cup$\{(G,A) $\supseteq$ (F,A) : (G,A)is a soft multi b-closed set of (U,E)\}.
  (ii) soft multi b-interior (briefly smb-Int(F,A)) of a soft multi set (F,A) is defined by smb$-$Int(F,A) = $\cap$\{(H,A) $\subseteq$ (F,A) : (H,A)is a soft multi b-open set of (U,E)\}.

Clearly smb-Cl(F,A) is the smallest soft multi b-closed set over (U,E) which contains (F,A) and smb-Int(F,A) is the largest soft multi b-open set over (U,E) which is contained in (F,A).

Theorem 3.4 If (F,A) be any soft multi set in a soft multi topological space (U,E), then
  (i) smb$-$Cl((F,A)^c) = (U,E)\smb$-$Int(F,A)
  (ii) smb$-$Int((F,A)^c) = (U,E)\smb$-$Cl(F,A)

Proof: (i)Let smb-open set (H,A) $\subseteq$ (F,A) and smb-closed set (G,A) $\supseteq$ (F,A)^c. Then, smb$-$Int(F,A) = $\cup$\{(G,A)^c : (F,A) is a soft multi b-closed set and(G,A) $\supseteq$ (F,A)^c\}
    = (U,E)\$\cup$\{(G,A) : (G,A) is a smb-closed set and(G,A) $\supseteq$ (F,A)^c\}
    = (U,E)\smb$-$Cl((F,A)^c).
 Therefore smb$-$Cl(F,A)^c = (U,E)\smb$-$Int(F,A).
(ii) Let (G,A) be smb-open set. Then for a smb-closed set (G,A) $\supseteq$ (F,A), (G,A) $\subseteq$ (F,A)^c smb$-$Cl(F,A) = $\cap$\{(G,A)^c : (F,A) is a soft multi b-open set and(G,A) $\subseteq$ (F,A)^c\}
    =(U,E)\$\cup$\{(G,A) : (G,A) is a smb-open set and(G,A) $\subseteq$ (F,A)^c\}
    = (U,E)\smb$-$Int((F,A)^c).
Therefore smb$-$Int(F,A)^c = (U,E)\smb$-$Cl(F,A). Hence the proof.

Theorem 3.5 A soft multi topological space (U,E) is soft multi b-closed iff for every soft multi finite intersection property Φ in (U,E), $\cap_{(G,B)\in\Phi}$ smb$-$Cl(G,B) = $\emptyset$.

Proof: Let \{(F,E)$_i$: i $\in$ I\} be a soft multi b-open cover of (U,E) and let for every finite collection of \{(F,E)$_i$: i $\in$ I\}, $\cap_{i\in I_0}(F, A_i)$ $\subseteq$ (U,E) for some i $\in$ I$_0$. Then $\cap_{i\in I_0}(G, B)^c_i \supseteq \emptyset$ for some i $\in$ I$_0$. Then \{smb$-$Cl(F,A$_i$)^c : i $\in$ I\} = Φ forms a soft multi b-open finite intersection property in (U,E). For \{(F,E)$_i$: i $\in$ I\} is a soft multi b-open set cover of (U,E), then $\cap_{i\in I_0}(F, A_i)$ = $\emptyset$ which implies $\cap_{i\in I}$smb$-$Cl(smb$-$Cl(G,B))^c = $\emptyset$ which is a contradiction. Then every soft multi b-open \{(F,E)$_i$: i $\in$ I\} of (U,E) has a finite subfamily I$_0$ such that $\cap_{i\in I_0}$ smb $-$ Cl (F,A$_i$) = (U,E) for every i $\in$ I$_0$. Hence (U,E) is soft multi b-closed space.
        Conversely, assume that there exists a soft multi b-open finite intersection property Φ in (U,E) such that$\cap_{(G,B)\in\Phi}$smb$-$Cl(G,B) = $\emptyset$. This implies$\cup_{(G,B)\in\Phi}$(smb$-$ Cl(G,B))^c = (U,E) for every i $\in$ I and hence \{(F,E)$_i$: i $\in$ I\} = \{smb$-$Cl(G,B) : (G,B) $\in$ Φ\} is a soft multi b-open cover of (U,E). For (U,E) is soft multi b-closed,

by definition $\{(F,E)_i : i \in I\}$ has a finite subfamily I_0 such that $\bigcap_{i \in I_0} smb - Cl(G,B))^c = (U,E)$ for every $i \in I_0$ and hence $\bigcap_{i \in I_0} smb - Cl(smb-Cl(G,B))^c)^c = \emptyset$. Thus $\bigcap_{i \in I_0}(G,B) = \emptyset$ is a contradiction. Therefore $\bigcap_{(G,B) \in \Phi} smb-Cl(G,B) = \emptyset$.

Theorem 3.6 If f : $(U,E) \to (V,E)$ be a soft multi b-irresolute surjection. If (U,E) is soft multi b-closed space, then (V,E) is soft multi b-closed space.

Proof: Let $\{(F,E)_i : i \in I\}$ be a soft multi b-open cover of (V,E). Since f is soft multi b-irresolute, $f^{-1}(F,E)_i : i \in I$ is soft multi b-open cover of (U,E). By hypothesis, there exists a finite subset I_0 of Φ such that, $\bigcup_{i \in I_0} smb - Cl(f^{-1}(F,E)_i) = (U,E)$. For f is surjective and by theorem, $(V,E) = f(U,E) = f(\bigcup_{i \in I_0} smb - Cl(F^{-1}(F,E)_i)) \subseteq \bigcup_{i \in I_0} smb - Cl(f(f^{-1}(F,E)_i)) = \bigcup_{i \in I_0}(smb - Cl(F,E)_i)$. Hence (V,E) is soft multi b-closed space.

Definition 3.7 A soft multi mapping f : $(U,E) \to (V,E)$ is said to be
   (i)   Soft multi b-continuous (briefly smb-continuous) if the inverse image of each soft multi b-open set of (V,E) is a soft multi b-open set in (U,E).
   (ii)   Soft multi b-irresolute (briefly smb-irresolute) if the inverse image of each soft multi b-open set of (V,E) is a soft multi b-open set in (U,E).
   (iii) Soft multi b∗-open if the image of each soft multi b-open set of (U,E) is soft multi b-open set in (V,E).

Definition 3.8 A soft multi topological space $((U,E),\tau)$ is soft multi compact space if each soft multi open cover of (U,E) has a finite sub cover.

IV. Soft Multi b-Compact Spaces

The most important of all covering properties is compactness. In this section we introduce the concept of soft multi b-compactness and study some of its basic properties. We now consider a soft multi b-compact space constructed around a soft multi topology.

Definition 4.1 A collection $\{(G,A)_i : i \in I\}$ of soft multi b-open sets in a soft multi topological space $((U,E),\tau)$ is called a soft multi b-open cover of a soft multi set (F,A) if $(F,A) \subseteq \bigcup\{(G,A)_i : i \in I\}$ holds. If $(F,E) = (U,E)$, then the collection $\{(G,A)_i : i \in I\}$ is said to be soft multi b-open covering of $((U,E),\tau)$. A finite subfamily of a soft multi b-open cover $\{(G,A)_i : i \in I\}$ of (U,E) is called a finite sub cover of $\{(G,A)_i : i \in I\}$, if it is also a soft multi b-open cover of (U,E).

Definition 4.2 A soft multi topological space $((U,E),\tau)$ is called a soft multi b compact space if every soft multi b-open cover of (U,E) has a finite sub cover.

Definition 4.3 A soft multi subset (F,A) of a soft multi topological space $((U,E),\tau)$ is called a soft multi b-compact in (U,E) provided for every collection $\{(G,A)_i, i \in I\}$ of soft multi b-open sets of (U,E) such that $(F,A) \subseteq \bigcup\{(G,A)_i : i \in I\}$ there exists a finite subset I_0 of I such that $(F,A) \subseteq \bigcup\{(G,A)_i : i \in I_0\}$.

Definition 4.4 A soft multi topological space $((U,E),\tau)$ is called soft multi b-space if every soft multi b-open set of (U,E) is soft multi open set in (U,E).

The following three results immediately from the above definitions.

Result 4.5 If $((U,E),\tau)$ is a soft multi b-compact space and soft multi b-space, then (U,E) is soft multi compact space.

Proof: Let $\{(F,A)_i : i \in I\}$ be a soft multi open cover of (U,E). since any soft multi open set is soft multi b-open set, $\{(F,A)_i : i \in I\}$ is a soft multi b-open cover of (U,E). Since (U,E) is soft multi b-compact space and soft multi b-space, there exists a finite subset I_0 of I such that $(U,E) \subseteq \cup \{(F,A)_i : i \in I_0\}$. Hence (U,E) is soft multi compact space. Hence the proof.

Result 4.6 If $f : (U,E) \to (V,E)$ is a soft multi b-continuous function and soft multi b-space, then f is soft multi continuous function.

Proof: Let $\{(G,A)_i : i \in I\}$ be a soft multi open set of (V,E). For f is soft multi b-continuous function, $\{f^{-1}((G,A)_i) : i \in I\}$ is a soft multi b-open set of (U,E) and for (U,E) is soft multi b-space, $\{f^{-1}((G,A)_i) : i \in I\}$ forms a soft multi open set of (U,E). Hence f is a soft multi continuous function.

Result 4.7 Let $((U,E),\tau)$ be a soft multi topological space. If $((U,E), \tau_{e_{u_i}})$ is a soft multi b-compact space, for each $e_{u_i} \in E$, then $((U,E),\tau)$ is a soft multi b-compact space.

Proof: Let $E = \{e_{u_1}, e_{u_2},... e_{u_n}\}$ be a parameters set and $((U,E), \tau_{e_{u_i}})$ is soft multi b-compact space, for every i= 1,2,...n. Let $\{(F,A) : i \in I\}$ be a soft multi b-open cover of (U,E). For $\{\cup_{i\in I}(F,A)_i(E_{u_i})\} = (U,E)$, for every $e_{u_i} \in E$, and $((U,E), \tau_{e_{u_i}})$ is soft multi b-compact space, there is a finite subset I_0 of I that $\{\cup_{i\in I_0}(F,A)_i(e_{u_i})\}$ = (U,E). Hence $\{(F,A)_i : i \in I_0\}$ is a finite sub cover of $\{(F,A)_i : i \in I\}$. Hence $((U,E),\tau)$ is soft multi b-compact space.

Theorem 4.8 A soft multi topological space $((U,E),\tau)$ is soft multi b-compact space iff for every family $\{(F,A)_i : i \in I\}$ of soft multi b-closed sets of (U,E) having the finite intersection property, $\{\cap_{i\in I}(F,A)_i\} \neq \emptyset$.

Proof: Let $\{(F,A)_i : i \in I\}$ be a family of soft multi b-closed sets with the finite intersection property. Assume that $\{\cap_{i\in I}(F,A)_i\} = \emptyset$. Then $\{\cup_{i\in I}(F,A)c_i\} = (\tilde{}U,E)$. For $\{(F,A)c_i : i \in I\}$ is a collection of soft multi b-open sets covering (U,E), then from the definition of soft multi b-compactness of (U,E) it follows that there exists a finite subset I0 $\subseteq$ I such that $\{\cup_{i\in I_0}(F,A)_i^c\} = (U,E)$. Then $\{\cap_{i\in I_0}(F,A)_i\} = \emptyset$, which gives a contradiction. Therefore $\{\cap_{i\in I}(F,A)_i\} \neq \emptyset$.

     Conversely, let $\{(F,A)_i : i \in I\}$ be a family of soft multi b-open sets covering (U,E). Suppose that for every finite subset $I_0 \subseteq I$, we have $\{\cup_{i\in I_0}(F,A)_i^c\} = (U,E)$. Therefore $\{\cap_{i\in I_0}(F,A)_i^c\} = \emptyset$. Then by definition we have $\{\cap_{i\in I_0}(F,A)_i\} \neq \emptyset$ which implies $\{\cup_{i\in I_0}(F,A)_i^c\} \neq (\tilde{}U,E)$ and this contradicts that $\{(F,A)_i : i \in I\}$ is a soft multi b-cover of (U,E). Thus (U,E) is soft multi b-compact space.

Theorem 4.9 Let $f : (U,E) \to (V,E)$ be a soft multi b-continuous function. If (U,E) is soft multi b-compact space, then the image of (U,E) under the f is soft multi compact.

Proof: Let $f : (U,E) \to (V,E)$ be a soft multi b-continuous function from a soft multi b-compact soft multi topological space $((U,E),\tau_1)$ to $((V,E),\tau_2)$. Take a soft multi open cover $\{(G,A)_i : i \in I\}$ of (V,E). For f is soft multi b-continuous, $\{f^{-1}((G,A)_i) : i \in I\}$ is a soft multi b-open cover of (U,E) and for (U,E) is soft multi b-compact, there exists a finite subset I_0 of I such that $\{f^{-1}((G,A)_i) : i \in I_0\}$ forms a soft multi b-open cover of (U,E). Thus, $\{((G,A)_i) : i \in I_0\}$ forms a finite soft multi open cover of (V,E).

Theorem 4.10 A soft multi topological space $((U,E),\tau)$ is soft multi b-compact space iff every family Φ of soft multi sets with the finite intersection property, $\cap_{(G,A)\in\Phi} smb-Cl(G,A) \neq \emptyset$.

Proof: Let $((U,E),\tau)$ be soft multi b-compact space and if possible let $\cap_{(G,A)\in\Phi} smb-Cl(G,A) = \emptyset$ for some family Φ of soft multi sets with the finite intersection property. Thus $\cup_{(G,A)\in\Phi}(smb-Cl(G,A))^c = (\tilde{}U,E) \Rightarrow \gamma = \{(smb-Cl(G,A))^c : (G,A) \in \Phi\}$ is a soft multi b-open cover of (U,E). Then by definition of soft multi b-compactness of (U,E), at least one finite sub cover ω of γ. That is $\cup_{(G,A)\in\omega}(smb-Cl(G,A))^c = (\tilde{}U,E) \Rightarrow \cup_{(G,A)\in\omega}(G,A)^c = (\tilde{}U,E) \Rightarrow \cap_{(G,A)\in\omega}(G,A) = \emptyset$ a contradiction. Hence $\cap_{(G,A)\in\Phi} sm-bCl(G,A) \neq \emptyset$.

Conversely, let $\cap_{(G,A)\in\Phi}sm-bCl(G,A)\neq\emptyset$, for every collection Φ of soft multi sets with finite intersection property. Suppose $((U,E),\tau)$ is not soft multi b-compact space. Then at least one collection of soft multi b-open soft multi sets covering (U,E) without a finite sub cover. Hence for every finite subfamily ω of γ we have $\cap_{(G,A)\in\omega}(G,A)\neq(\tilde{}U,E)\Rightarrow\cap_{(G,A)\in\omega}(G,A)^c\neq\emptyset\Rightarrow\{(G,A)^c:(G,A)\in\gamma\}$, is a family of soft multi sets with finite intersection property. Now $\cap_{(G,A)\in\gamma}(G,A)=(\tilde{}U,E)\Rightarrow\cap_{(G,A)\in\gamma}(G,A)^c=\emptyset\Rightarrow\cap_{(G,A)\in\gamma}sm-bCl(G,A)^c=\emptyset$. This is a contradiction. Hence the proof.

Theorem 4.11 Let f be a soft multi b^* continuous function carrying the soft multi b-compact space $((U,E),\tau)$ onto the soft multi topological space $((V,E),\tau^*)$. Then $((V,E),\tau^*)$ is soft multi b-compact space.
Proof: Let $\{(G,A)_i:i\in I\}$ be a soft multi b-open cover of (V,E). Then $\{f^{-1}((G,A)_i):i\in I\}$ is a cover of (U,E). For f is soft multi b-irresolute, $f^{-1}((G,A)_i)$ is soft multi b-open set, and hence $\{f^{-1}((G,A)_i):i\in I\}$ is a soft multi b-open cover of (U,E). Since (U,E) is soft multi b-compact space, there exists a finite subset $I_0\subseteq I$ such that $(U,E)\subseteq\cup\{f^{-1}((G,A)_i):i\in I_0\}$. Thus $f(U,E)\subseteq f(\cup\{f^{-1}((G,A)_i)':i\in I_0\})=\cup\{f(f^{-1}((G,A)_i)):i\in I_0\}=\cup\{(G,A)_i:i\in I_0\}$. Since f is surjective, $(V,E)=f(U,E)\subseteq\cup\{(G,A)_i:i\in I_0\}$. Hence (V, E) is soft multi b-compact space.

Theorem 4.12 Let (F,A) be a soft multi b-closed subset of a soft multi b-compact space (U,E). Then (F,A) is also soft multi b-compact in (U,E).
Proof: Let (F,A) be any soft multi b-closed subset of (U,E) and $\{(G,A)_i:i\in I\}$ be a soft multi b-open cover of (U,E). For $(F,A)^c$ is soft multi b-open, $\{(G,A)_i:i\in I\}\cup(F,A)^c$ is a soft multi b-open cover of (U,E). For (U,E) is soft multi b-compact space, there exists a finite subset $I_0\subseteq I$ such that $(U,E)\subseteq\{(G,A)_i:i\in I_0\}\cup(F,A)^c$. But (F,A) and $(F,A)^c$ are disjoint, hence $(F,A)\subseteq\{(G,A)_i:i\in I_0\}$. Therefore (F,A) is soft multi b-compact in (U,E).

Theorem 4.13 A soft multi b-closed subspace of a soft multi b-compact space is soft multi b-compact.
Proof: Let (V,E) be a soft multi b-closed subspace of a soft multi b-compact space $((U,E),\tau)$ and $\{(G,A)_i:i\in I\}$ be a soft multi b-open cover of (V,E). For each $(G,A)_i$, at least one soft multi b-open soft multi set (G,A) of (U,E) such that $(G,A)=(G,E)\cap(V,E)$. Then the family $\{(G,A)_i:i\in I\}\cup((U,E)\backslash(V,E))$ is a soft multi b-open cover of (U,E), which has a finite sub cover. So $\{(G,A)_i:i\in I\}$ has a finite subfamily to cover (V,E). Hence (V,E) is soft multi b-compact space.

Theorem 4.14 Let (F,A) and (G,A) be soft multi subsets of a soft multi topological space $((U,E),\tau)$ such that (F,A) is soft multi b-compact in (U,E) and (G,A) is soft multi b-closed set in (U,E). Then $(F,A)\cap(G,A)$ is soft multi b-compact in (U,E).

Proof: Let $\{(G,A)_i:i\in I\}$ be a cover of $(F,A)\cap(G,A)$ consisting of soft multi b-open subsets of (U,E). Since $(G,A)^c$ is a soft multi b-open set, $\{(G,E)_i:i\in I\}\cup(G,A)^c$ is a soft multi b-open cover of (F,A). Since (F,A) is soft multi b-compact in (U,E), there exists a finite subset $I_0\subseteq I$ such that $(F,A)\cap(G,A)\subseteq\{(G,E)_i:i\in I_0\}\cup(G,A)^c$. Therefore $(F,A)\cap(G,A)\subseteq\{(G,E)_i:i\in I_0\}$. Hence $(F,A)\cap(G,A)$ is soft multi b-compact in (U,E).

Theorem 4.15 Let $f:(U,E)\to(V,E)$ be a soft multi b-continuous function, soft multi b-open and injective mapping. If a soft multi subset (G,E) of (V,E) is soft multi b-compact in (V,E), then $f^{-1}(G,A)$ is soft multi b-compact in (U,E).

Proof: Let $\{(H,B)_i:i\in I\}$ be a soft multi b-open cover of $f^{-1}(G,A)$ in (U,E). Then $f^{-1}(G,A)\subseteq\cup\{(H,B)_i:i\in I\}$ and hence $(G,A)\subseteq\cup f(f^{-1}(G,A))\subseteq f(\cup\{(H,B)_i:i\in I\})=\{f(H,B)_i:i\in I\}$. Since (G,A) is soft multi b-compact in (V,E), there is a finite subset $I_0\subseteq I$ such that $(G,A)\subseteq\cup\{f(H,B)_i:i\in I_0\}$. So $f^{-1}(G,A)\subseteq f^{-1}(\cup\{f((H,B)_i:i\in I_0)\})=\cup\{f^{-1}(f(H,B)i):i\in I\}=\cup\{(H,B)_i:i\in I_0\}$. Hence the proof.

In the following theorem it is shown that image of a soft multi b-compact space under a soft multi b-irresolute mapping is soft multi b-compact.

Theorem 4.16 If a function $f:(U,E)\to(V,E)$ is soft multi b-irresolute and (F,A) is soft multi b-compact

relative to (U,E), then the image f(F,A) is soft multi b-compact in (V,E).

Proof: Let $\{(G,E)_i : i \in I\}$ be a soft multi b-open cover of f(F,A) in (V,E). For f is soft multi b-irresolute function, $\{f^{-1}(G,E)_i : i \in I\}$ is soft multi b-open cover of (F,A) in (U,E). For (F,A) is soft multi b-compact relative to (U,E), there is a finite subset $I_0 \subseteq I$ such that $(F,A) \subseteq \cup\{f^{-1}((G,E)_i) : i \in I_0\}$. Then f(F,A) $\subseteq \cup\{(G,E)_i : i \in I_0\}$. Therefore f(F,A) is soft multi b-compact in (V,E). The pre image of a soft multi b-compact space under soft multi b∗ -open bijective mapping in soft multi b-compact space.

Theorem 4.17 If a function f : (U,E) → (V,E) is soft multi b∗-open bijective mapping and (V,E) be a soft multi b-compact space, then (U,E) is soft multi b compact space.

Proof: Let $\{(F,E)_i : i \in I\}$ be a collection of soft multi b-open covering of (U,E). Let $\{f((F,E)_i) : i \in I\}$ be a soft multi b-open cover of a collection of soft multi b-open sets covering (V,E). For (V,E) is soft multi b-compact space, by definition there exists a finite family $I_0 \subseteq I$ such that $\{f((V,E)_i) : i \in I_0\}$ covers (V,E). Also since f is bijective we have $(U,E) = f^{-1}(V,E) = f^{-1}(f(\cup_{i \in I_0}(F, A)_i) = \cup_{i \in I_0}(F, A)_i$. Then (U,E) is soft multi b-compact space.

V. Soft Multi Generalized b-Compact Space

We shall define the concept of soft multi generalized b-compact spaces.

Definition 5.1 A collection$\{(F,A)_i : i \in I\}$of soft generalized b-open sets in (U,E) is called soft multi generalized b-open cover of a soft multi set (G,B) in (U,E) if $(G,B) \subseteq\cup_{i \in I}(F,A)_i$.

Definition 5.2 A soft multi topological space (U,E) is called soft multi generalized b-compact if every soft multi generalized b-open cover of (U,E) has a finite sub cover.

Definition 5.3 A soft multi set (F,A) in (U,E) is said to be soft multi generalized b-compact relative to (U,E) if for every collection $\{(F,A)_i : i \in I\}$ of soft multi generalized b-open sets of (U,E) such that $(F,A) \subseteq\cup_{i \in I}(F,A)_i$, there exists a finite subset I_0 of I such that $(F,A) \subseteq\cup_{i \in I_0}(F, A)_i$.

Definition 5.4 A soft multi set (F,A) of (U,E) is said to be soft multi generalized b-compact if (F,A) is soft multi generalized b-compact relative to (U,E).

Theorem 5.5 Let (U,E) be soft multi generalized b-compact and (F,A) be soft multi generalized b-closed set in (U,E). Then (F,A) is soft multi generalized b-compact space.

Proof: Let (F,A) be any soft multi generalized b-closed subset of (U,E) and $\{(G,A)_i : i \in I\}$ be a soft multi generalized b-open cover of (U,E). Since $(F,A)^c$ is soft multi generalized b-open, $\{(G,A)_i : i \in I\}\cup(F,A)^c$ is a soft multi generalized b-open cover of (U,E). For (U,E) is soft multi generalized b-compact space, there exists a finite subset $I_0 \subseteq I$ such that $(U,E) \subseteq\{(G,A)_i : i \in I\}\cup(F,A)^c$. But (F,A) and $(F,A)^c$ are disjoint, hence $(F,A)\cup\{(G,A)_i : i \in I_0\}$. Therefore (F,A) is soft multi generalized b-compact space in (U,E).

Theorem 5.6 A soft multi generalized b-continuous image of a soft multi generalized b-compact space is soft multi compact space.

Proof: Let f : (U,E) → (V,E) be a soft multi generalized b-continuous function from a soft multi generalized b-compact space $((U,E),\tau_1)$ to $((V,E),\tau_2)$. Take a soft multi open cover $\{(G,A)_i : i \in I\}$ of (V,E). For f is soft multi generalized b continuous, $\{f^{-1}((G,A)_i) : i \in I\}$ is a soft multi generalized b-open cover of (U,E) and for (U,E) is soft multi generalized b-compact, there exists a finite subset I_0 of I such that $\{f^{-1}((G,A)_i) : i \in I_0\}$ forms a soft multi generalized b-open cover of (U,E). Thus $\{(G,A)_i : i \in I_0\}$ forms a finite soft multi open cover of (V,E).

VI. Conclusion

In this paper, we introduce a new class of generalized soft multi open sets in soft multi generalized topological spaces as a generalization of compact spaces called soft multi b-compact spaces. We characterize soft multi b-compact space and study some of their basic properties.

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