Modeling of Abnormal Blood Flow through Arteries: Presence of Obstacle with Slip Condition and Magnetic Field

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Abstract: In this precise mathematical calculation we have got modeling of abnormal blood flow. This fluid model is used to get the axial velocity of blood with slip condition and magnetic field in the presence of obstacle i.e. stenosis. Abnormality results of Non-Newtonian condition of blood are displayed graphically for different flow properties with different parameters like axial velocity, fluid index, viscosity coefficient, Hartmann Number, magnetic field and volumetric flow rate etc. The significance of the present study may be useful to identify cardiovascular deceases.

IndexTerms- **Velocity, Fluid Index, Viscosity Coefficient, Magnetic Field**

I. Introduction

Under usual circumstances, blood movement in the man circulatory system relies upon the pumping act of the heart and influenced a pressure gradient over all the artery and vein network system. Pressure gradient is having two mechanisms, first of which is regular said to be non-fluctuating and the other is fluctuating known as pulsatile [1]. The main purpose of this study is to get knowledge of blood flow by arteries in a lot of heart diseases mainly in atherosclerosis. The regular thing of blood flow is uneasy due to a few odd developments like stenosis in the lumen of the human being's artery in pulse rate etc [2].

In the present time, the outcome of the magnetic area on the flow of viscous liquid in the process of a regular absorbent liquid has been the topic of frequent submissions. RBC is a main bio-magnetic matter, and the blood flow may be subjective by the magnetic field. In a broad manner, biological systems are concerned by the submission of an external magnetic field on blood flow, through human being arterial structure[3]. The occurrences of the magnetic field assist to augment in the resistance of moving blood. This analysis depends on a modest replica in which the convective acceleration terms in the Navier-Stokes solution are ignored.A finite element explanation for the Navier-Stokes solutions for steady movement through a paired branched in a two-dimensional section of three-dimensional replicas of the canine aorta is got by them[4].

The mathematical procedure involves the transformation of the physical organized to a curvilinear border fitted coordinate arrangement. The shear stress is considered for Reynolds digit of1000with the division to key aortic movement rate relation as a constraint [5]. The consequences are evaluated with a previous mechanism involving investigational information and it is studied that the consequences are very close to their explanation.Some scientist has done a study to examine the blood move in the large arteries in the company of a homogeneous magnetic area [6].

The blood pipes were understood to be cylindrical tubes of even cross-section along non-conducting walls. He states that "in the existence of a magnetic area the nature of the blood flow rate fluctuates and with steady expenditure, the pressure gradient augments with an increase in the field potency". Few mathematicians explained some significant hydrodynamic reasons; consist of pressure drop, turbulence,and separation. They have taken locally constricted tubes for their study [7]. They stated that both plastic models, having differed area fraction and magnitude of constriction. Data was achieved for Reynolds numbers fluctuating from range 100 to 5000. The area of separated fluid flow was examined and the critical Reynolds range for the transition to the turbulent movement was measured by hot-film probes [8].

Rheological actions of blood are supposed as Newtonian for values of a shear rate higher than 100 per second and such condition generally functioning in bigger arteries.Exceed gathering of cholesterols, fats and soft muscle cells in fluid arteries that's why partial occlusion said as stenosis [9].Thick and hard walls may reduce the quantity of the blood that movements through the blood arteries [10].The magnetic field is forced externally has constant strength and the electromagnetic field is very small.The stenosis develops symmetrically about the axis of the artery but not symmetric radial coordinates [11].

II. DEVELOPMENT OF THE MODEL

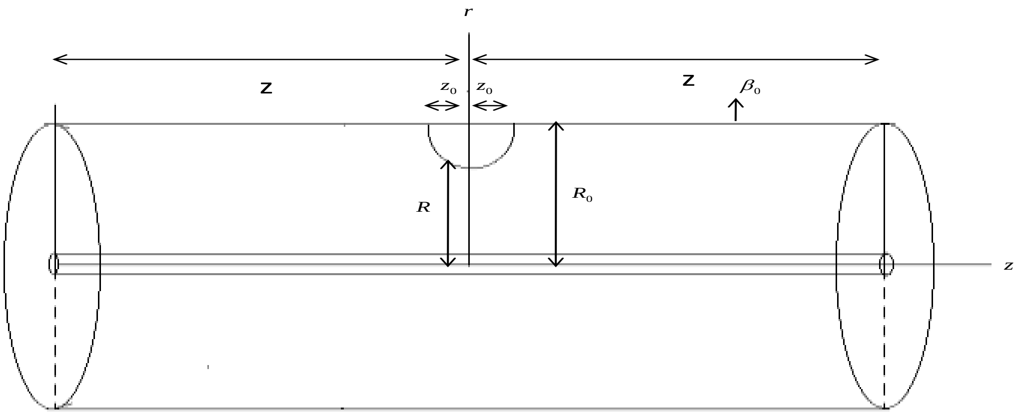


Figure 1: Geometry of Stenosis in an artery with a magnetic field

A steady laminar entirely developed one-dimensional movement of blood following the Herschel–Bulkley mathematical model through a diseased artery. The radius of the blood vessel depends upon stenosis and a mathematical geometry of bell-shaped stenosis is

$$R = R_0\left[1-\frac{\psi}{R_0}\mathrm{e}^{\frac{-m^2\alpha^2z^2}{R_0^2}}\right] \qquad (1)$$

Where R_0 is the radius of an artery, R(z) is the radius of an artery at stenosed segment , Z is the half magnitude of artery whereas (z_0) is the half magnitude of stenosed artery, n is fluid index, ψ is stenosed depth, m is a parametric constant, r and z are radial and axial coordinate respectively, α characterizes the concerned magnitude of the constriction, described as the ratio of radius to half magnitude of stenosis,

$$\alpha = \frac{R_0}{z_0} \qquad (2)$$

Assuming the stenosis precise geometry to be

$$\mathrm{R(z)}/R_0 = \left[1-\upsilon e^{-\beta z^2}\right] \qquad (3)$$

$$Where \quad \upsilon = \psi/R_0 \text{ AND } \beta = m^2\alpha^2/R_0^{\,2}$$

If blood is a power law fluid then the constitutive equation is

$$\tau = \mu e^n \qquad \Rightarrow \left(\frac{\tau}{\mu}\right)^{\frac{1}{n}} = e = \left(\frac{\frac{1}{2}\mathrm{Pr}}{\mu}\right)^{\frac{1}{n}} - Ha^2u = -\frac{du}{dr} \qquad (4)$$

$$\frac{du}{dr} = -\left(\frac{P}{2\mu}\right)^{1/n} r^{1/n} - Ha^2u \qquad (5)$$

$$u = -\left(\frac{\mathrm{P}}{2\mu}\right)^{\frac{1}{n}}\frac{n}{n+1}r^{\frac{1}{n}+1} - Ha^2ur + C \qquad (6)$$

Subject to the slip conditions

τ is finite at $r = 0$ (7)

And u = u_s at $r = r(z)$ (8)

III. ANALYTICAL SOLUTION OF THE PROBLEM

$$C = u_s + \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1} R^{\frac{1}{n}+1} + Ha^2 uR \quad (9)$$

From equation (6)

$$u = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left[R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right] + Ha^2 u\left[R - r\right] + u_s \quad (10)$$

For the slip condition

$$u = \frac{\left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left[R_0^{\frac{1}{n}+1} - R^{\frac{1}{n}+1}\right] + u_s}{\left[1 - Ha^2 (R_0 - R)\right]} \quad (11)$$

The fluid movements between two parallel non conducting plates positioned at $x = -a$ and $x = +a$, under the functioning of a constant pressure gradient $\left(\partial P/\partial z\right)$. These plates have an infinite breadth. An external transverse magnetic field is applied along the horizontal direction and an external electric field along the vertical direction.
Here Ha is the Hartmann number.

$$H_a^{\ 2} = \beta_0^{\ 2} R_0^{\ 2}\left(\frac{\sigma}{\mu}\right) \quad (12)$$

$$\beta_0 = \sqrt{\frac{H_a^{\ 2}}{R_0^{\ 2}\left(\frac{\sigma}{\mu}\right)}} \quad (13)$$

Flow rate

$$Q = \int u 2\pi R dR \Rightarrow 2\pi \int uRdR \quad (14)$$

$$Q = 2\pi \int \left[\left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left[R_0^{\frac{1}{n}+1} - R^{\frac{1}{n}+1}\right]\right] RdR + 2\pi \int \left[Ha^2 u\left[R_0 - R\right]\right] RdR + 2\pi \int u_s RdR \quad (15)$$

$$Q = 2\pi \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1} \int \left[R_0^{\frac{1}{n}+1} . R - R^{\frac{1}{n}+2}\right] dR + 2\pi Ha^2 u \int R_0 . RdR - 2\pi Ha^2 u \int R^2 dR + 2\pi u_s \int RdR \quad (16)$$

$$Q = 2\pi \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1}\left[R_0^{\frac{1}{n}+1} . \frac{R^2}{2} - \frac{R^{\frac{1}{n}+3}}{\frac{1}{n}+3}\right] + 2\pi Ha^2 u R_0 \frac{R^2}{2} - 2\pi Ha^2 u \frac{R^3}{3} + 2\pi u_s \frac{R^2}{2} \quad (17)$$

$$Q = 2\pi \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1} R^2 \left[\frac{1}{2} R_0^{\frac{1}{n}+1} - \frac{n}{3n+1} R^{\frac{1}{n}+1}\right] + \pi R^2 \left[\frac{1}{3} Ha^2 u (3R_0 - 2R) + u_s\right] \quad (18)$$

Or $$\left(\frac{P}{2\mu}\right)^{\frac{1}{n}} = \frac{Q-\pi R^2\left[\frac{1}{3}Ha^2u(3R_0-2R)+u_s\right]}{2\pi\frac{n}{n+1}R^2\left[\frac{1}{2}R_0^{\frac{1}{n}+1}-\frac{n}{3n+1}R^{\frac{1}{n}+1}\right]} \tag{19}$$

Or $$\left(\frac{P}{2\mu}\right) = \left[\frac{Q-\pi R^2\left[\frac{1}{3}Ha^2u(3R_0-2R)+u_s\right]}{2\pi\frac{n}{n+1}R^2\left[\frac{1}{2}R_0^{\frac{1}{n}+1}-\frac{n}{3n+1}R^{\frac{1}{n}+1}\right]}\right]^n \tag{20}$$

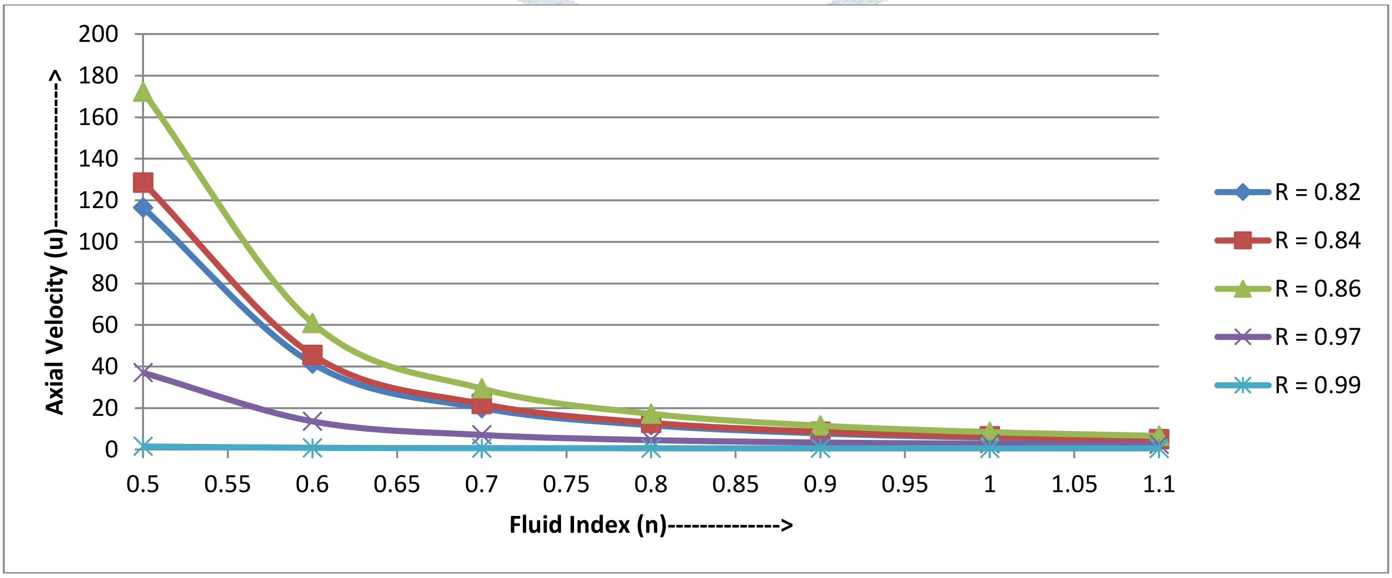
$$P = 2\mu\left[\frac{Q-\pi R^2\left[\frac{1}{3}Ha^2u(3R_0-2R)+u_s\right]}{2\pi\frac{n}{n+1}R^2\left[\frac{1}{2}R_0^{\frac{1}{n}+1}-\frac{n}{3n+1}R^{\frac{1}{n}+1}\right]}\right]^n \tag{21}$$

IV. RESULTS AND DISCUSSION

The objective of this study is to discuss the result of many parameters on the physiologically significant flow quantities like axial velocity, viscosity coefficient, fluid index, radius of artery, radius of stenosed artery, stenosis height, pressure drop, volumetric flow rate, magnetic field, Hartmann number, electrical conductivity, characteristic length scale etc.
The ranges of parameters are shown in the following figures and tables.

Table 1

R	Variation of axial velocity at different value of radius of stenosed artery as well as fluid index P = 0.15, u_s = 0.5, μ = 0.003						
	n						
	0.5	0.6	0.7	0.8	0.9	1	1.1
0.817214	116.41	41.4388	20.00505	11.70949	7.801613	5.692531	4.436216
0.84424	128.5125	45.6067	21.99829	12.88749	8.605072	6.297896	4.925536
0.860465	172.2414	61.02832	29.43479	17.26365	11.55037	8.475535	6.648113
0.97892	36.98072	13.59812	7.071822	4.589489	3.435474	2.819103	2.455028
0.998419	1.576279	0.888032	0.696497	0.623797	0.590053	0.572053	0.561431

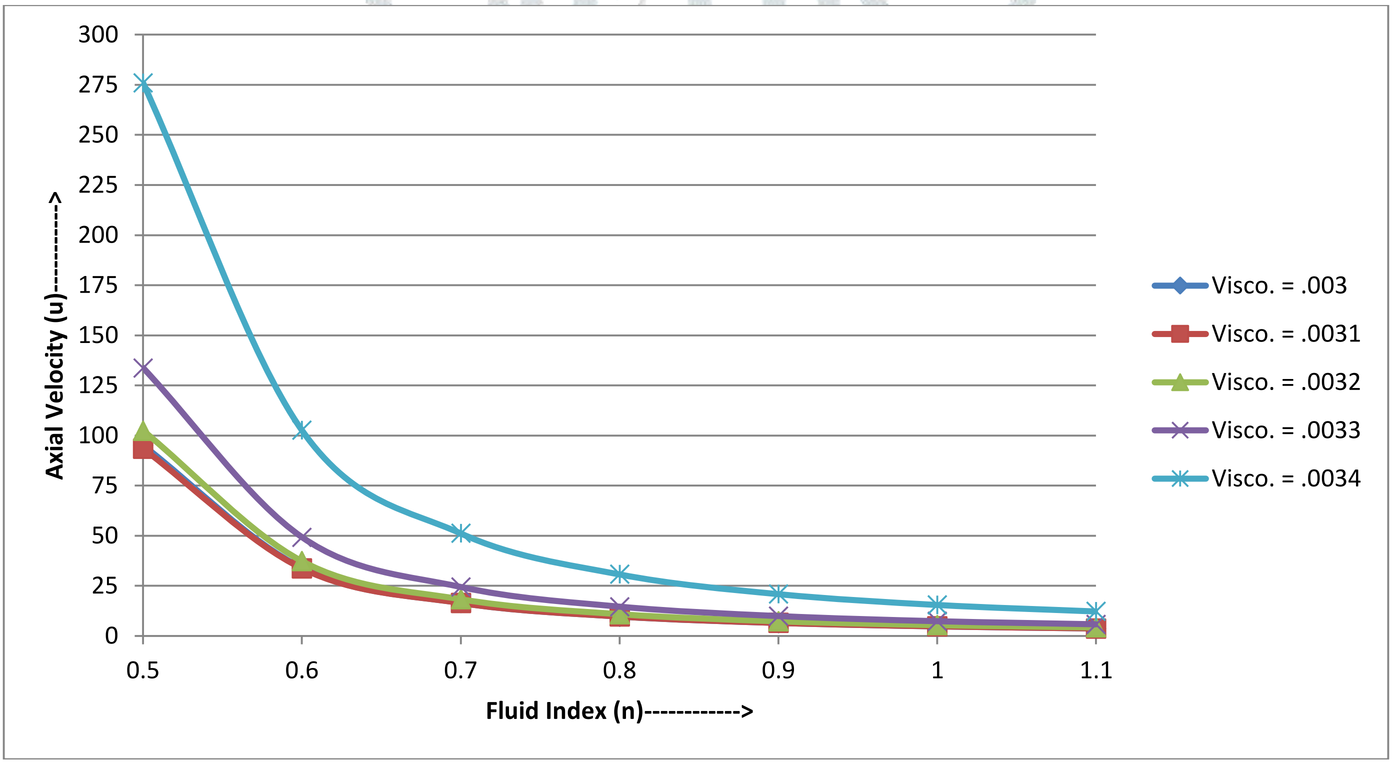


Graph 1: Variation of axial velocity at a different value of the radius of the stenosed artery as well as a fluid index P = 0.15, u_s = 0.5, μ = 0.003

The rate, or velocity, of blood movement, is inversely proportional with the cross-sectional region of the blood cylindrical vessels. As the sum of the cross-sectional region of the cylindrical vessels increases, the velocity of blood decreases. Blood movement is slowest in the capillaries. A lesser amount of invasive alternative is to compute a concern measure of blood movement acknowledged as the blood flow index (BFI), which is relied solely on the NIRS ICG curve, thus opposing the require for arterial cannulation. The dependency of axial velocity (u) on the radius of the stenosed artery (R_0) and fluid index (n) are shown in table (1) and graph (1). It is evident that the radius of the stenosed artery (R_0) and fluid index (n) are directly proportional but their connection is inversely proportional to result i.e. axial velocity (u). As these parameters are increased, axial velocity (u) is decreased.

Table 2

μ	Variation of axial velocity at different value of viscosity coefficient as well as fluid index P = 0.15, us = 0.5, R = 0.817						
	n						
	0.5	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**1.1**
0.003	95.13184	33.86436	16.34841	9.569158	6.375587	4.652016	3.625338
0.0031	93.39276	33.62656	16.37126	9.645729	6.460601	4.734427	3.702752
0.0032	102.3875	37.27522	18.29699	10.84937	7.304089	5.374987	4.21829
0.0033	133.6911	49.1972	24.34215	14.5237	9.826509	7.260627	5.717277
0.0034	275.8923	102.5916	51.15578	30.7067	20.87639	15.48615	12.23409



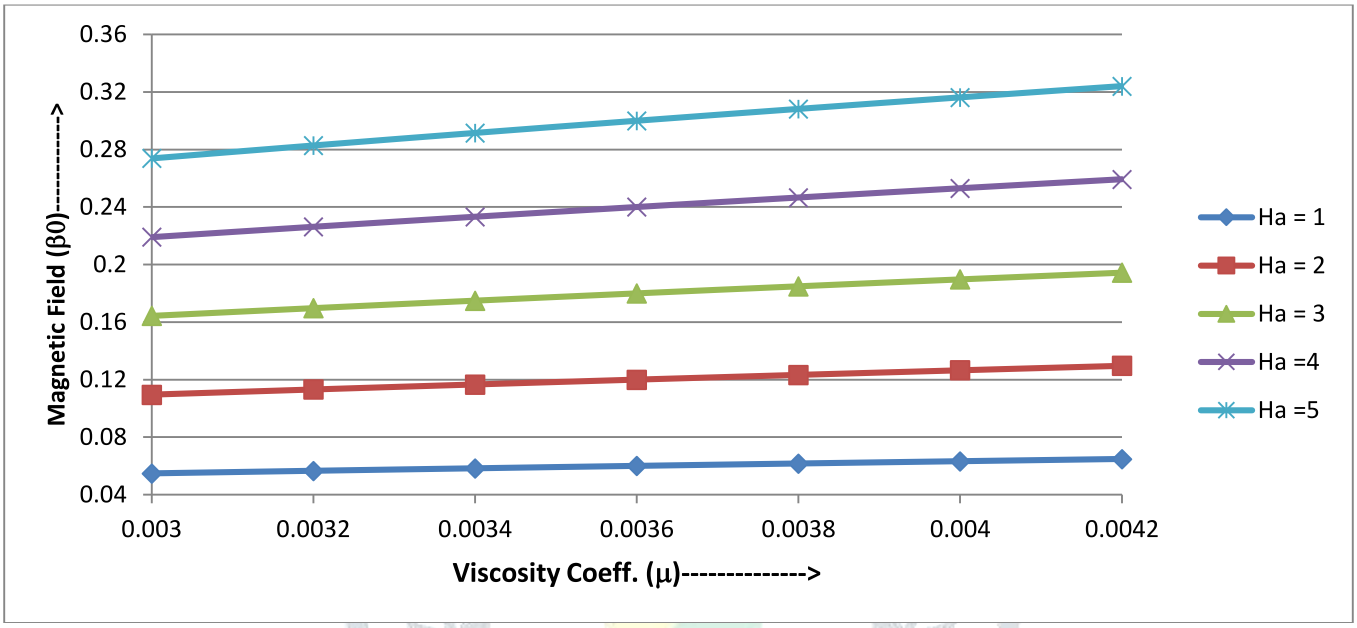
Graph 2: Variation of axial velocity at a different value of viscosity coefficient as well as a fluid index
P = 0.15, u_s = 0.5, R = 0.817

A careful look at table (2) and graph (2) reveals that viscosity coefficient (μ) and fluid index (n) has a similar connection but this mathematical connection is just reciprocal to required result i.e. axial velocity (u). When these parameters are increased, axial velocity (u) decreases. Above observation are very crucial for some big arteries due to the requirement of high axial velocity (u).

Here, the rate, or velocity, of blood movement is inversely proportional with the cross-sectional region of the blood cylindrical vessels. As the sum of the cross-sectional region of the cylindrical vessels increases, the velocity of blood decreases. Blood movement is slowest in the capillaries and blood viscosity is a calculation of the thickness as well as the stickiness of an individual particle of blood. It is a straight computation of the capability of blood to move through the cylindrical blood vessels. Augmented blood viscosity is a major independent predictor of cardiovascular activities. A lesser amount of invasive alternative is to compute a concern measure of blood movement acknowledged as the blood flow index (BFI), which is relied solely on the NIRS ICG curve, thus opposing the require for arterial cannulation.

Table 3

Ha	Variation of the magnetic field at a different value of Hartmann Number as well as Viscosity Coefficient $\sigma = 1, R_0 = 1$						
	μ						
	0.003	**0.0032**	**0.0034**	**0.0036**	**0.0038**	**0.004**	**0.0042**
1	0.054772	0.056569	0.05831	0.06	0.061644	0.063246	0.064807
2	0.109545	0.113137	0.116619	0.12	0.123288	0.126491	0.129615
3	0.164317	0.169706	0.174929	0.18	0.184932	0.189737	0.194422
4	0.219089	0.226274	0.233238	0.24	0.246577	0.252982	0.25923
5	0.273861	0.282843	0.291548	0.3	0.308221	0.316228	0.324037

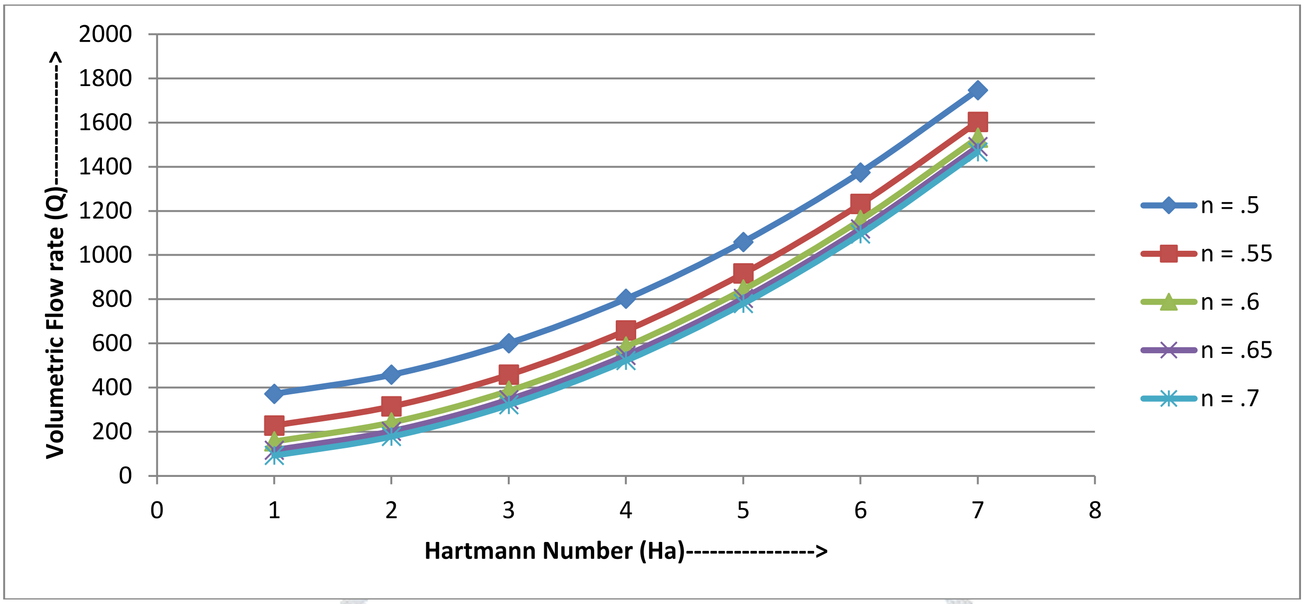


Graph 3: Variation of the magnetic field at a different value of Hartmann Number as well as Viscosity Coefficient $\sigma = 1, R_0 = 1$

The dependency of the magnetic field on Hartmann number (Ha) and viscosity coefficient (μ) is displayed in table (3) and graph (3) respectively. This observation states that the precise values of the Hartmann number (Ha) and viscosity coefficient (μ) affect the magnetic field. Table (3) and graph (3) reveals that if Hartmann number (Ha) and Viscosity coefficient (μ) increases, consequently magnetic field (β_0) are also increased. So, Magnetic field (β_0) is directly proportional to the Hartmann number (Ha) and viscosity coefficient (μ).

Table 4

n	Variation of volumetric flow rate at different value of Hartmann Number as well as fluid index $P = 0.15, u = 30, \mu = 0.003, R_0 = 1, R = 0.817$						
	Ha						
	1	**2**	**3**	**4**	**5**	**6**	**7**
0.5	371.3969	457.4841	600.8231	801.4137	1059.2558	1374.3496	1746.6950
0.55	227.9516	314.0389	457.3778	657.9684	915.8105	1230.9043	1603.2497
0.6	155.7899	241.8771	385.2160	585.8066	843.6488	1158.7426	1531.0880
0.65	115.7481	201.8353	345.1742	545.7648	803.6069	1118.7008	1491.0462
0.7	91.756	177.8429	321.1819	521.7724	779.6146	1094.7084	1467.0538

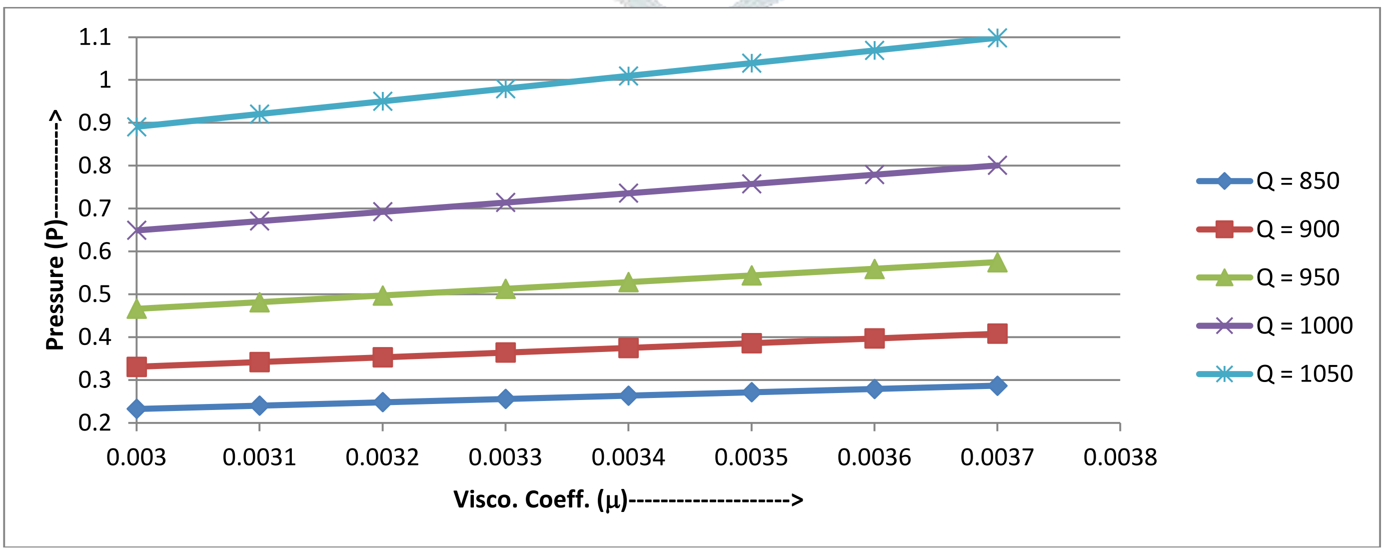


Graph 4: Variation of the volumetric flow rate at a different value of Hartmann number as well as a fluid index
P = 0.15, u = 30, μ = 0.003, R_0 = 1, R = 0.817

The dependency of volumetric flow rate (Q) on Hartmann number (Ha) and fluid index (n) is displayed and depicted in table (4) and graph (4) respectively. This observation states that the precise values of the Hartmann number (Ha) and fluid index (n) affect the volumetric blood flow rate (Q). This result reveals that if Hartmann number (Ha) and fluid index (n) increases consequently magnetic field (β_0) are also increased. So, Magnetic field (β_0) is directly proportional to the Hartmann number (Ha) and fluid index (n).

Table 5

Q	Variation of pressure drop (P) at different viscosity coefficient (μ) as well as volumetric flow rate (Q) u = 30, R_0 = 1, R = 0.817							
	μ							
	0.003	**0.0031**	**0.0032**	**0.0033**	**0.0034**	**0.0035**	**0.0036**	**0.0037**
850	0.2324	0.2402	0.2479	0.2557	0.2634	0.2711	0.2789	0.2866
900	0.3306	0.3417	0.3527	0.3637	0.3747	0.3858	0.3968	0.4078
950	0.4660	0.4816	0.4971	0.5126	0.5282	0.5437	0.5592	0.5748
1000	0.6491	0.6707	0.6924	0.7140	0.7357	0.7573	0.7789	0.8006
1050	0.8906	0.9203	0.9500	0.9797	1.0094	1.0391	1.0688	1.0985



Graph 5: Variation of pressure drop (P) at different viscosity coefficient (μ) as well as volumetric flow rate (Q)
u = 30, R_0 = 1, R = 0.817

The dependency of pressure drop (P) on different viscosity coefficient (μ), as well as the volumetric blood flow rate (Q), is displayed and depicted in table (5) and graph (5) respectively. This observation states that the precise values of different viscosity coefficient (μ), as well as volumetric flow rate (Q), affect to pressure drop (P). This study reveals that if different viscosity coefficient (μ), as well as volumetric flow rate (Q), increases consequently pressure drop (P) is also increased. So, pressure drop (P) is directly proportional to different viscosity coefficient (μ) as well as the volumetric flow rate (Q).

V. CONCLUSION

This study reveals the effect of various parameters in the stenosed artery. This investigation is simply focused to get precise calculations of abnormal blood flow in the presence of slip condition and magnetic field. Further studymay be helpful to prevent stenosis. It may be helpful for cardiovascular diseases.

VI. ACKNOWLEDGMENT

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