CASSON FLUID FLOW: FREE CONVECTIVE HEAT TRANSFER OVER TRUNCATED CONE WITH CONVECTIVE HEATING

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Abstract : This paper purely examines the variable convective heating effect on MHD Casson viscoplastic flow past a truncated vertical cone. The governing partial differential equations of the viscoplastic flow field are transformed in terms of a system of non-linear coupled non-similarity ordinary differential equations. Employing finite difference technique followed by Keller Box method, the system is solved numerically. The different impacts of the several physical parameters are reported in the flow field on the velocity and temperature profiles are calculated by computer and represented graphically. A contrast with published facts has been finished, and appropriate agreements are observed. The study is highly relevant to petro-chemical engineering (polymer based),nd the corresponding processing systems.

***IndexTerms*: Lorentz force; Convective boundary condition; Keller-box numerical method; Casson Viscoplastic model.**

INTRODUCTION

Magnetic field effects on a conducting fluid received good attention from researchers, because in view of industrial aspects Hydromagnetic flow and Heat transmissions are highly essential concepts. For an instance several metallurgical methods like drawing, tinning of copper wires and annealing processes involve uninterrupted cooling of strips and filaments by drawing these materials through a fluid which is in quiescent state. The characteristics of final product will be affected when the rate of cooling is controlled during these processes. So this can be resolved with the usage of an electrically conducted fluid with the application of magnetic field. Mahapatra et al. [1] explained about the natural convection on a horizontal regular plate with the impact of magnetic field. Rushi kumar et al. [2] discussed about the effects of diffusion and radiation on a plate in the field of magnetism. Although an extensive study was done for boundary layer flow in the field of thermal radiation, the thermal conductivity of the fluid in such instance is considered as constant. These processes include drilling of petroleum muds [3], biological gels [4] and polymer processing [5]. Amanulla et al. [6] described a wide number of numerical solutions for Magneto-hydrodynamic and non-Newtonian fluid flow through a vertical cone with the presence of slip effects, using finite difference method. Cheng [7] investigated heat transfer of power law nanofluid passed above a truncated cone embedded in a porous medium. Elbashbeshy et al. [8] analyzed radiation effects non-Newtonian fluid flow past a vertical truncated cone. Amanulla et al. [9] discussed the velocity slip and temperature jump conditions on hydrodynamic fluid flow over a circular cylinder. Reddy and Pradeepa [10] studied viscous dissipation and sorret effects on natural convection flow over a truncated cone in the presence of Biot number. Several authors [11-13] investigated heat transfer of non-Newtonian fluid of various physical problems.

The present work, we inspect theoretically and also computationally the steady-state transport things in magneto-hydrodynamic non-Newtonian fluid flow from a truncated cone by means of Convective heating effects. It has been found that the Magnetic field is extremely influence the characteristics of heat transfer and velocity in the flows over curved body. Relevant examples are expressed by Bég et al. [14] (used for cylindrical geometries), Alkasasbeh et al. [15] who addressed radiative effects, and also considered the drag effects of porous medium and Kasim et al. [16] who utilized a prominent viscoelastic model.

MATHEMATICAL VISCOPLASTIC FLOW MODEL

Consider the steady, viscous, two-dimensional, incompressible, free convection flow from a non-isothermal permeable vertical cone embedded in a Casson non-Newtonian fluid. Figure 1(a) depicts the flow model and physical coordinate system.

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right)e_{ij}, \pi \geq \pi_c \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, \pi < \pi_c \end{cases} \quad (1)$$

In which $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i, j)th component of deformation rate, represents the product of the product base deformation rate by means of itself, πc represents a critical value of this product on the basis of non-Newtonian model, μ_B denotes plastic dynamic viscosity of the non-Newtonian fluid and p_y represents yield stress of the corresponding fluid.

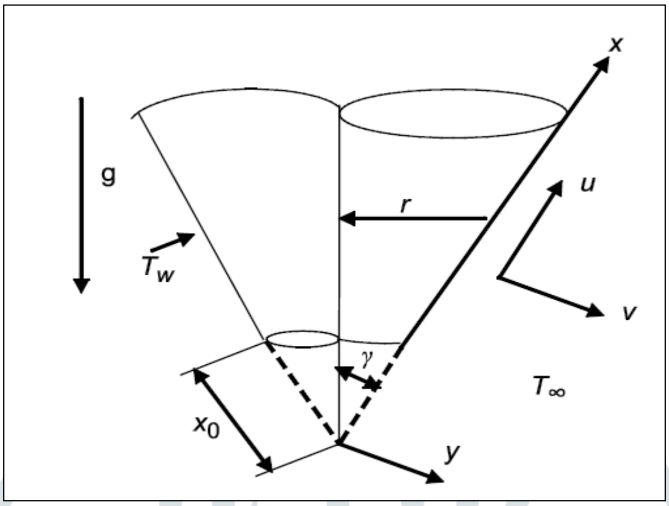


Fig. 1. Schematic diagram of the problem

The equations for mass continuity, momentum and energy, can be represented as follows:

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + g\Lambda(T-T_\infty)\cos\gamma - \frac{\sigma B_0^{\,2}}{\rho}u \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions for the considered flow with conductive heating:

At $y=0, u=0,\ v=0,\ -k\dfrac{\partial T}{\partial y} = h_w(T_w - T)$

As $y\to\infty,\ u\to 0,\ v\to 0,\ T\to T_\infty$ (5)

The stream function ψ is defined by $ru = \partial\psi/\partial y$ and $rv = -\partial\psi/\partial x$, and therefore, the equation of continuity is accordingly satisfied. In order to express the governing equations with the concern boundary conditions in non-dimension mode, the following dimensionless quantities are defined:

$$\xi = \frac{\bar{x}}{x_0} = \frac{x-x_0}{x_0},\ \eta = \frac{y}{\bar{x}}(Gr_x)^{1/4},\ \psi = r\nu(Gr_x)^{1/4} f(\xi,\eta),\ \theta(\xi,\eta) = \frac{T-T_\infty}{T_w-T_\infty},\ Gr_x = \frac{g\beta_T\cos\gamma(T_w-T_\infty)\bar{x}^3}{\nu^2} \tag{6}$$

The transformed boundary layer equations for momentum and energy emerge as:

$$\left(1+\frac{1}{\beta}\right)f''' + \left(\frac{3}{4}+\frac{\xi}{1+\xi}\right)ff'' - f'^2 + \theta - Mf' = \xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right) \tag{7}$$

$$\frac{\theta''}{\Pr} + \left(\frac{3}{4}+\frac{\xi}{1+\xi}\right)f\theta' = \xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right) \tag{8}$$

Along the transformed concerned boundary conditions (4) are

At $\eta=0,\ f=0,\ f'=1, \theta' = -\xi^{1/4}\gamma(1-\theta(0))$

As $\eta\to\infty,\ f'\to 0,\ \theta\to 0$ (9)

The skin-friction coefficient and Nusselt number (rate of heat transfer) can be determined by using the transformations defined earlier by means of the following expressions:

$$\frac{1}{2}Gr_x^{1/4}C_f = \left(1+\frac{1}{\beta}\right)f''(0) \quad (10)$$

$$Gr_x^{-1/4}Nu = -\theta'(0) \quad (11)$$

COMPUTATIONAL FINITE DIFFERENCE SOLUTIONS

In this study, the efficient implicit (finite difference) Keller-Box method has been employed to solve the general flow model defined by equations (6) – (7) with boundary conditions (8). This method was actually developed in view of aerodynamic boundary layers of low speed by Keller [17]. This method has been used widely and efficiently for over three decades in a large spectrum of nonlinear fluid mechanics problems. These include laminar transport phenomena (2017) and Viscoplastic boundary layer flows (2017).

RESULTS AND DISCUSSIONS

Comprehensive solutions have been found with Keller Box Method and are represented in Figs. 2 to 9. The numerical problem consists of three dependent thermo-fluid dynamic variables (f,θ) and five multi-physical control parameters, *We, M, Pr and* γ . The influence of stream wise space variable ξ is also investigated. For accuracy, our results are compared with Yih [20] and Reddy and Pradeepa [10] and we observed that a very good refinement with previous results as shown in Table1.

Figure 2-3 illustrates the variations of velocity & temperature profiles for numerous values of Casson parameter (β). From figure 2, we tend to determined that the velocity profile decreases once an increasing β. An uplifting the values of β, leads to an increase in dynamic viscosity that produces resistance within the flow of fluid and a decrease in fluid velocity is determined. Figure 3 presents that the larger values of Casson parameter produce an enhancement within the temperature profile and associated boundary layer thickness.

Table 1: Comparison analysis of $-\theta'(\xi,0)$, for distinct values of Pr when $M=0$ and $(\beta, Bi)\to\infty$.

Pr	$-\theta'(\xi,0)$		
	Yih [20]	Reddy and Pradeepa [10]	Present
0.01	0.057	0.05954	0.05965
0.1	0.1629	0.16273	0.16281
1	0.4012	0.40103	0.40112
10	0.8266	0.82684	0.82692
100	1.5493	1.54953	1.54963

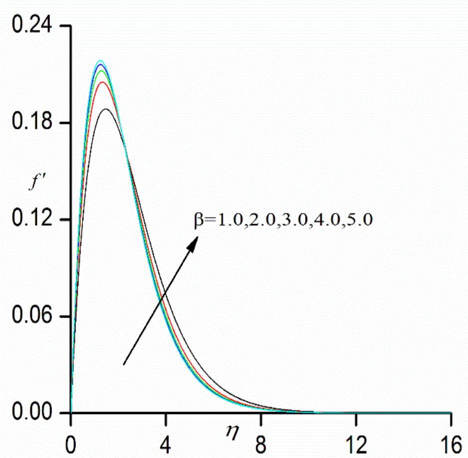


Fig.2 Effect of β on velocity profiles.

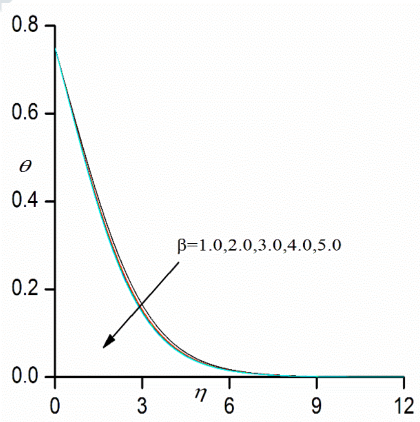


Fig.3 Effect of β on temperature profiles

In figure 4, it's determined that the velocity profile decelerates with the rise of the magnetic parameter values, as a results of the existence of a magnetic field in an electrically conducting fluid generates a force referred to as the Lorentz force, that acts opposite to the flow when the magnetic field is operated in an usual direction, as within the gift study. This resistive force decelerates the fluid velocity. The temperature profile will increase by increasing the values of magnetic parameter(M) as shown in figure 5. Figure 6-7 illustrate the deviation of velocity & temperature with transverse coordinate (η), for distinct values of Convective heating (γ). Convective heating is imposed in the augmented wall boundary condition in eqn. (9). With increasing Convective heating, more heat is transmitted to the fluid and this energizes the boundary layer. This also leads to a general acceleration as observed in fig.5a and also to a more pronounced restoration in temperatures in fig. 7, in particular near the wall. The effect of Biot number is progressively reduced with further distance from the wall (cone surface) into the boundary layer and vanishes some distance before the free stream.

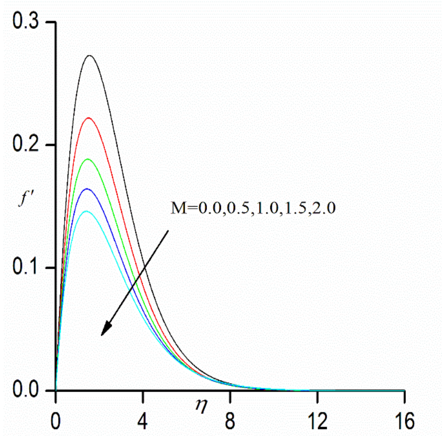


Fig.4 Effect of M on velocity profiles

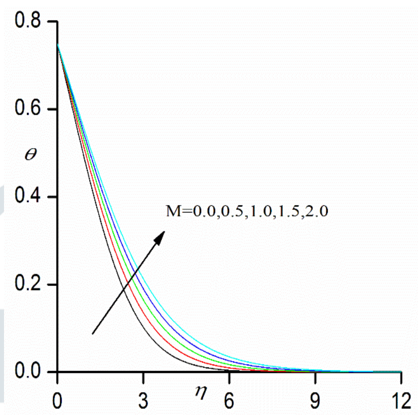


Fig.5 Effect of *M* on temperature profiles

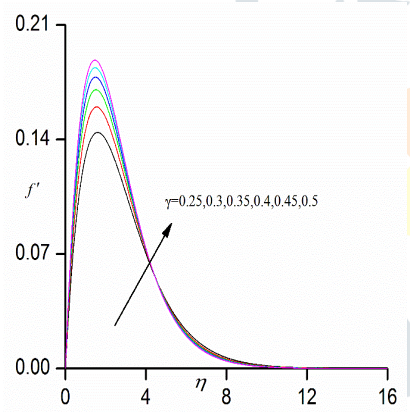


Fig.6 Effect of γ on velocity profiles.

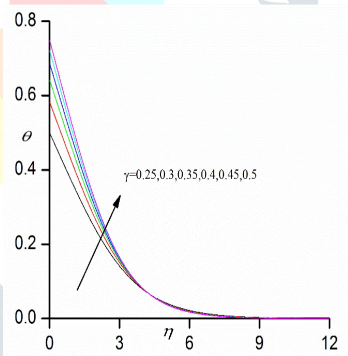
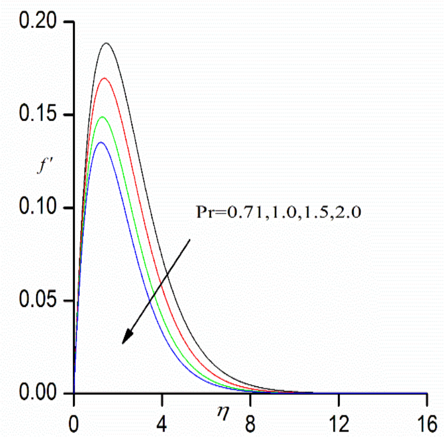


Fig.7 Effect of γ on temperature profiles

Figure 8-9 represents the variation of the velocity & temperature profile for several values of Prandtl number (*Pr*). Fig. 8 shows that with rising Prandtl number there is a strong deceleration in the flow. The Prandtl number expresses the fraction of momentum diffusion rate and thermal diffusion rate. Fig. 9 shows the deviation of the temperature profile for numerous values of Prandtl number*(Pr)*. The result shows that an increases of Prandtl number leads to the reducing of thermal boundary layer thickness. The reason is that smaller values of *Pr* are similar to increasing the thermal conductivities, and so, heat is ready to differ far away from the heated surface quicker than for greater values of *Pr*. Hence, the boundary layer is thickness raised and also there is a reduction of rate of heat transfer, for gradient are reduced.



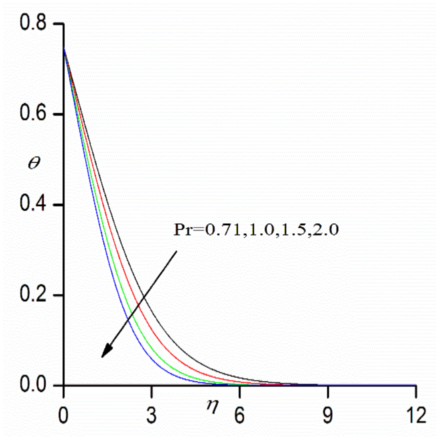


Fig.8 Effect of *Pr* on temperature profiles Fig.9 Effect of *Pr* on temperature profiles

CONCLUSIONS

A new mathematical model has been generated for the convection free boundary layer flow of a Viscoplastic fluid through a vertical truncated cone with convective boundary conditions. The Casson non- Newtonian model have been used to simulate different rheological characteristics. The transformed boundary layer conservation equations are solved with pre-determined boundary conditions by means of the finite difference implicit Keller–box method which has second order accuracy. The most important results in summary are:

1. Magnetic field (M) and Casson parameter (β) increase the velocity profile whereas reducing the temperature & concentration profiles;
2. Increasing Prandtl number (Pr) slowdowns the flow and also strongly weakens temperatures, for the entire boundary layer regime.
3. Increasing Convective heating (γ), increases velocity & temperature for each value of radial coordinate that is for the entire boundary layer regime.

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