On left bipotent Γ semi near ring

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Abstract

The concept Γ - near ring was introduced by Bhavanarisathyanarayana and studied by several authors like Yong ukcho, R.Balakrishnan with P(r,m) near rings.It was later extended to semi near ring by R.Perumal and P.Chinnaraj.Also, the left bipotent semi near ring was introduced by R.Balakrishnan and R.Perumal. In this paper, we have defined a structure called Left Bipotent Γsemi near ring, M. It is proved that a left bipotent Γ semi near has no non zero nilpotent elements; M is left bipotent if and only if every radical is left ideal. Further it is proved that in a left bipotent Γ semi near ring with identity $x^3=x^2$, every element is idempotent; A left bipotent Γsemi near ring is simple whenever it is right cancellative.

Introduction:

An algebraic structure (N,+,.) is said to be a seminear-ring if i)(N,+) is a semigroup ii)(N,.) is a semigroup.iii)(a+b)c=ac+bc for all a,b,c∈N.Let M be an additive semigroup and Γ a nonempty set. Then M is called a right Γ-seminear-ring if there exists a mapping M× Γ×M→M satisfying the following conditions: i)(a+b)γc=aγc+bγc ii)(aγb)βc=aγ(bβc) for all a,b,c∈M and $\gamma, \beta \in \Gamma$.Let M be a Γ-seminear-ring under the mapping f:M× Γ ×M→M. A right Γ-seminear-ring M is said to have an absorbing zero '0' if i)a+0=0+a=a ii)aγ0=0γa=0,hold for all a∈M and γ∈ Γ.$M_0 = \{m$ ∈M/mγ0=0 γ for all γ∈ Γ} is called the zero-symmetric part of M.A Γ-seminear-ring M is called zero-symmetric, if M= M_0 .A Γ-near ring is said to satisfy left bipotency if N Γ a=N Γ a^2 for each a in N.

Keywords: Left normal Γ semi near ring, Mate function, Idempotent, Nilpotent, Simple and left annihilator.

Notations:

We furnish below the notations that we make use of throughout this paper.1.E= {e∈R/e^2 =e}- set of all idempotents of R.

2. L={x∈R/x^k =0 for some positive integer k} - set of all nilpotent elements of R.

3. C(R) = {r ∈R /rx =xr for all x ∈R}- centre of R.

2 Preliminaries

Result 1:[3]A semi near ring R has no non zero nilpotent elements if and only if x^2 =0⇒x = 0 for all x in R.

Result 2:[16]A map 'f ' from M into M is called a mate function for if x = x γ f(x) γ x for all x in M and γ ∈ Γ (Here f(x) is called a mate of x).

Result 4:[20]A Γ semi near ring R is called a P(1,2) Γ semi near ring if $x\gamma M = M\gamma x^2$ for all x in R.

Result 5:[2]A Γ semi near ring M is called left (right) normal $a\in M\gamma a$ ($a\gamma M$) for each $a\in M$ and $\gamma\in\Gamma$.M is normal if it is both left and right normal.

Definition:[15]For any subset A of M we define the radical of A as$\sqrt{A}=\{x \in M \in x^k\in A$ for some positive integer k}.

3Left Bipotent Γ semi near ring

In this section we define the concept of left bipotent Γ semi near ring and using the above preliminary results,we prove the following main theorems

.**Definition 3.1**

We say that the Γ seminear ring M is called *left bipotent* if $M\gamma a = M\gamma a^2$for all$a \in M$.

Definition 3.2

If *S* is any nonempty subset of *R*, then the *left annihilator* of *S* in *R* is$l(S) = \{x \in R/xs= 0$ for all $s \in S\}$. (We observe that $l(S)$ is a left ideal of R).

Theorem 3.3

Let M be a left normal left bipotent Γ semi near-ring. Then M has no non zero nilpotent elements.

Proof:

Let $x \in M - \{0\}$. Since M is left normal, $x \in M\alpha x$ $=M\alpha x^2$. Clearly then $x = r\alpha x^2$ for some r in R. Thus $x^2 = 0 \Rightarrow x = 0$.Now result(1) guarantees that $L = \{0\}$.Hence M has no non-zero nilpotent elements.

Theorem 3.4

Let M be a left normal Γ seminear-ring. Then M is left bipotent if and only if $A =\sqrt{A}$ for every left ideal of M.

Proof: For the only if part, let $x \in\sqrt{A}$. Then there exists some positive integer k such that $x^k\in A$.Since M is a left normal left bipotent Γsemi near-ring we have, $x\in R\alpha x = R\alpha x^2 \Rightarrow x = y\alpha x^2$ for some $y \in M$. Then $x=y\alpha x x =y(y\alpha x^2)x =y^2\ \alpha x^3=$ . . . $= y^{k-1}\ \alpha x^k\in MA\subseteq A$.i.e.$x\in A$. Therefore$\sqrt{A}\subseteq A$....(1). But it is obvious that $A \subseteq\sqrt{A}$.......(2).From (1) and (2) we get $A =\sqrt{A}$.For the if part, we observe that Ma^2 is a left ideal of Mα.If $a \in M\alpha$, $a^3 = aa^2\in M\ \alpha a^2$. This implies $a \in\sqrt{M\alpha a^2} = M\alpha a^2$.Hence for any $x \in M\alpha$, $xa=x(ya^2)\in M\alpha a^2$forsome $y \in M\alpha$.Therefore $M\alpha a \subseteq M\alpha a^2$.Consequently M is left bipotent.

Theorem 3.5

In a left normal left bipotent Γ seminear-ring with identity $x^3=x^2$,every element is idempotent.

Proof:

Suppose $x^3 = x^2$ for x in M. Since M is left normal left bipotent Γ semi near ring,$x \in M\alpha x = M\alpha x^2$. Hence $x = y\alpha x^2$ for some y in M, so that $x^2 = y\alpha x^3 =y\alpha x^2 = x$.Therefore it is an idempotent.

Theorem 3.6

Let M be a left bipotent Γ seminear-ring. If M is right cancellative then M is simple.

Proof:Let *A* be a non-zero left ideal of *M*. If *'a'* is any non-zero element of *A* then $M\alpha a = M\alpha a^2$. If $r \in M$ then $r\alpha a = s\alpha a^2$ for some s in M. Therefore $r\alpha = s\alpha a$ (as M is right cancellative). Clearly then $r \in A$ and so $A = M$. Thus M is simple.

Theorem 3.7

Let M be a left normal P(1,1) Γseminear-ring. Then M isleft bipotent if and only if M is P_1 ' Γseminear-ring.

Proof:

Let M be a left bipotent $P(1,1)$ Γ semi near-ring. Since M is left normal, $a \in M\alpha a = M\alpha a^2 = (M\alpha a)a = a\alpha Ma$.Therefore$M\alpha a \subseteq a\alpha Ma$.Clearly $a\alpha Ma \subseteq M\alpha a$.Thus $M\alpha a = a\alpha Ma$ for every $a \in M$. Therefore M is a P_1' Γ seminear-ring.Conversely M be a P_1 ' Γ seminear-ring.Then $M\alpha a = a\alpha Ma = (a\alpha M)a = M\alpha a^2$.Consequently M is left bipotent.

Theorem 3.8

If M has no non-zero nilpotent elements, l(S) is an ideal for every non empty subset S of M.

Proof: It is sufficient to show that $l(S)$ Γ $R \subseteq l(S)$. Let $x \in l(S)$ and $s \in S$, $\alpha \in \Gamma$.Then $x\alpha s = 0$ implies $(s\alpha x)^2 = s\alpha(xs)\ \alpha x = 0$. Hence $s\alpha x = 0$ by assumption. For any $r \in R$, $((x\ \alpha\ r)\ \alpha\ s)^2 = x\ \alpha\ r(s\ \alpha\ x)r\ \alpha\ s = 0$ implies $(x\ \alpha\ r)\ \alpha\ s = 0$. Thus $x\ \alpha\ r \in l(S)$.

Hence $xr \in l(S)$. Therefore$l(S)\ \Gamma R \subseteq l(S)$.

Theorem 3.9

In a left normal left bipotent Γ seminearing M, left annihilators are ideals.

Proof: In view of theorem 3.3, M has no non-zero nilpotent elements. Now theorem 3.8 guarantees that left annihilators are ideals.

Theorem 3.10

Let M be a left normal left bipotent Γ semi near-ring. If M is prime then M has no non-zero zero divisors.

Proof:

Let $a\gamma b = 0$ for a, b in M,γ in Γ . Then for any *'x'* in M, $(b\gamma x\gamma a)^2 = (b\gamma x\gamma a)(b\gamma x\gamma a) = b\gamma x\gamma(ab)\gamma x\gamma a = 0$.Now theorem 3.3 demands that $b\gamma x\gamma a = 0$ and this implies$Mb\gamma Ma = \{0\}$.

Since $M$is prime and $M\gamma b$, $M\gamma a$are left ideals we have that,$M\gamma b = \{0\}$ or $M\gamma a = \{0\}$.

Since M is left normal, we get either $b = 0$or $a = 0$.

Bibliography

[1] Bhavanari Sathyanarayana, A talk on gamma near rings, Proceedings of the National Seminar on ALGEBRA AND ITS APPLICATION, JMJ College, Tenali, A.P.,(July 11-12,2011) (Editors: Bhavanari Satyanarayana and A.V. Vijaya Kumari) PP: 01-16.

[2]D.Radha and C. Raja Lakshmi, *A study on semicentral Seminear - rings*, Proceedings; National Seminar on New Dimensions in Mathematics and its Applications, October 17, 2018, ISBN No:(yet to receive).

[3]D.Radha and C. Raja Lakshmi, *On Weakly π - Subcommutative Γ - Near Rings*, Proceedings on National Conference on Innovations in Mathematics (NCIM - 2018), Feburary 2018, ISBN : 978-81-935198-5-1, Page (10-18).

[4]D.Radha and C.Raja Lakshmi, *On Zero - Symmetric Semicentral Γ - Near Rings*, International Journal of Science, Engineering and Management (IJSEM) Vol 3, Issue 4, April 2018, ISSN (Online) 2456-1304.

[5]D.Radha and C.Raja Lakshmi, *On Zero - Symmetric Semicentral Γ - Near Rings*, International Journal of Science, Engineering and

Management (IJSEM) Vol 3, Issue 4, April 2018, ISSN (Online) 2456-1304.

[6]D.Radha and M.Kavitha, *Pseudo Symmetric Ternary Γ - Semiring*, Proceedings on National Conference on Innovations in Mathematics (NCIM - 2018), Feburary 2018, ISBN : 978-81-935198-5-1, Page (19-24).

[7] D.Radha and M.Parvathi Banu, Left Singularity and Left Regularity in Near Idempotent Γ - Semigroup , International Journal of Science, Engineering and Management (IJSEM) Vol 3, Issue 4, April 2018, ISSN (Online) 2456-1304.

[8]D.Radha and P.Meenakshi, *Some Structures of Idempotent Commutative Semigroup*, International Journal of Science, Engineering and Management (IJSEM) Vol 2, Issue 12, December 2017, ISSN (Online) 2456-1304.

[9]D.Radha and S.Suguna, *Normality in Idempotent Commutative Γ - Semigroup*, International Journal of Science, Engineering and Management (IJSEM) Vol 3, Issue 4, April 2018, ISSN (Online) 2456-1304.

[10]D.Radha and V.Selvi, *Stable and Pseudo Stable Gamma Near Rings*, Proceedings on National Conference on Recent Trends in Pure and Applied Mathematics, September 2017, ISBN: 978-81-935198-1-3.

[11]J.L.Jat and S.C.Choudhary , *On Left Bipotent Near rings* ,Proceedings of the Edinburgh Mathematical society (1979), 22,99-107.

[12]R.Balakrishnan and S.Suryanarayanan, P(r,m) Near-Rings ,Bull. Malasiyan Math.Sc. Soc (Second Series) 23(2000) 117-130

[13] R.Balakrishnan and R.Perumal ,Left Bipotent Semi near ring , International Journal of Algebra, Vol. 6, 2012, No. 26, 1289-1295.

[14]Park.Y.Sand Kim. W. J, *On structures of Left Bipotent Near Rings*, Kyungbook Math. J,20(2),(1980), 177-181.

[15]Pilz Gunter.,Near Rings,North-Holland, Amsterdam, second edition, (1983).

[16] R.Perumal and P.Chinnaraj, *Ideals in P(r,m) gamma semi near rings*, International Journal of Pure and Applied Mathematics Volume 106 No. 2 2016,533-541.

[17]Van Hoorn. W.G. and Van Rootselar.B., *Fundamental notions in the theory of semi near rings.*, composito mathematic., 18 (1967),65-78.

[18]Volety V.S. Ramachandram, *Commutativity of Semi near rings* ,Journal of Science and Arts Year 11,No. 4(17),pp.367-368,2011.

[19]V.R. Chandran, On right bipotent rings, *Abstract in Notices* Amer. Math.Soc. Nov. (1970).

[20]Yong Uk Cho, T.Tamizh Chelvam and N. Meenakumari *P(r,m) Γ-NearRings*, J. korea Soc. Math. Educ Ser B. Pure Appl. Math. Vol.13, No.2, (2006)PP 113-120.

[21]Yong Uk Cho; *Some Results on Γ near rings*, Journal of the Chungcheong Mathematical Society volume 19, No.3, September 2006.